

Serious Gaze

(Invited Paper)

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Abstract—In this paper I review gaze-based interaction, distinguishing eye movement analysis from synthesis in virtual reality and games for serious applications. My focus is on four forms of gaze-based interaction: diagnostic, active, passive, and expressive. In discussing each, I briefly review seminal results and recent advancements, highlighting outstanding research problems.

1. Introduction

Motivation for this paper was found in the somewhat sudden inclusion of eye tracking technology in virtual reality. Recent acquisitions of eye tracking companies Eye Tribe, Eyefluence and SMI by Facebook (Oculus), Google, and Apple, respectively, were notable events. Other eye tracking developments in helmet-mounted displays (HMDs) include the FOVE, and SMI or Pupil Labs add-ons to the HTC Vive. Interestingly, these HMDs are affordable ($\sim \$600$) compared to what was available some 15 years ago ($\sim \$60,000$) [1]. Most of these systems (including the one I used ca. 2002) feature binocular eye tracking at sampling rates 60 Hz or better. The newer systems sport a larger number of infra-red LEDs, e.g., surrounding each eye, and are more comfortable than my HMD custom-built by Virtual Research and ISCAN.

Why has eye tracking suddenly become so popular, or, perhaps more importantly, how is tracked gaze being exploited in virtual reality and serious games? A useful taxonomy for reviewing these applications is shown in Figure 1, which splits gaze-based interaction into four forms, namely diagnostic (off-line measurement), active (selection, look to shoot), passive (foveated rendering, a.k.a. gaze-contingent displays), and expressive (gaze synthesis). Diagnostic interaction is the mainstay of eye-tracked serious applications, such as training or assessment of expertise, and is possibly the longest standing use of gaze due to its mainly offline requirements. Diagnostic analysis of gaze is still very much in demand, especially in serious training situations such as flight or surgery training. Active interaction is rooted in the desire to use the eyes to point and click, with gaze gestures growing in popularity. Passive interaction is the manipulation of scene elements in response to gaze direction, e.g., to improve frame rate. Expressive eye

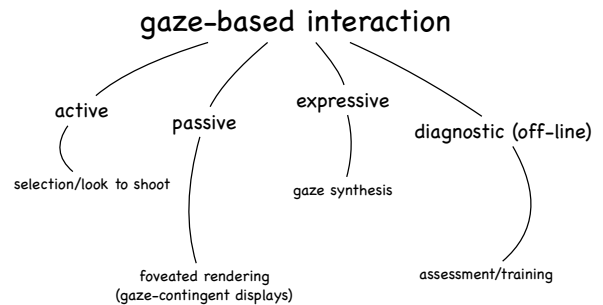


Figure 1. Gaze interaction taxonomy.

movement is drawn from its synthesis, which can make use of a procedural (stochastic) model of eye motion driven by goal-oriented tasks such as reading.

Before reviewing the four forms of gaze-based interaction, a short review of eye movement basics offers some nomenclature and characteristics of gaze.

2. Eye Movement Basics

Detailed human vision is limited to the central 2° visual angle, about the dimension of one's thumbnail at arm's length. Outside of this range, visual acuity drops sharply, e.g., about 50% during photopic (day-time) conditions. High visual acuity within the central 2° is due to the tight packing of cone photoreceptors in the central *foveal* region of the retina. Outside foveal vision, the visual field can be delineated further into *parafoveal* vision (out to about 5°), then *perifoveal* vision (10°), and then *peripheral* vision (all the way out to about 80° on either the temporal or nasal side of each eye). Sundstedt showed a nice depiction of the human visual field in her course notes [2] and subsequent book [3].

Because of the fovea's limited spatial extent (2°), in order to visually inspect the entire 160° – 180° (horizontal) field of view, one needs to reposition the fovea along successive points of *fixation*. Fixations are characterized by tremor, drift, and microsaccades, which are used in concert to stabilize gaze on the point of interest on the one hand, but keep the eyes in constant motion on the other, so as to prevent adaptation [4]. This is a consequence of the directional selectivity of retinal and

cortical neurons implicated in visual perception [5], [6]. If the eyes were perfectly still, the visual image would fade from view.¹ Most of viewing time (about 90%) is spent in fixations, which is why detection of these eye movements is of particular importance.

The fovea is repositioned by large jumps of the eyes known as *saccades*. Saccade amplitudes generally range between 1°–45° visual angle (but can be larger; at about 30°, the head starts to rotate [8]).

When tracking an object, *smooth pursuits* are used to match the motion of the moving target. When fixating an object, the semi-circular canals of the inner ear provide signals to counter-rotate the eyes when the head turns—this is known as *Vestibular-Ocular Response*, or VOR. The eyes may also rotate in opposite direction during *vergence* movements; when looking close, the eyes converge, when looking far, they diverge. Vergence eye movements are used for depth perception and are tightly coupled to *accommodation*, the focusing of the eye’s lens. Further details can be found elsewhere [9].

3. Diagnostic Applications

Diagnostic analysis of eye movements generally relies on detection of fixations in an effort to discern what elements of the visual scene attracted the viewer’s attention. The sequential pattern of fixations is referred to as the *scanpath* [10]. What is perhaps most relevant to serious applications is the observation made classically by Yarbus [11]: the pattern of fixations is task-dependent. That is, vision is largely *top-down*, directed by viewing strategy and task demands. However, vision is also *bottom-up*, drawn often involuntarily by eye-catching elements in the scene [12]. Being able to visualize and analyze an expert’s strategy, e.g., during inspection or monitoring, is of prime importance to the understanding of expertise.

Ericsson et al. [13] surveyed experts’ gaze and noted that experts tend to make shorter fixations, make better use of extrafoveal/peripheral information, and make use of a larger visual span (area around the fixation). Because experts’ visual search strategies develop with training, it makes sense to not only analyze visual patterns, e.g., to assess expertise, but also as a means of its development via training.

Following a review of literature related to the use of scanpaths in training, we showed that *Gaze-Augmented Think-Aloud* can be particularly effective [14]. This protocol records the eye movements of an expert as they verbalize whatever task they are expert in, and then the video playback is shown to novices as a means of training of the same task. This is a fairly straightforward application of eye tracking, yet it holds a number of important advantages over alternatives where pointing

(e.g., with a laser pointer) is involved. Eye movements are faster than hand/limb movements, and perhaps for this reason seem more effortless than pointing. The expert is therefore free to look and make verbal *deictic references* (e.g., “look at this” [15]) without having to consciously think about pointing at something.

Recorded eye movements of both expert and novice can be used to assess the effectiveness of training. A particularly good example of a serious game application is flight simulation, where the study of visual monitoring is especially important [16].

Analysis of eye movements depends to a large extent on detection of fixations in the recorded (or real-time) (x, y, t) eye movement signal. Outstanding problems include better algorithms for scanpath comparison, and better visualizations. Recent contributions include visualization of dynamic ambient/focal visual attention [17], and transition matrix analysis of eye movement [18], but more advanced developments are sure to come.

4. Active Applications

As soon as eye trackers matured sufficiently to produce a real-time signal of the viewer’s (x, y, t) gaze point, they were investigated for their interactive potential. Two seminal contributions from this time are those of Jacob [19] and Starker and Bolt [20]. Both contributions focused on some means of using *dwell time* to effect some kind of system response. Jacob used dwell time as a means of disambiguating gaze-based selection, while Starker and Bolt used it as an interest metric, prompting the system to provide more detail when something was fixated for a prolonged period of time. What is especially notable about Jacob’s contribution was his observation of the *Midas Touch* problem—anything looked at can trigger a response unless some mechanism can be used to prevent it, e.g., Jacob’s introduction of *dwell time*.

Although dwell time is still heavily relied upon for gaze-based selection, gaze-based gestures have also become popular. Along with a review of various gaze-based gestural interfaces, we compared two leading approaches and showed that boundary-crossing gestures can be faster than dwell time, depending on the given task [21]. This is obvious as dwell time incurs an artificial delay whereas boundary crossing does not.

In gameplay, a tempting form of interaction is to use the eyes to point at something to aim or shoot at, as in a first-person shooter [22]. This is particularly effective for arcade-style games (e.g., missile command), as it reduces the amount of mouse movement (although perhaps spinning that large trackball was part of the fun of the old arcade game). Gaze in this context can also be used to orient the viewpoint, as in Tobii’s (an eye tracking company) version of *Rise of the Tomb Raider*.

Sundstedt [3] reviewed various issues of gaze-based gameplay. The Midas Touch is an ever-present consideration. Multi-modality as well as gaze gestures are also interesting emerging alternatives.

1. An impressive simulation of this phenomenon was demonstrated by Mahowald and Mead [7] in the design of a silicon retina based on physiological principles—when held still the image faded.

5. Passive Applications

Passive use of gaze suggests that the eyes are not used to actively select something, rather, the system responds to gaze in a continuous manner. Passive interaction can be considered more natural than active, since, as Zhai et al. [23] put it, the eyes are a perceptual organ, and are not well suited as interactive motor devices (like the hands). Possibly the best example of passive use of gaze is the foveated, gaze-contingent display [24].

There are generally two types of approaches to foveated displays: model- and pixel-based. The model-based approach manipulates the graphics geometry prior to rendering, e.g., by reducing the number of triangles to render outside the foveal region. A classic example of this was demonstrated by Luebke et al. [25] although earlier proposals also exist [26]. The pixel-based approach deals with reducing spatiotemporal complexity of pixel data just prior to rendering, e.g., via MIP-mapping [27] or Laplace filtering [28]. Often the common objective of both approaches is to match human retinal or visual acuity resolutions in an attempt to increase frame rates without the user noticing.

Recent examples of foveated rendering include those of Guenter et al. [29] and Patney et al. [30]. Guenter predicted a 100-fold increase in rendering speed at a 70° field of view using three delineations for resolution degradation: foveal, middle, and outer. The effect of these as well as most other gaze-contingent displays is a region of high resolution, with resolution degrading progressively outwards. How the resolution degrades varies—it can be discretized into three levels (as per Guenter et al.), or it can follow a more smoother function resembling that of visual acuity or contrast sensitivity.

The greatest obstacle to the practical utility of foveated rendering is eye tracking latency leading to a delay in the appearance of the central, high-resolution inset. To be indistinguishable from a full-resolution display, the inset should appear within 7 ms of fixation onset [31]. Greater delays (e.g., 15 ms following fixation onset), while detectable, have minimal impact on performance of visual tasks when the radius of the foveal inset is 4° or greater. Due to saccadic suppression, which raises perceptual thresholds for low spatial frequencies and motion signals just before, during, and for about 20–80 ms after each saccade, delays as long as 60 ms do not significantly increase blur detection [32]. Use caution when interpreting these results: the latter pertains to the time following saccade termination (60 ms), the former to the time following fixation onset (7 ms). Either way, there is precious little time within which the foveal region must be updated before the update is noticed. With regard to visual performance, however, Loschky and McConkie’s [31] point was that in certain circumstances the user will tolerate the delay in order to complete whatever task they were trying to accomplish.

One of the most promising recent approaches to beating the gaze-contingent lag was demonstrated via

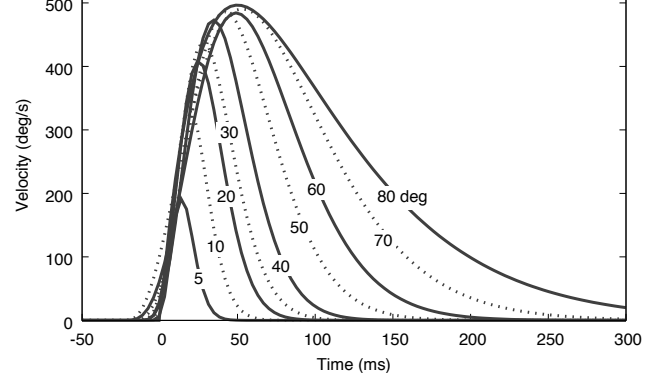


Figure 2. Models of horizontal saccade velocity profiles ranging from 5°–80°, using the Gamma shaping function provided by Van Opstal and Van Gisbergen [33] to illustrate data depicted by Collewyn et al. [34].

saccade endpoint prediction [35]. The basic premise is straightforward, dating back to Anliker [36] who suggested predicting saccade termination by mirroring the saccade velocity profile once peak velocity was detected. The assumed symmetry of the velocity profile is an oversimplification, as saccades of different amplitudes have differently shaped velocity profiles. The velocity profile of small saccades is symmetrical but is skewed for large saccades, and can be modeled by the expression

$$V(t) = \alpha \left(\frac{t}{\beta} \right)^{\gamma-1} e^{-t/\beta}$$

where time $t \geq 0$, $\alpha, \beta > 0$ are scaling constants for velocity and duration, respectively, and $2 < \gamma < 15$ is the shape parameter that determines the degree of asymmetry [33]. Small values of γ yield asymmetrical velocity profiles and as γ tends to infinity, the function assumes a symmetrical (Gaussian) shape, see Figure 2 for an illustration. To handle asymmetric velocities, Arabadzhiyska et al. [35] built a kind of saccade velocity lookup table for each viewer following calibration. The scheme appears effective at predicting the landing position of saccades in mid-flight and thereby offsetting any lag due to latency incurred by the eye tracker.

Apart from rendering speedup, a perhaps more important application of the gaze-contingent display is to promote viewing comfort of 3D displays (e.g., stereo or virtual reality). 3D displays break the natural coupling between vergence and accommodation (focal distance) by rendering images with non-zero disparity (stimulating vergence) at a fixed display distance [38], [39], [40]. This dissociation—referred to as the *accommodation-vergence conflict*—has been considered to be the primary reason for discomfort (asthenopia) felt by viewers of 3D (stereoscopic) displays, with its source tied to eye strain and fatigue [41], [42].

Figure 3, adapted from Shibata et al. [37], shows results from their experiment using a dual-lens haploscope monitor arrangement to demarcate a visual comfort zone

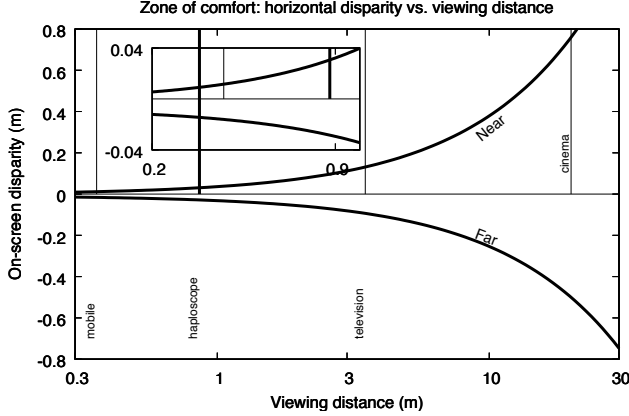


Figure 3. The zone of comfortable stereo display viewing (based on Shibata et al. [37]) augmented to include short-view distances such as the desktop and haploscope.

for various stereoscopic display types, including mobile, desktop, television, and cinema displays. A key insight from their study is that comfortable perception of on-screen disparity is dependent on viewing distance. In cinema, the range extends from 1.6 m to the full screen width, producing a relatively wide range of disparities. A mobile device’s range, 0.28–0.44 m, narrows the comfortable on-screen disparity range considerably.

We examined vergence response via gaze disparity, measured at the screen depth, at two mid-range viewing distances: a typical desktop display at a distance of 0.5 m and a haploscope at a distance of 0.86 m, see Figure 4. We found that vergence error increases away from the $z=0$ screen plane [43], which we conjectured as evidence of the accommodation-vergence conflict.

To reduce visual discomfort, local disparity of the 3D display can be adjusted at the 3D gaze location [44], or alternatively, peripheral blur can be simulated via gaze-contingent depth-of-field [45]. We implemented a real-time depth-of-field display based on the work of Riguer et al. [46]. Peripheral blur is simulated through estimation of the Circle of Confusion (CoC) radius

$$\text{CoC} = a \cdot \left| \frac{f}{d_0 - f} \right| \cdot \left| 1 - \frac{d_0}{d_p} \right|$$

where $a = 1.0$ is modeled lens aperture diameter, $f = 2.2$ is the lens focal length, d_0 is the distance between

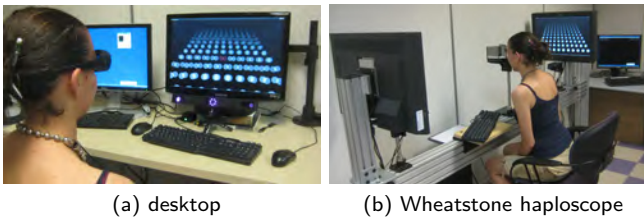


Figure 4. Desktop and Wheatstone haploscope stereo displays.

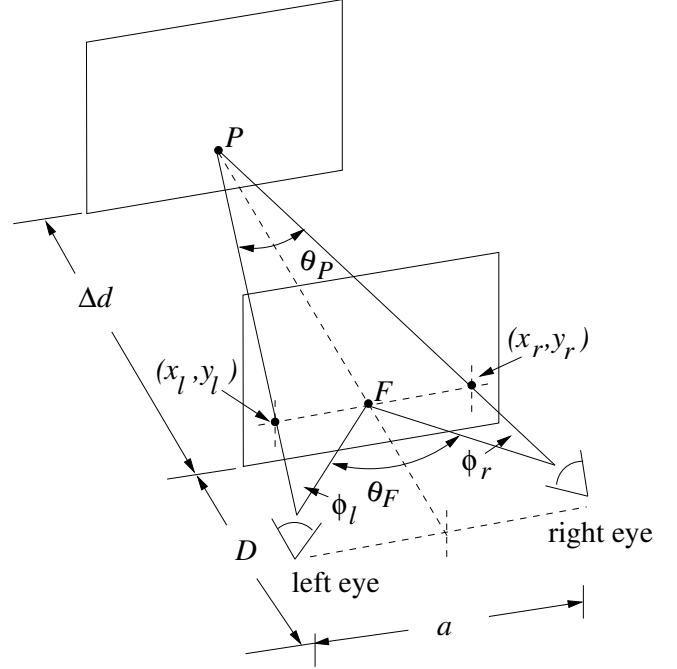


Figure 5. Binocular disparity of point P w.r.t. fixation point F at viewing distance D with (assumed) interocular distance a [41]. Given binocular gaze coordinates on the image plane, (x_l, y_l) and (x_r, y_r) , distance Δd (gaze depth) is found via triangle similarity.

the focal plane and the lens (objects in this plane at this distance are in sharp focus), and d_p is the distance from the rendered object to the lens. Unlike Mantiuk et al. [47], we did not estimate d_0 as the depth value of the current pixel at the viewer’s gaze point, rather, we measured gaze depth z directly, and set the depth-of-field focal plane to this distance.

Gaze depth estimation relies on deriving a fixed mapping of 2D coordinates to 3D gaze depth, requiring a 3D calibration procedure. A binocular eye tracker delivers two eye gaze points, (x_l, y_l) for the left eye and (x_r, y_r) for the right, measured in screen coordinates. The horizontal disparity $\Delta x = x_r - x_l$, between the left and right gaze coordinates, is sufficient to estimate the gaze depth coordinate $z = (\Delta x D) / (\Delta x - a)$ where D is the viewing distance and a is the inter-ocular distance (e.g., 6.3 cm) [48], see Figure 5.

Latency and visual comfort issues are at the forefront of gaze-contingent display research, but several notable approaches have already been proposed to alleviate both. Gaze-contingent depth-of-field is software-based. Recent advancements in hardware e.g., focal surface displays [49], are starting to produce similar effects.

6. Expressive Applications

To bridge Mori’s [50] *Uncanny Valley*, avatars, whether acting in film or games for entertainment or more serious applications, should be modeled with as

realistic eye motion as possible. Indeed, Garau et al. [51] found a strong subjective interaction effect between the realism of a character and its gaze: for a more realistic character, more elaborate gaze behavior was preferred. Gaze behavior, e.g., of a game character influences perceived trust [52]. Gaze behavior of conversational agents can also be used to convey emotion and expression [53], [54]. In virtual reality, eye gaze is critical for correct identification of deictic reference [55]. In film, extreme close shots of the eyes are important for conveying the character’s emotional or perhaps cognitive state, e.g., as in Neil Burger’s feature film *Limitless*.

Thus far, eye movements have been modeled at a fairly coarse grain of motion, largely based on Lee et al.’s [56] seminal *Eyes Alive* model, which focused on saccades, implementing what is known as the saccadic *main sequence* [57], [58], [59] (see below). According to Ruhland et al.’s [60] state-of-the-art report on eye modeling, beyond the rapid saccadic shifts of gaze, previous work on eye motion has also included smooth pursuits, vergence, and the coupling of eye and head rotations.

How are avatar fixations animated? Recall that fixations are characterized by tremor, drift, and microsaccades, and that the eyes are in constant motion to prevent adaptation [4]. The eyes are thus never perfectly still. Meanwhile, the perceptual system is sensitive to and amplifies small fluctuations [61], hence when viewing synthetic eye motion it makes sense to consider the jitter and drift that characterize gaze fixation [4].

Rapid advancement of eye tracking technology has revitalized interest in recording eye movements for inclusion in computer graphics [62], [63]. Why not simply use eye trackers to record eye motion and map that motion onto the eyes of an avatar? This is indeed possible (see Figure 6), however, an eye tracker and an actor emoting expressive eye motions are not always available. Moreover, eye trackers typically inject noise into the recorded signal, which is difficult to separate from the underlying gaze jitter that may be of interest [64].

We have developed a straightforward stochastic model of gaze jitter using $1/f^\alpha$ pink noise as an effective means of simulating microsaccadic jitter [64], [65]. Why

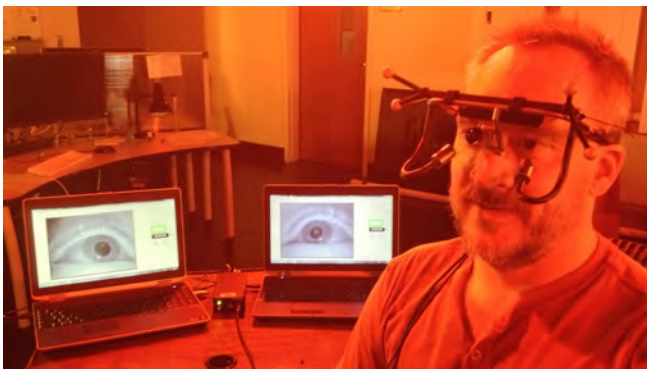


Figure 6. Eye tracking and motion capture for gaze synthesis.

pink noise and not white noise? Three possible signals could trigger microsaccades: fixation error, neural noise, and insufficient retinal motion [66]. Evidence suggests that the three possibilities might not be mutually exclusive, i.e., fixation error and neural noise combine to trigger microsaccades. Recorded neural spikes are superimposed with noise that exhibits non-Gaussian characteristics and can be approximated as $1/f^\alpha$ noise [67].

Pink noise is also suitable for describing physical and biological distributions, e.g., plants [68] and galaxies [69], as well as the behavior of biosystems in general [70].² Aks et al. [72] suggest that memory across eye movements may serve to facilitate selection of information from the visual environment, leading to a complex and self-organizing (saccadic) search pattern produced by the oculomotor system reflecting $1/f^\alpha$ pink noise.

Microsaccadic (fixation) jitter can thus be modeled by pink noise perturbation around the fixation point $\mathbf{p}_{t+h} = \mathbf{p}_t + \mathcal{P}(\alpha=0.6, \omega_0=0.85)$, where $\mathcal{P}(\alpha, \omega_0)$ defines a pink noise filter as a function of two parameters with $1/f^\alpha$ describing the filter’s power spectral density and ω_0 the filter’s unity gain frequency [73]. The pink noise filter takes as input a white noise signal, e.g., modeled by Gaussian noise, $\mathcal{N}(\mu=0, \sigma=12/60)$ arcmin visual angle. Setting $\alpha=1$ produces $1/f$ noise, which has been observed as characteristic of pulse trains of nerve cells belonging to various brain structures [61].

Saccades are modeled by advancing the fixation point $\mathbf{p}_t$ at simulation time t from one look point $\mathbf{P}_{i-1}$ to the next $\mathbf{P}_i$, i.e.,

$$\mathbf{p}_t = \mathbf{P}_{i-1} + H(t)\mathbf{P}_i$$

following fixation duration at point $\mathbf{p}_t$ with $H(t) = \frac{1}{10}t^5 - \frac{1}{4}t^4 + \frac{1}{6}t^3$, a Hermite blending function on the normalized interval $t \in [0, 1]$ used to smoothly advance position $\mathbf{p}_t$. Saccade durations follow the main sequence

$$\Delta t = 2.2\theta + 21 \text{ ms}$$

which relates saccade duration to amplitude.³ Fixation durations are modeled by normal distribution which can be adjusted to the given task, e.g., $\mathcal{N}(\mu=250, \sigma=50)$ ms for reading.

The procedural eye movement simulation is illustrated in Figure 7 by a *gristmill* which accepts as input a sequence of fixations with fixation durations. Such a sequence can be produced by simulation, or obtained from a sequence of eye movement data recorded by an eye tracker. If the latter, then fixations and their durations are extracted via event detection, i.e., filtering. Given a sequence of fixations, jitter can be modeled as pink noise perturbation, as described above, or perhaps as a self-avoiding random walk [75].

2. See Zhou et al. [71] for a discussion of different colors of noise and their point sampling implementations.

3. Other variations of the linear main sequence include $\Delta t = 2.7\theta+37$ ms [58], $\Delta t = 2.7\theta+23$ ms [34], and $\Delta t = [2, 2.7]\theta+[20, 30]$ ms [56], [74].

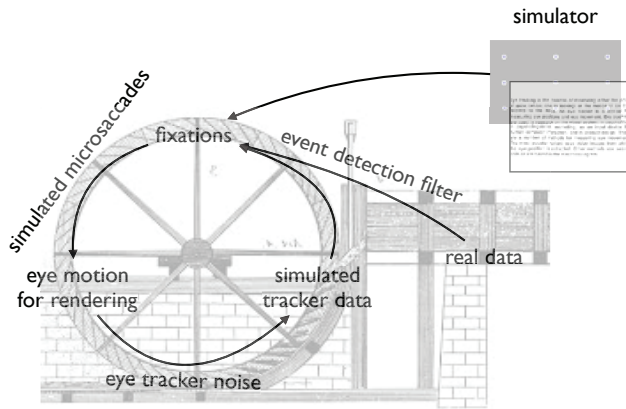


Figure 7. Procedural eye movement simulation “gristmill”.

Realistic eye motion is clearly important for promoting the believability of virtual characters, be they human(oid), robotic, or something else. There is still much left to be done to render lifelike eyes and eye movements. Care should be taken to animate pupil diameter and blinks [65]. Pupil diameter can also be modeled as a pink noise procedural simulation of *pupil unrest*, or *hippus* [76], augmented with a model of light response [77] or based on small random variations to light intensity [78]. Blinks, following Trutoiu’s [79] work, can also be modeled procedurally with fast down and slow up phases.

7. Conclusion

Analysis and synthesis of eye movements in virtual reality and serious games presents interesting diagnostic and interactive possibilities with exciting challenges. Gaze opens an additional bidirectional channel of information to the user. Because gaze is associated with cognitive processing (e.g., Just and Carpenter’s [80], [81] *eye-mind assumption*) and emotional expression [82], [83], it is a particularly rich source of information.

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