Implementing Photonic Crystals in Transformation Media as a Step to Optical Invisibility

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Abstract – **Developing transformation media for optical range cannot be based on lossy metamaterials. We propose, instead, all-dielectric photonic crystals capable to support superluminal wave propagation and provide prescribed spatial dispersions of refractive index components. Demonstrated below employing these materials in cylindrical invisibility cloaks confirms their applicability.**

Keywords—anisotropic photonic crystals, spatial dispersion of indices, superluminal wave propagation

I. Introduction

The concepts of Transformation Optics (TO) [1] have opened up perspectives for the development of new photonic devices, including invisibility cloaks. Realizing such devices required transformation media with specific properties, such as, in particular, an ability to transmit waves with superluminal phase velocities. First media with desirable properties were developed for microwave range by using resonance metamaterials (MMs), however, scaling MM-based invisibility cloaks to higher frequencies met problems because of inherent for MMs losses. In this report we continue earlier launched investigations of the possibility to replace MMs in transformation media by photonic crystals, which are lossless in optical range [2].

Perspectives of employing photonic crystals (PhCs) in transformation media were first mentioned in [3], while our studies in [2] have shown that these perspectives could become real at using 2D PhCs of high permittivity dielectric rods capable to support superluminal wave propagation. An opportunity to realize such propagation in 2D PhCs has been found at studying transformations of PhC dispersion diagrams at increasing the dielectric permittivity of crystal "atoms". It has been revealed that at enough high permittivity the 2nd transmission band in PhC dispersion diagram sinks into the fundamental band with formation of the bandgap, which transforms the upper part of fundamental band into a new transmission band. It has been shown that this new band supports the wave propagation with indices changing from zero at the band bottom up to more than 1 at the band upper boundary [2]. That makes the waves propagating in lower part of transmission band capable to demonstrate superluminal phase velocity, which is necessary for the wave front restoration in invisibility cloaks.

The cylindrical cloak in [2] has been composed of concentrically placed rolled-up fragments of PhCs with different lattice constants. Such arrangement allowed to obtain requested by the TO radial dispersion of the azimuthal index components - $n_\theta(r)$ which control the speed of wave propagation along circular arrays of rods in PhC fragments (see section II). For a simplicity we used PhCs with square lattices that made radial index components equal to azimuthal ones and, so, belonging to the superluminal range. It excluded realization of TO prescribed radial dispersion for radial index components – $n_r(r)$, which are responsible for bending the wave paths around the hidden object (Fig.1). Therefore, instead of proper dispersion of $n_r(r)$ we were compelled to rely on the self-collimation of waves propagating along circular arrays of rods in round PhC fragments. Involvement of this phenomenon gave us finally an opportunity to obtain cloaking effect at building the transformation medium of PhCs with square lattices. However, increased scattering from boundaries of PhC fragments and from distorted unit

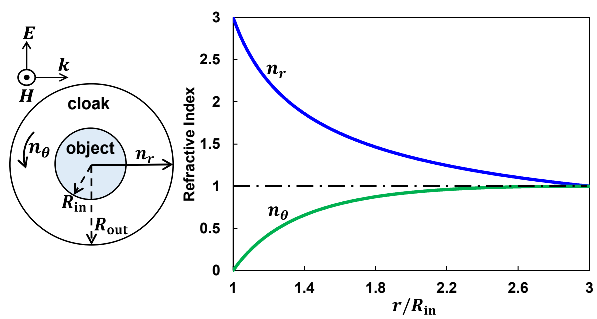

Fig. 1. (a) Schematic of cylindrical cloak; (b) example of TO prescribed dispersions for $n_\theta(r)$ and $n_r(r)$ at R_{out}/R_{in} = 3.0:

cells prevented total scattering cross-width (SCW) of the cloaked object from proper decrease compared to bare object. In addition, fine tuning of self-collimation effect was found challenging because of significant distortions of periodicity in round crystal fragments.

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Thus, it became obvious that the request to provide prescribed dispersion of $n_r(r)$ could not be omitted and should be provided for advancing an implementation of PhCs in transformation media. In current work, we show the possibility to realize this request by using PhCs with rectangular lattices.

II. Realizing TO-prescribed Dispersions for Index Components in Two Orthogonal Directions

As seen from Fig.1, required by the TO dispersions of $n_\theta(r)$ and $n_r(r)$ in cylindrical cloak can be represented by opposite functional dependencies. While dispersion of $n_\theta(r)$ has to be ascending from near-zero values in the vicinity of cloaked object up to values close to 1 at the outer boundary of the cloak, the dispersion of $n_r(r)$ has to be descending from much higher than 1 values near the object down to 1 at the outer cloak boundary. Thus, for complete realizing the TO prescriptions it is not enough to provide some difference between directional indices, i.e. anisotropic wave transfer in the cloak medium. It is necessary, in addition, to provide opposite changes of n_θ and n_r at increasing the radial distance from the hidden object.

In [4,5] it has been shown that an anisotropic wave transfer in two orthogonal directions could be easily achieved in PhCs with rectangular lattices. However, for obtaining opposite radial dispersion of n_θ and n_r in such PhCs there has to be developed special approach. In [2] we provided TO prescribed dispersion of n_θ by changing the lattice constants α of PhC fragments used to form the cloak. The idea to use lattice changes for obtaining index dispersion in the cloak media appeared after investigating frequency dependencies of index values characterizing wave transfer in the 2nd transmission band of 2D PhCs composed of dielectric rods with high permittivity. It has been found that increasing α in such PhCs leads to red shift of above mentioned frequency dependencies (Fig.2). As a result the PhCs with α values taken in some range

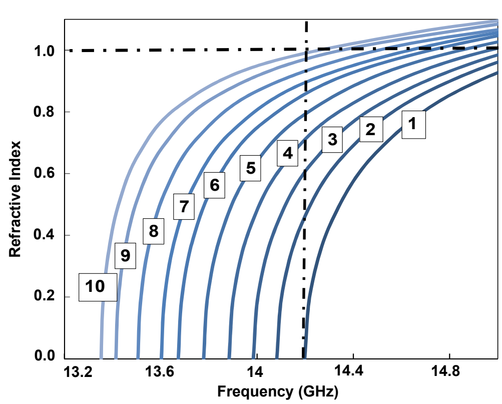

Fig. 2. Dependencies $n(f)$ for the 2nd band of PhCs with r=1.5mm, ε_r =35. Lattice parameters for curves 1-10, respectively, are in mm: 5.17; 5.35; 5.55; 5.76; 6.0; 6.25; 6.52; 6.81; 7.14; 7.5.

demonstrate at some chosen frequency (exemplified by 14.2 GHz in Figure) a set of index values between 0 and 1. It follows that fragments of such PhCs arranged in sequence within the cloak medium could be used for obtaining any desirable dispersion of index values.

In more complicated current case with the need to control two index components n_θ and n_r, we had to investigate if changes of two lattice constants in PhCs with rectangular lattices could be used for obtaining requested by TO variations of n_θ and n_r. To clarify the problem we calculated the dispersion diagrams for wave propagation in X and Y directions in PhCs with rectangular lattices and extracted from these diagrams the values of directional index components by using the expression from [6]: $n_{eff} = \mathrm{sgn}(v_g \cdot k)(\frac{c}{\omega}|k|)$, where c is the speed of light in free space and $v_g = \frac{d\omega}{dk}$ is the group velocity, while k and ω are wave vector and angular frequency, respectively.

Fig. 3 presents extracted from dispersion diagrams frequency dependencies of index components for X and Y directions in four PhCs with different lattice

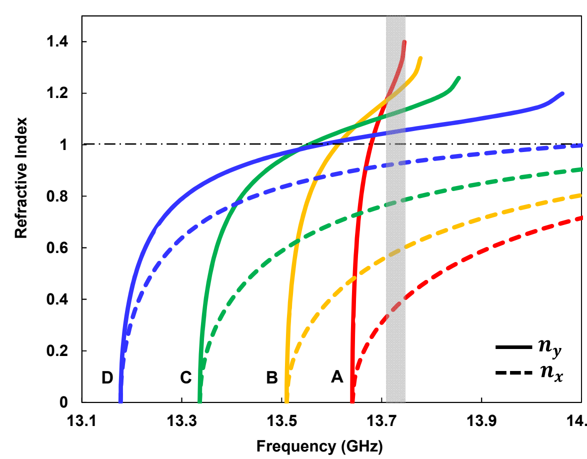

Fig. 3. Frequency dependencies of directional index values in the second transmission bands of four PhCs with rod parameters: ε = 35, R = 1.5 mm, and lattice constants a_x and a_y (in mm): A (5 ; 7.8), B (5.5 ; 8.15), C (6.5 ; 8.6), and D (8 ; 8.9).

constants α_x and α_y. As it is clear from captions we used step-increase of α_x in the PhCs sequence from A to D. It was done to provide the red shift of frequency dependencies for n_x as in Fig.2. The values of α_y also experienced the step- increase, however, since we started in PhC-A with α_y essentially exceeding α_x, this increase was less than in the case of α_x. Therefore, the difference between α_x and α_y, decreased in the line of PhCs from A to D. As seen from Fig.3, while the used approach provided big difference between frequency dependencies for n_x and n_y in PhC-A, in next PhCs of the A, B, C, D sequence this difference gradually drops down to significantly less degrees. These specifics pointed out on an opportunity to employ similar the A,B,C,D sequence of PhCs or similar one for obtaining at some chosen frequency (exemplified by 13.72 GHz in Fig.3) a set of n_x values ascending from close to zero values up to the values close to 1 and a set of n_y values descending from 1.4 down to the values close to 1 (watch index values along vertical line marking 13.72 GHz). This finding allowed us to suggest employing rolled-up fragments of PhCs belonging to A, B, C, D or similar series for building up the cylindrical cloak medium. Such employment should be accompanied by replacing X and Y directions on another orthogonal pair representing, respectfully, azimuthal and radial directions in the cloak. Due to concentric design of the cloak medium the A, B, C, D series of PhCs fragments along the radius of cloak was supposed to provide ascending with radius increase dependence of $n_\theta(r)$ and descending with it dependence of $n_r(r)$. Taking into account the possibility to control, in addition, radial dimensions of PhC fragments the proposed design promised the possibility to realize TO prescriptions for index dispersions quantitatively.

III. Solving Numerical Problems preventing Complete Realization of TO Prescriptions

Further investigations, however, have shown that obtaining thus high values of n_r, as shown in Fig.1 near the inner boundary of the cloak is challenging for PhCs with rectangular lattices since taking much bigger a_y compared to a_x led to degrading the dependencies of n_y on frequency into vertical lines i.e. to an extinction of the 2nd branch in dispersion diagrams for Y direction. Further increase of a_y restored lost transmission band but at account of switching n_y to negative values. Modifying the rod radiuses or permittivity aslo did not allow to approach index values exceeding 2 [7]. Therefore, we investigated the possibility to reduce prescribed values of n_r near the inner boundary of the cloak. It has been found that these values drop down at increase of the cloak thickness, however, even at maximal acceptable thicknesses of the cloak the TO requested up to 1.75 times higher index values than those, which could be achieved in Y direction of of PhCs blocks. Taking into account these restrictions, we proposed to replace the prescribed dispersion for n_r (represented by the n_r curve in Fig.1) by the reduced dependence with achievable in PhCs n_r values of about 2 near the inner boundary of the cloak. To obtain comparable with TO prescriptions effect of reduced n_r dependence on bending the wave paths in the cloak it has been decided to compensate smaller index values near inner surface of the cloak by proper increase of index values in outer cloak layers. Thus, instead of prescribed dispersion for n_r we took as a goal the curve which crossed the former n_r curve in Fig.1 starting from lower than 2 index values at inner boundary of the cloak down to values well exceeding 1 in outer layers of the cloak. Full-wave simulations of the cloak performance at reduced dispersion of n_r by using COMSOL Multiphysics software have confirmed the possibility of obtaining almost the same cloaking effects as at TO prescribed dispersion of n_r. Fig.4 allows to compare snap-shots of the wave propagation in two models of the cloak: one – with TO prescribed dispersion of n_r., and another – with reduced dispersion starting from $n_r=2$ at inner boundary of the cloak. As seen two models provide comparable wave front reconstruction beyond the cloak and have similar distortions of the wave front around the object. The model with reduced dispersion demonstrates differs a bit more reflections than the first model and insignificantly higher scattering beyond the object within the cloak.

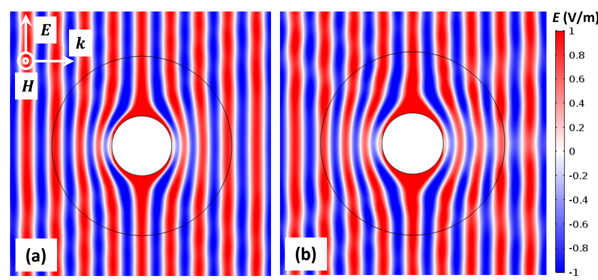

Fig.4. Snap-shots of the wave propagation patterns at TM wave incidence from left to right on two models of the cloak: (a) – with TO prescribed dispersions for n_r and n_θ ; (b) – with the same dispersion for n_θ, but reduced dispersion for n_r. $R_{out}/R_{in}=3.0$, f=11.2 GHz.

IV. Fighting the Scattering provided by PhCs Fragments

At implementation PhCs with square lattice in cylindrical cloak in [2] we have met serious challenge while trying to obtain real cloaking effect, i.e. significantly decreased scattering cross-width of cloaked object in comparison with the SCW of the bare object. The main reason of enhanced scattering was seen in big percentage of distorted PhC cells in crystal fragments. However, neglecting the TO prescriptions for n_r was also named as possible reason. Since studied by our team sets of PhCs with rectangular lattices provided for an opportunity for realizing the adequate dispersion of n_r in addition to TO prescribed dispersion of n_θ we decided, at first, to test the role of neglecting the TO prescription for n_r in scattering by using the same design solutions, which have been used in [2]. In particular, we approximated chosen for realization in the cloak spatial dispersions of refractive index components by step-functions, each of 4 steps, positions of which along Y axis were used to define exact values of index components, which have to be provided by PhCs fragments from the same type series as A, B, C, D one (Fig.5). Then exact values of index components were used to determine the lattice constants of respective PhCs. This reversed finding employed similar to presented in Fig.3 sets of frequency dependences for index components calculated at various combinations of lattice constants. After lattice constants have been found we could form circular arrays for building respective PhCs fragments. Following [2] we formed, at least, 3 identical circular array for each of fragments. Simulations of the performance of cloak designed by using the presented approach has shown that the care about providing proper dispersions of both index components gives an opportunity to get more corresponding to TO expectations wave patterns than it was observed in [2]. In particular, there was achieved complete wave front reconstruction beyond the cloak and less articulated diffraction phenomena. However, scattering cross-width of the cloaked object did not demonstrate still desirable drop against the SCW of the bare object thus confirming excessive scattering of incident waves by the cloak substructures.

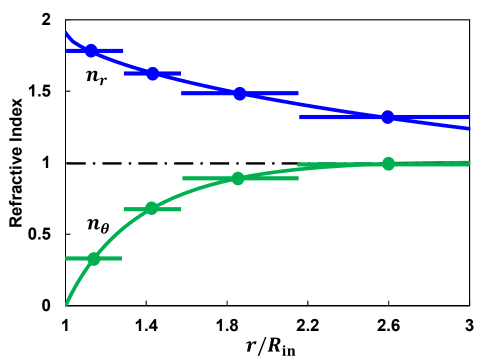

Fig.5. Spatial dispersions of reduced index components chosen for realization in cylindrical cloak composed of 4 PhC fragments approximated by four steps.

The obtained results underlined the necessity to decrease in the cloak the quantity of scattering centers, produced, by rolling-up the PhCs fragments and their combining in the cloak since these procedures could not be done without degrading neighboring circular arrays and caused by it distortions of unit cells. The most radical solution of scattering problem was seen in decreasing the number of circular arrays and, so, the thickness of the cloak. It was decided, for the first tries, to shorten the number of circular arrays used to form each of PhC fragments from 3 down to 2. Such shorting caused concerns about the possibility to provide in reduced fragments the same index values as those, which we extracted from dispersion diagrams for infinite arrays. However, performing the reduced cloaks did not confirm these troubles and have demonstrated, in addition, expected decrease of scattering. Further investigations have shown the possibility to reduce the number of circular arrays in fragments even more. In particular, in the next series of tries we replaced formation of two PhC fragments near the inner boundary of the cloak (A and B) by placing there instead 2 circular arrays with different lattice constants calculated, respectfully, for fragments A and B. The 3^{d} fragment (C) was presented by 2 circular arrays, and the 4th – by 3 ones.

Fig.6 demonstrates snap-shots of wave propagation at incidence on the bare object and at meeting the object shelled by the cloak with described above design. As seen in the Figure, the designed cloak provide perfect restoration of the flat wave front beyond the cloak and corresponding to the TO expectations wave front transformations at passing the cloak. It is also seen that there is no directional scattering usually accompanying the performance of cloaks composed of more bulk PhC fragments. Calculations of the scattering cross-width for two cases have shown essential decreasing the SCW

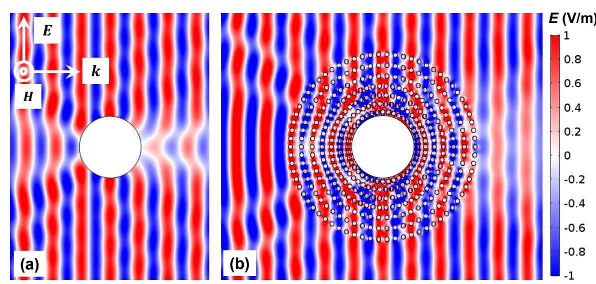

Fig. 6: Simulated snap-shots of the field patterns at TM wave incidence on (a) bare metal cylindrical object and (b) object covered by PhC-based cloak at the frequency of 11.47 GHz.

for cloaked object compared to the SCW of the bare object.

The conducted studies, in whole, confirmed the possibility to implement PhCs instead of MMs in transformation media, even with such complicated geometry as in invisibility cloaks. In difference from MMs the PhCs has no loss related restrictions for applications in optical range.

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