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A Modification to the Weissinger Vortex Arrangement for the Prediction of Stability Derivatives Due to Sideslip for Low-Aspect-Ratio Wings

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Traditional inviscid theories and CFD simulations are unable to accurately predict the stability derivatives of low-aspect-ratio (Æ) wings across the attached- and separated-flow flight envelope. This comes at a critical time when unmanned aerial vehicles (UAVs) are tasked with low-speed maneuvering in energetic background flows. The inaccuracy of stability derivative prediction, specifically for lifting-line and lifting-surface theories, is attributed in part to the vortex arrangement used to model the wing in sideslip. In this work, we show that the historical Weissinger vortex arrangement is inconsistent with the flow physics over low-Æ wings with large tip chord lengths, such as that of a low-Æ rectangular wing. Specifically, the Weissinger vortex arrangement does not acknowledge the presence of separated flow on the windward portion of the wing in sideslip. We propose a new vortex arrangement that is consistent with these flow physics and for preliminary validation, we derive an analytical expression for the lateral static stability derivative, C_{l_β}, for flat-plate rectangular wings. The resulting expression for C_{l_β} is directly compared with experiment and prior theory and is shown to more accurately predict C_{l_β} across the reattached-flow flight envelope.

Nomenclature

LAR	=	low aspect ratio
b	=	wingspan
c	=	root chord
S	=	wing area
ρ	=	density of air
U_∞	=	freestream velocity
Γ	=	circulation
C_L	=	lift coefficient
C_{l_β}	=	lateral stability derivative, deg.$^{-1}$
α	=	angle of attack, deg.
β	=	side-slip angle, deg.

I. Introduction

Tasks such as aircraft carrier launch and recovery, flight in urban environments, and aerial refueling require low-speed maneuvering in turbulent background flows. For unmanned aerial vehicles (UAVs) these tasks must be accomplished autonomously without the safety net of an experienced pilot. Part of the

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challenge in designing an autopilot for these flight regimes, is that the unsteady background flow exploits unmodeled uncertainties in the aircraft dynamics. At low speeds the aircraft aerodynamics are nonlinear due to the existence of mixed regions of attached flow with unorganized separated flow over the wing and control surfaces.[1] Moreover, these aircraft often have low-aspect-ratio (Æ) wings (e.g. X-47B and X-45C) which increases the aircraft susceptibility to side gusts and corresponding flow-field asymmetry. These features and their impacts on the dynamics and stability of the wing are not fully captured by conventional inviscid theory or CFD simulations (see AVT-161[2] and AVT-201[3]). Until these plant dynamics can be properly modeled, an autopilot must be made robust which can stunt UAV performance and/or limit the operational flight envelope of these fliers. To this end, an improved understanding of the low-speed stability and control of low-Æ fliers would be beneficial.

A first step toward improving the stability and control of low-Æ fliers at low speeds is to understand the flow physics around the wing when it is perturbed from trim. With high trim angles of attack required for low speed flight, the flow often separates from the edges of the wing. This separated flow organizes into coherent structures such as the side-edge 'tip' vortices and a leading-edge separation region where the resulting fluid-wing interactions are well recognized as the source of nonlinear lift, drag, and pitching moments with respect to angle of attack perturbations.[5–11] In order to grasp the lateral-directional stability properties, one must understand the fluid dynamics over the wing in sideslip. The smoke wire visualizations of Shields and Mohseni[4] and subsequent direct flow measurements of DeVoria and Mohseni[10] exposed the asymmetric nature of separated vortex flow over low-Æ unswept wings in sideslip. The smoke wire visualization image of an Æ $= 1$ rectangular wing at a sideslip angle of $\beta = 10°$ from Ref.[4] is provided in Fig. 1 for reference. From Fig. 1, the vortex axis of the windward tip vortex is parallel to the freestream such that separated flow exists on a significant portion of the sideslipped wing. Ultimately, these flow physics must be acknowledged in order to accurately model the stability derivatives due to sideslip of low-Æ fliers.

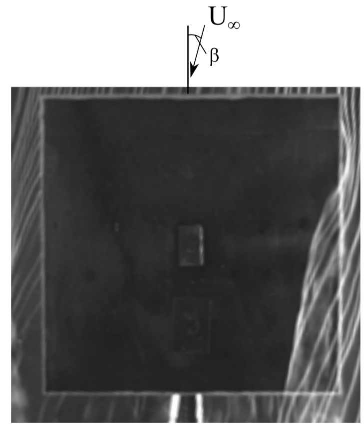

Figure 1: Smoke-wire visualization depicting the asymmetric nature of separated flow over the sideslipped wing rectangular Æ $= 1$ wing. Picture from Shields and Mohseni.[4]

For the rest of this manuscript, we focus our attention to modeling the static roll moment generated by low-Æ wings in sideslip, such as that produced by a side gust. Under the small perturbation assumption, the static roll moment about trim can be represented to first order by the lateral static stability derivative, C_{l_β}, as $C_l = C_{l_\beta}\beta$. Accurate prediction of C_{l_β} is of significant interest due to the slender inertia and low roll damping of low-Æ wings which makes them uniquely sensitive to roll accelerations. The theoretical work of Weissinger was one of the first to capture the scaling trends of C_{l_β} with respect to lift coefficient, C_L, as a function of Æ and taper ratio, λ, for unswept planar wings with straight leading and trailing edges.[12] Thereafter, in the late 1940s and throughout the 1950s a large experimental and theoretical campaign was conducted by NACA. Among this work, Queijo extended Weissinger's theory to capture the effects of quarter-chord sweep angle on C_{l_β}.[13] Both Queijo and Weissinger used a lifting-line analysis for which Weissinger's vortex arrangement (see Fig. 2) was used to model the wing in sideslip. In the Weissinger vortex arrangement, the sideslipped wing is replaced with an infinite series of horseshoe vortices located along the quarter-chord line across the wingspan. Each horseshoe vortex consists of bound vortex segments which remain parallel to the chord of the wing and trailing legs oriented parallel to the freestream velocity vector. A circulation distribution is prescribed along the $c/4$ vortex where the strength of the bound and trailing legs are equivalent to the local rate of change of circulation along the $c/4$ vortex. The general theories of Queijo and Weissinger were validated against experimental measurements of high aspect-ratio swept and tapered wings and were shown to be in agreement. However, these equations

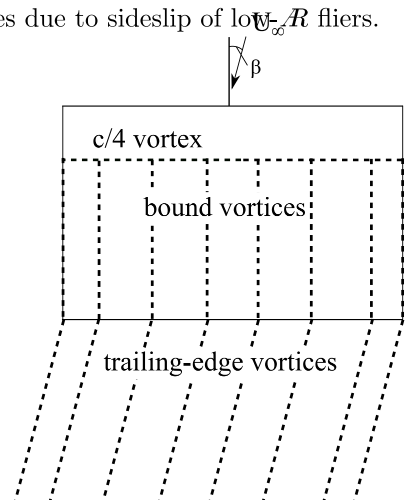

Figure 2: Weissinger's vortex arrangement for a wing in side-slip.

breakdown for certain classes of low-Æ wings. In this manuscript we will show that these equations fail to accurately predict C_{l_β} for low-Æ rectangular wings.

When comparing Fig 2 to Fig. 1, one can get an immediate sense of why the Queijo and Weissenger models breakdown for low-Æ rectangular wings; the Weissinger vortex arrangement does not acknowledge the occurrence of separated flow over the wing in sideslip. As the Weissinger vortex arrangement forms the basis for many inviscid lifting-line and lifting-surface theories for lateral-directional stability derivative calculation (e.g. USAF Stability and Control DATCOM, AVL, and LinAir[14]), recovering a more general vortex arrangement that is consistent with the flow physics of low-Æ wings would be beneficial in improving the accuracy of these relatively inexpensive aerodynamic modeling techniques.

In this paper, we present new vortex arrangement that acknowledges the flow physics over the low-Æ rectangular wing in sideslip. Using this vortex arrangement, we derive an equation for C_{l_β}/C_L which is shown to accurately predict C_{l_β} for both low and high Æ rectangular wings to a lift coefficient of roll stall and/or lift stall, whichever occurs at a lower angle of attack. This work forms the basis for future modeling of the effects of taper ratio and sweep angle on C_{l_β} both at angles of attack of attached and separated flow. The manuscript is organized as follows. The model formulation and derivation is presented in Section II. A discussion section comparing theory and experiment is given in Section III. Lastly, concluding remarks and work is given in Section III.

II. Model formulation

The rectangular wing in sideslip is replaced with a single line vortex of constant strength Γ as shown in Fig. 3. The line vortex consists of three vortex segments that remain bound to the wing and two trailing legs which extend to infinity. The orientations of these vortex structures are discerned from Fig. 1 and the direct flow measurements of DeVoria and Mohseni.[10] The leeward bound vortex segment is oriented parallel to the chord of the wing as the flow remains attached on the leeward side edge. Hereafter, we refer to this vortex filament as the side-edge vortex with subscript SE. The windward bound vortex segment is oriented parallel to the in-plane freestream velocity vector (i.e. $U_\infty \cos(\alpha)$) to mimic the reattachment line imposed by the tip vortex in Fig. 1. The joining bound vortex segment is oriented parallel to the leading edge of the wing and placed at the quarter-chord point. Hereafter, we refer to this vortex segment as the $c/4$ vortex. Two trailing vortices begin at the wing's trailing edge and extend downstream toward infinity oriented parallel to the freestream velocity vector.

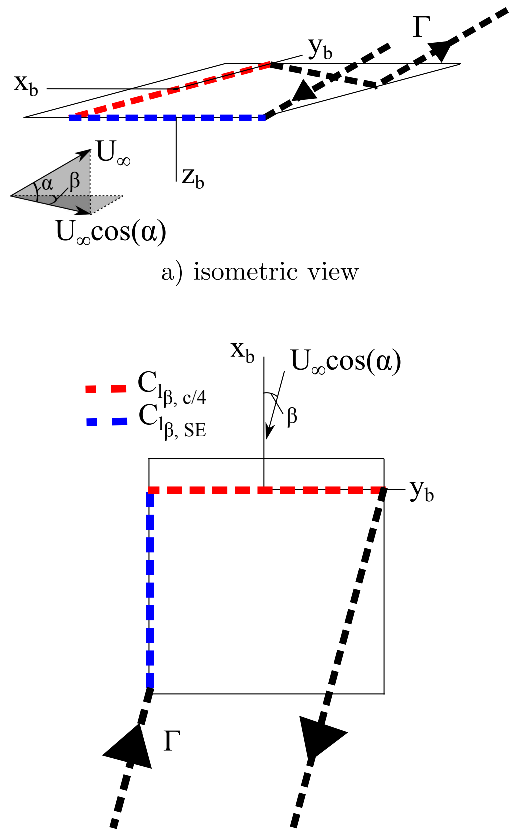

Figure 3: Isometric and top view of vortex arrangement used to model the low-Æ rectangular wing in sideslip.

Notice that this vortex representation differs from the Weissinger arrangement due to the orientation of the windward bound vortex. In this case, the windward bound vortex is oriented parallel to the in-plane velocity vector consistent with the flow physics around the low-Æ rectangular wing. The orientation of this vortex with respect to sideslip will be shown to have a significant influence on C_{l_β} of the low-Æ wing.

Using this vortex arrangement we now derive an expression for C_{l_β}. To begin, we outline the assumptions involved in this derivation. First, we assume small sideslip angle deviations $|\beta| < 10^\circ$ about a symmetric trimmed flight condition, $\beta = 0^\circ$. Under this assumption, we assume that the total lift of the wing at small sideslip is approximately equal to that at zero sideslip (i.e. $C_L \approx C_{L_{\beta=0}}$). This is a valid assumption for the rectangular wing except at high angles of attack near that of lift stall as shown in the experimental measurements of Shields and Mohseni.[15] The circulation of the vortex line at zero sideslip, $\Gamma_{\beta=0^\circ}$, is that which is required to sustain the lift of the wing. This value is given by the Kutta-Joukowski theorem as $L_{\beta=0^\circ} = \rho U_\infty \Gamma_{\beta=0^\circ} b$, where b is the span of the wing. As we have assumed that the lift of the wing does not change for small sideslip, the same must be true for circulation, therefore the circulation of the wing at small sideslip $\Gamma = \Gamma_{\beta=0^\circ}$. Nondimensionalizing and reorganizing terms, the circulation of the wing in sideslip is related to the lift coefficient at zero sideslip, $C_{L_{\beta=0^\circ}}$, of the wing by:

$$\Gamma = \frac{1}{2}\frac{C_{L_{\beta=0^\circ}}}{b}U_\infty S. \tag{1}$$

.

From the vortex-line model of the wing in sideslip, Fig. 3, we note that C_{l_β} of the wing can be interpreted as the summation of the individual contributions of the vortex segments to C_{l_β}. As we are interested only in the forces due to sideslip, hereafter, we need only to refer to Fig. 3b. The vortex force due to sideslip has a magnitude and direction given by the Kutta-Joukowski theorem. Simply stated, in order for a vortex segment to generate a force due to sideslip there must be a component of the local in-plane velocity vector that is perpendicular to that vortex. The magnitude of this local velocity vector is simply $U_\infty \cos(\alpha)$ due to the fact that the bound vortex segments lie in the same plane and thus do not induce side wash on one another. It is now immediately clear that the windward and trailing-leg vortex filaments do not generate a force due to sideslip as they remain parallel to the in-plane freestream velocity. Therefore, these vortex segments do not influence the lateral static stability of the wing in the framework of this model and the $c/4$ and side-edge vortex filaments are the sole contributors to a force due to sideslip. The force due to sideslip from the $c/4$ vortex is given by:

$$(NF)_{c/4} = \rho U_\infty \cos(\alpha)\cos(\beta)\Gamma b, \tag{2}$$

where b is the length of the vortex segment, i.e. the wingspan for the rectangular wing. The abbreviation NF is used to distinguish that this force acts normal to the wing's surface. The force due to sideslip for the bound vortex on the leeward wing's side edge is given by:

$$(NF)_{SE} = -\rho U_\infty \sin(\beta)\Gamma\left(\frac{3c}{4}\right), \tag{3}$$

where the length of the vortex segment is $\frac{3c}{4}$. Employing the small sideslip assumption ($\cos(\beta) \approx 1$ and $\sin(\beta) \approx \beta$) and then taking the partial derivative of the normal force with respect to sideslip, we obtain the the derivative of normal force with respect to sideslip angle for these vortex segments as,

$$(NF_\beta)_{c/4} = 0, \tag{4}$$

and

$$(NF_\beta)_{SE} = -\rho U_\infty \cos(\alpha)\Gamma(\frac{3c}{4}). \tag{5}$$

We note that for the rectangular wing with its unswept leading edge, the side-edge vortex is the only contributer to a force under the assumptions of the model. Nondimensionalizing by standard aerodynamic definitions $C_{NF} = \frac{2NF}{\rho U_\infty^2 S}$ we obtain,

$$(C_{NF_\beta})_{SE} = -2\frac{\Gamma}{U_\infty S}(\frac{3c}{4})\cos(\alpha). \tag{6}$$

Taking the semispan, $b/2$, as the moment arm for the vortex force of the side-edge bound vortex, combining Eq. 1, and employing the definition of $A\!\!\!R$ for rectangular wings $A\!\!\!R \equiv b^2/S = b/c$ we obtain an expression for the change in roll moment due to sideslip or the lateral static stability derivative for rectangular wings as:

$$C_{l_\beta} = C_{l_{\beta_{SE}}} = -\frac{1}{b}(C_{NF_{\beta_{SE}}}\frac{b}{2}) = -\frac{3}{8}\frac{C_L}{AR}\cos(\alpha). \tag{7}$$

Here, the $1/b$ term arises from the nondimensionalization of roll moment which is $C_l = \frac{2l}{\rho U_\infty^2 Sb}$. In the low angle of attack limit the stability ratio, C_{l_β}/C_L, becomes:

$$\frac{C_{l_\beta}}{C_L} = -\frac{3}{8}\frac{1}{AR}. \tag{8}$$

III. Discussion

In Fig. 4 we compare the Weissinger,[12] Queijo,[13] and the current model (Eq. 8) for C_{l_β}/C_L directly with experimental measurements of rectangular wings of $\mathcal{R} = 0.75$ - 3 from Ref.[16] For the reader's convenience, the theory of Weissinger for the rectangular wing is given as:

$$\left(\frac{C_{l_\beta}}{C_L}\right)_{Weissinger} = -\frac{1}{2}\frac{\kappa}{AR} + 0.05. \tag{9}$$

The exact theory of Weissinger fixes the value of κ to $\kappa = 1.5$, however, Weissinger noted that $\kappa = 1$ led to more accurate prediction of C_{l_β}/C_L. The value of 0.05 is used as a correction factor to account for what was deemed to be a change in the circulation of the wing due to sideslip. The theory of Queijo for the rectangular wing is equivalent to Eq. 9 with $\kappa = 1.5$. From Fig. 4, the equation derived in this manuscript (Eq. 8) is seen to accurately predict the slope C_{l_β}/C_L up to a lift coefficient $C_L \approx 0.7$. The Weissinger and Queijo models are seen to over predict C_{l_β}/C_L in this lift coefficient range. The exception to this is the $\mathcal{R} = 3$ wing, Fig. 4d, where the equation derived herein and the Weissinger model are shown to be in close agreement. The agreement seen for high $\mathcal{R}$ rectangular wings corroborates the notion that while a lifting-line analysis using the Weissinger vortex arrangement may predict the stability derivative of high $\mathcal{R}$ wings to sufficient accuracy it will be unable to do so for certain low-$\mathcal{R}$ wings. The core reason behind this is due to the vortex arrangement used to model the wing in sideslip and specifically the orientation of the bound vortices on the windward portion of the wing. In the Weissinger arrangement, the windward bound vortices contribute to the lateral static stability of the wing as they remain parallel to the chord of the wing. This feature causes the over prediction of C_{l_β}/C_L for the theories of Queijo and Weissinger which utilize the Weissinger vortex arrangement.

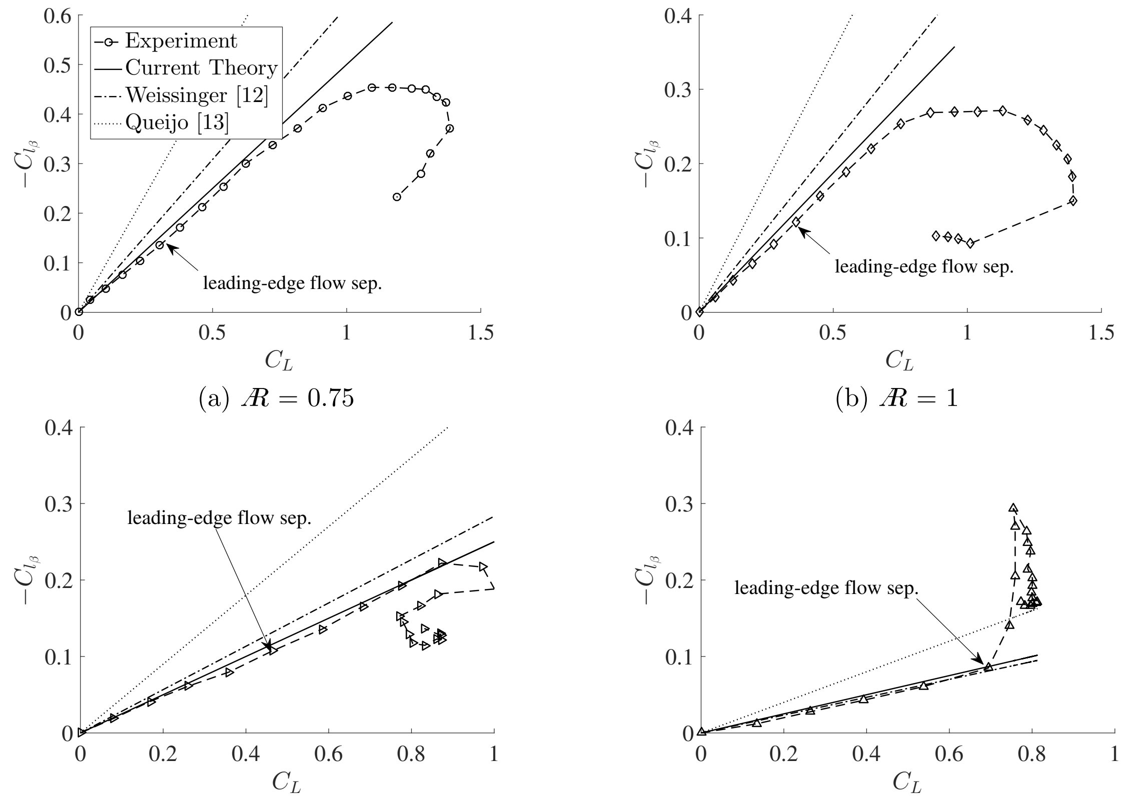

Figure 4: Lateral static stability derivative C_{l_β} as a function of lift coefficient C_L for various $\mathcal{R}$ rectangular wings. Comparison between experiment and theory. The angle of attack of leading-edge flow separation is marked. Experimental stability derivative measurements from Ref.[16]

Notice that throughout the derivation of Eq. 8 we did not acknowledge the lift distribution of the wing.

We were able to get away with this due to the fact that under the small sideslip assumption the $c/4$ vortex does not contribute to C_{l_β} for the rectangular wing with its unswept leading edge. When generalizing this model to incorporate the effects of sweep, a distribution of horseshoe vortices like that of Fig. 5 will need to be used and a $c/4$ vortex circulation distribution prescribed.

The lift coefficient corresponding to leading-edge flow separation as determined from the experiments of Linehan and Mohseni[16] is also marked in Fig. 4. From Fig. 4, leading-edge flow separation and the resulting loss of leading-edge suction, is seen to have little influence on the lateral stability trends which is why this model remains accurate to such high lift coefficients. Recall that under the small sideslip assumption the $c/4$ line vortex did not contribute to the lateral static stability of the wing. The introduction of leading-edge sweep, and the corresponding contribution of $c/4$ vortex to lateral stability, is likely to introduce nonlinear behavior in C_{l_β}/C_L at angles of attack of leading-edge flow separation when leading-edge suction is lost.

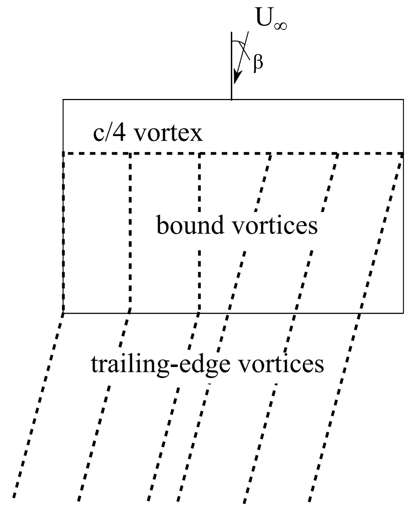

Figure 5: A modified Weissinger vortex arrangement for a wing in sideslip.

The sustained accuracy Eq. 8 to such high lift coefficients suggests that a key parameter of the lateral stability of planar wings at angles of attack involving separated leading-edge flow is the length of the wing's side edge. For the rectangular wing the length of the wing's side edge is directly connected to the aspect-ratio of the wing. However, one can envision that changing the wing's tip chord length by varying the wing's taper ratio on a swept wing of fixed Æ may be an effective design tool to sustain lateral stability at angles of attack involving leading-edge flow separation. The investigation of wing tip length and sweep on the lateral dynamics at angles of attack involving leading-edge flow separation is an active research thrust for the authors.

Lastly, at very high lift coefficients, $C_L > 0.7$ the accuracy of Eq. 8 diminishes as the stability curves becomes nonlinear. This is the roll stall regime, where the associated flow physics in this regime was analyzed in detail by Linehan and Mohseni.[16] In that work it was shown that this nonlinear stability behavior was due to the development of the recirculatory leading-edge separation and the destabilizing stall characteristics of the low-Æ wing. The model in its current form does not have provisions to capture these effects.

IV. Conclusion

A new vortex arrangement was proposed which modifies the classical Weissinger vortex arrangement to capture the flow physics around low-aspect-ratio (Æ) wings with large tip chords in sideslip. The proposed vortex arrangement acknowledges the occurrence of separated flow on the windward portion of the wing by reorienting the windward bound vortex segment of a horseshoe vortex element to be parallel to the in-plane freestream velocity vector. As such, the windward bound vortex does not generate a force due to sideslip and therefore does not contribute to the lateral static stability of the wing. Preliminary validation of this vortex arrangement was made for low-Æ rectangular wings for which an analytical expression for the lateral static stability derivative, C_{l_β}, was derived by modeling the wing in sideslip as a single skewed horseshoe vortex. This expression was seen to accurately predict C_{l_β} to lift coefficients $C_L < 0.7$. Notably, for a wing of sufficiently low Æ, this lift coefficient range includes angles of attack for which the flow separates from the leading edge of the wing. The accuracy of the model in this flow regime suggests that the tip chord length may be a key design tool in modifying C_{l_β} at angles of attack involving leading-edge flow separation.

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