BIT ERROR RATE ANALYSIS OF CODE-MULTIPLEXED COULTER SENSOR NETWORKS

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ABSTRACT

Efficient in spatially manipulating particles with microscale precision, microfluidic devices have long been sought for biophysical/chemical analysis. We have recently introduced the Microfluidic CODES, a scalable electrical sensor technology that combines Coulter detection with the code division multiple access to spatially track particles processed on microfluidic devices from a single electrical readout. Here, we develop a mathematical model based on telecommunications theory and calculate the error rate for the Microfluidic CODES sensor network as a function of the sample and sensor network properties, to estimate device performance for a given sample.

KEYWORDS: Microfluidics, Microfluidic CODES, Multiplexing, Sensor network, Coulter sensing

INTRODUCTION

Electrical sensors have been commonly employed in lab-on-a-chip devices for integrated sensing and characterization of suspended particles. Simple fabrication process combined with the ability to directly digitize and process the sensor data are among several advantages of electrical sensors over other sensing modalities. We recently introduced an electrical sensor technology, called Microfluidic CODES [1], which combines the Coulter detection with the code division multiple access (CDMA) for distributed sensing of particles on a microfluidic device from a single electrical waveform. The simplicity of the hardware makes this technique well suited to implement electrically readable, low-cost chemical/biological assays as integrated lab-on-a-chip devices to be used in mobile or resource-limited settings.

In the Microfluidic CODES (Figure 1), we use micromachined electrodes to create a code-multiplexed sensor network, where each sensor is defined by a distinct electrode pattern. When a suspended particle flows over these electrodes, it produces a signature waveform, dictated by the underlying electrode pattern, via the Coulter principle. Because all sensors are connected, coincident particles lead to interference of different sensor signals and this interference appears as noise in the output of a decoder designed to receive information solely from a specific sensor. In this paper, we analytically model this noise and predict the sensor network performance as a function of the sensor network size and the sample properties.

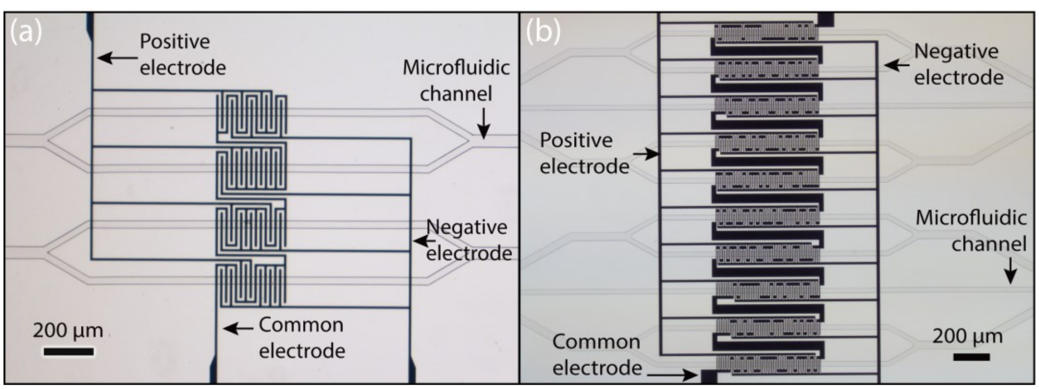

Figure 1: Layouts of fabricated Microfluidic CODES devices. Images of (a) a 4-sensor 7-bit device, and (b) a 10-sensor 31-bit device.

THEORY

When the sensor signals due to K coincident cells interfere (Figure 2), the output of each correlation-based decoder contains both the desired component from the target sensor, and an undesired component, which is the multiple access interference from the other (K-1) sensors in the network. Based on the CDMA telecommunications theory, the interference can be written as the sum of independent Bernoulli trials, and its effect on the bit error rate can be modeled using the improved Gaussian approximation (IGA) [2]. In IGA, we model the variance of the interference as a Gaussian random variable (Ψ), and use the mean and the variance of the sensor signal power levels (Figure 3) to calculate the mean and variance of Ψ as

$$\mu_\Psi = \frac{TN}{3}(K-1)\mu_p\,,$$
$$\sigma_\Psi^2 = (K-1)\frac{T^4}{4}\left[\left(\frac{7N^2+2N-2}{15}\right)\sigma_\mathrm{p}^2 + \left(\frac{1}{45}N^2 + \left(\frac{1}{9}K - \frac{4}{45}\right)N - \left(\frac{1}{9}K - \frac{4}{45}\right)\right)\mu_\mathrm{p}^2\right]$$

where T is the bit period, N is the bit number, μ_Ψ, σ_Ψ, μ_p, and σ_p are the mean and the variance of the interference and signal power, respectively.

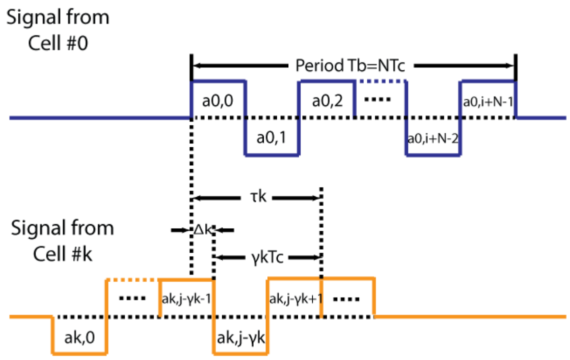

Figure 2: The schematic of the multiple access interference between sensors due to coincident particles. Sensor signals are modeled as digital waveforms with random delays. Here, N is the bit number, Tb is the sequence period, Tc is the bit period, γ and Δk are defined from the delay of sensor k relative to sensor 0.

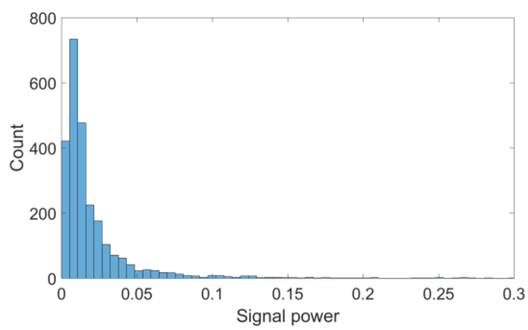

Figure 3: The histogram of the experimentally measured sensor signal power from processing of a cell suspension. The mean and the variance of the signal power was obtained from the histogram and used to estimate the multiple access interference.

Finally, the error probability can be evaluated as a function of Ψ as

$$P_{e0} = E\left[Q\left(\sqrt{\frac{P_0T^2}{2\Psi}}\right)\right]$$
$$\approx \frac{2}{3}Q\left(\sqrt{\frac{P_0T^2}{2\mu_\Psi}}\right) + \frac{1}{6}Q\left(\sqrt{\frac{P_0T^2}{2(\mu_\Psi+\sqrt{3}\sigma_\Psi)}}\right) + \frac{1}{6}Q\left(\sqrt{\frac{P_0T_b^2}{2(\mu_\Psi-\sqrt{3}\sigma_\Psi)}}\right)$$

where P_0 is power of the desired signal from the target sensor. The average error rate is calculated by averaging the equation above over all possible sensor signal power levels.

$$P_e = \int_0^\infty P_{e0}f_{P_0}(P_0)\mathrm{d}P_0$$

where, f_{P_0} is the probability density function for sensor signal power levels. By using experimentally measured sensor power levels, the error rate can be expressed as a function of the expected number of coincident cells (Figure 4).

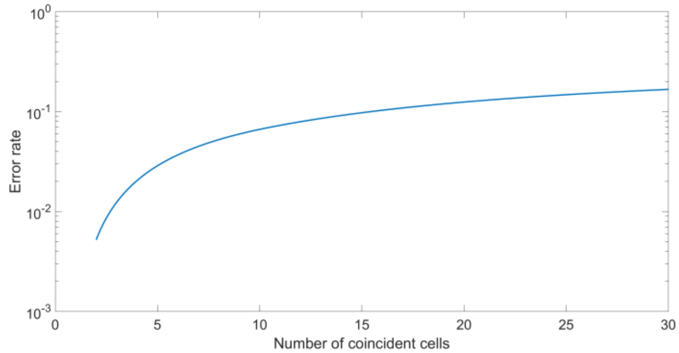

Figure 4: The estimated error rate in the decoder as a function of the number of coincident cells.

In the microfluidic system, the number of coincident cells is a random variable that follows a Poisson distribution, and the expected number of coincident cells in the sensing region, namely λ in the Poisson distribution, is the product of the sensing volume and the particle concentration. Therefore, the error rate of the decoder can be calculated as a function of the device and the sample properties (Figure 5).

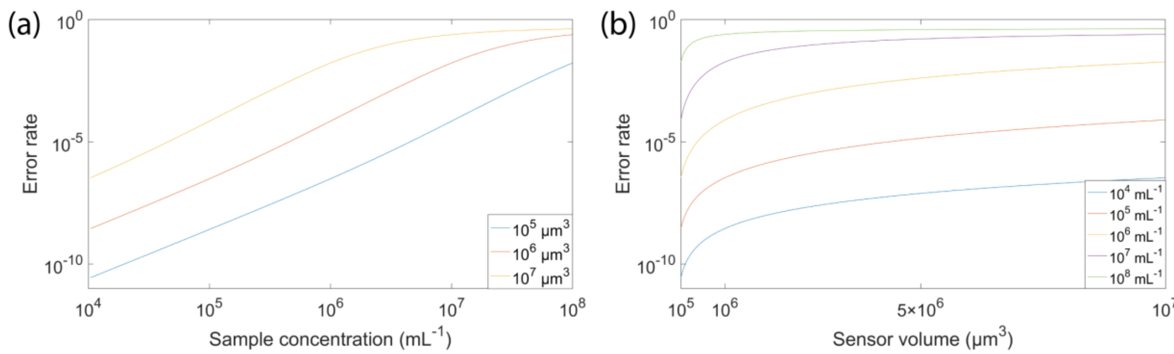

Figure 5: The estimated error rate in the decoder as a function of sample and sensor properties. (a) The error rate is calculated for three microfluidic devices with sensing volumes ranging from 10^5-10^7 µm^3 at different sample concentrations. (b) The error rate is calculated at 5 sample concentrations ranging from 10^4-10^8 cells/mL for different sensor geometries.

CONCLUSION

In this paper, we developed a mathematical model to calculate the error rate for the Microfluidic CODES sensor network. Taken together, the analysis presented in this paper can be used to optimize the code-multiplexed Coulter sensor networks and the decoder design for a given sample density to achieve the target error rate.

ACKNOWLEDGEMENTS

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