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Solving the linear inviscid shallow water equations in one dimension, with variable depth, using a recursion formula

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Abstract
When solving the linear inviscid shallow water equations with variable depth in one dimension using finite differences, a tridiagonal system of equations must be solved. Here we present an approach, which is more efficient than the commonly used numerical method, to solve this tridiagonal system of equations using a recursion formula. We illustrate this approach with an example in which we solve for a rectangular channel to find the resonance modes. Our numerical solution agrees very well with the analytical solution. This new method is easy to use and understand by undergraduate students, so it can be implemented in undergraduate courses such as Numerical Methods, Lineal Algebra or Differential Equations.

Keywords: resonance, numerical methods, tridiagonal system

1. Introduction

The understanding of shallow water flows is of great importance in the modelling of natural ecosystems. The concept of the shallow water approximation for fluid motion is present in atmospheric processes and in rivers, lakes and coastal environments that are delimited by topographical boundaries. Modelling of the shallow water phenomenon can be achieved by applying a much greater significance to the horizontal flow scale than to the vertical scale, both with respect to the position and velocity fields [1, 2]. Recent works that offer a clear mathematical implementation of the shallow water approximation in order to model wave–current interactions are given by [3–5]. With this simplistic and didactic model undergraduate students can easily understand physical phenomena that involve shallow waters. In addition, this model is very useful for studying ocean currents, port design, flood warning systems, coastal processes that involve changes in the coastline due to hurricanes and other coastal processes, and even climate prediction and reduction of marine pollution – all of them favorite subjects of our students.

Shallow water equations have been studied extensively both by mathematicians and people who have used these equations to apply problems in fluids, both in terms of its analytical and numerical solution [6–8].

One important application of shallow water models is the phenomenon of resonance in ports. This phenomenon can cause major damage to port structures, potential collisions between ships, and problems in the loading and unloading of cargo ships [9]. Several studies have demonstrated the great importance of bathymetry in the response of water bodies with semi-closed boundaries to the incident waves [10]. Considering a circular water body, major variations occur in the amplitude of the water level; however, the wavelengths of the incident waves present minor variations [10].

The linearized shallow water differential equations are a linear system that derive from the conservation of mass and momentum using different schemes, both in one and two dimensions. Many numerical methods have been developed to calculate their solutions to solve problems where the shallow water model can be applied (see, e.g. [11–18], and references therein). The shallow water equations can be represented by a tridiagonal system. This kind of system is commonly found in problems of partial differential equations, in cubic spline interpolation algorithms, and in other applications of engineering and sciences. In 1949, the Thomas algorithm was proposed to calculate a numerical solution for tridiagonal systems [19]. This algorithm is based on LU decomposition, and consists of two phases, a forward elimination and a backward substitution. However, because of its sequential nature, this algorithm is considered inefficient, because the computing time of this algorithm is of the order of N, where N is the number of equations to be solved. Several studies have proposed methods that solve tridiagonal systems in parallel: the cyclic reduction algorithm, proposed by Hockney in 1965 [12]; the recursive doubling algorithm that reduces the computation time to the order of $log_2(N)$, proposed by Stone in 1971 [20, 21]; or the parallel factorization [22], among others.

The purpose of this paper is to present a numerical method to solve the tridiagonal system that we obtain from linearized inviscid shallow water equations. It is based on the ANA method proposed by Gomez-Garcia in 1991 [23], but modified for use in the linearized equations for shallow water in one dimension (1D) with variable depth; Robin-type boundary conditions are used in the present case. This new way of solving a tridiagonal system can be used in undergraduate courses, such as Numerical Methods, Linear Algebra or Differential Equations, so that our students learn a new, easy and didactic method to solve this type of system, along with the existing Thomas algorithm. Thus, our students have the possibility to learn that they can approach the solution of the same problem using different methods, comparing their effectiveness, the precision of the results, and analyzing the advantages and disadvantages of each technique. This activity is of great importance in the teaching and learning process.

In section 2 we present the linearized inviscid shallow water equations for the elevation and the velocity of the water flow in a channel in 1D. In sections 3 and 4 we show the numerical solutions for both differential equations, corresponding to the elevation of the water level and the flow velocity, respectively. In both cases we use an iterative method to calculate the solution for the tridiagonal system that is formed from discretization of the differential equations by the finite differences method.

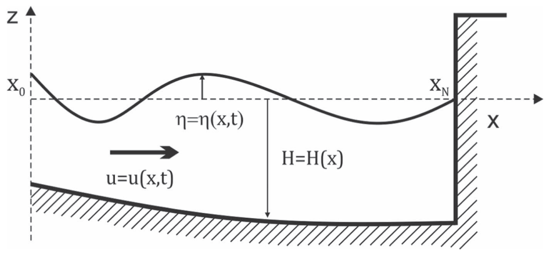

Figure 1. Model of fluid in shallow water.

In section 5 two numerical examples are presented. The first one corresponds to a channel whose bathymetry is flat, where we know the analytic solution. In the second example the bottom channel presents a bathymetric high modelled with a Gaussian function.

2. Shallow water equations equation in 1D

Figure 1 presents a model of a thin water layer flowing frictionless in a channel with variable bathymetry. We suppose that the horizontal dimension of the channel is much larger than the vertical one. The flow within the channel is uniform, and we consider that the longitudinal scale is greater than the horizontal width of the flow. The flow direction is from left to right and the channel has two fixed boundaries: open to the left, in x_0, which corresponds to the channel entrance, and closed to the right, in x_N, which can model a shoreline or a dock with a vertical solid wall. The boundary conditions for elevation and velocity will depend on the corresponding problem, and will be defined in the following sections.

The linearized inviscid shallow water theory reduces the problem to solving the following partial differential equations [10]:

$$\frac{\partial u}{\partial t} = -g\frac{\partial \eta}{\partial x}, \tag{1}$$

$$\frac{\partial \eta}{\partial t} = -H\frac{\partial u}{\partial x} - u\frac{\partial H}{\partial x},$$

where $u = u(x, t)$ is the average flow velocity that varies with position and time; $\eta = \eta(x, t)$ is the elevation above the mean sea level, also depending on position and time; $H = H(x)$ is the bottom topography; and g is the acceleration due to gravity. Although the shallow water model considers vertical scale variations to be less than horizontal scale, bathymetric changes should be considered important, but these changes should not be brusque but mild [10].

The method for solving the system of equations (1) depends on which one of the variables you want to obtain, either elevation or velocity.

For elevation, the second equation in the system (1) is used to differentiate with respect to time and the first equation is used to reduce the number of variables to one, obtaining

$$\frac{\partial^2 \eta}{\partial t^2} = gH\frac{\partial^2 \eta}{\partial x^2} + g\frac{\partial \eta}{\partial x}\frac{\partial H}{\partial x}. \tag{2}$$

Assuming that the elevation is given by a plane wave, $\eta = \hat{\eta}(x)e^{i\omega t}$, where $\hat{\eta}$ is the amplitude and ω is the angular frequency, equation (2) is transformed into

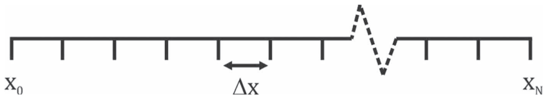

Figure 2. Mesh for a rectangular channel in 1D.

$$-\omega^2\hat{\eta} = gH\frac{\partial^2\hat{\eta}}{\partial x^2} + g\frac{\partial\hat{\eta}}{\partial x}\frac{\partial H}{\partial x}. \tag{3}$$

To solve the problem for velocity, we must take the first equation of the system (1) and differentiate it as a function of time, obtaining

$$\frac{\partial^2 u}{\partial t^2} = g\left\{2\frac{\partial H}{\partial x}\frac{\partial u}{\partial x} + H\frac{\partial^2 u}{\partial x^2} + u\frac{\partial^2 H}{\partial x^2}\right\}. \tag{4}$$

If the velocity is modelled as a plane wave, $u = \hat{u}(x)e^{i\omega t}$, where $\hat{u}(x)$ is the velocity and ω is its angular frequency, we obtain

$$-\omega^2\hat{u} = 2g\frac{\partial H}{\partial x}\frac{\partial\hat{u}}{\partial x} + gH\frac{\partial^2\hat{u}}{\partial x^2} + g\hat{u}\frac{\partial^2 H}{\partial x^2}. \tag{5}$$

To calculate $\hat{\eta}(x)$ and $\hat{u}(x)$, we propose two separate tridiagonal systems from equations (3) and (5), respectively, whose solutions are calculated using numerical methods. To do so, we suppose that the channel has a unitary length and we divide it into intervals of equal size, Δx, to create a mesh, as it is shown in figure 2.

The position of each node is given by

$$x_i = i\Delta x, \qquad i = 0{:}\ N, \tag{6}$$

where $i = 0{:}\ N$, and Δx is the size of the intervals,

$$\Delta x = \frac{1}{N}. \tag{7}$$

3. Numerical solution: elevation

If we replace $\eta_i \equiv \hat{\eta}(x_i)$ in the differential equation (3), and use the finite differences method, we obtain the following discrete equation:

$$\frac{H_i}{\Delta x^2}(\eta_{i+1} - 2\eta_i + \eta_{i-1}) + \frac{(H_{i+1} - H_i)(\eta_{i+1} - \eta_i)}{\Delta x^2} + \frac{\omega^2}{g}\eta_i = 0. \tag{8}$$

The above equation can be accommodated as a tridiagonal system of equations as

$$\eta_{i+1} + \phi_i\eta_i + \gamma_i\eta_{i-1} = 0, \tag{9}$$

where we are defining

$$\phi_i = \frac{-H_i - H_{i+1} + \frac{\omega^2\Delta x^2}{g}}{H_{i+1}}, \tag{10}$$

and

$$\gamma_i = \frac{H_i}{H_{i+1}}. \tag{11}$$

The boundary conditions that we use to solve the system (9) are

$$\eta_0 = 0.001,$$
$$\left(\frac{\partial \eta}{\partial x}\right)_N = 0. \tag{12}$$

In position $i = 0$, which corresponds to the channel entrance, we have set the unit value for the elevation in order to illustrate clearly the amplitude of the incident wave. At the end of the channel, for $i = N$, the Neumann boundary condition has been chosen. The latter condition can be reduced using the finite differences method for discretization, obtaining

$$\eta_{N-1} = \eta_N. \tag{13}$$

Taking into account the boundary conditions (12) and (13), the tridiagonal system (9) can be rewritten as

$$\begin{bmatrix} \Phi_1 & 1 & & & & \\ \gamma_2 & \Phi_2 & 1 & & & \\ & \gamma_3 & \Phi_3 & 1 & & \\ & & \cdots & \cdots & \cdots & \cdots \\ & & & \gamma_{N-2} & \Phi_{N-2} & 1 \\ & & & & \gamma_{N-1} & (\Phi_{N-1} + 1) \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \vdots \\ \eta_{N-2} \\ \eta_{N-1} \end{bmatrix} = \begin{bmatrix} -\gamma_1 \eta_0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}. \tag{14}$$

Solving for the first value of the system, we obtain

$$\eta_1 = \frac{-\gamma_1 \eta_0 - \eta_2}{\Phi_1}, \tag{15}$$

and doing the same for the second equation of the system, we obtain the second value for elevation

$$\eta_2 = \frac{-\gamma_2 \eta_1 - \eta_3}{\Phi_2}, \tag{16}$$

which depends on the previous point η_1 and the next point η_3. Substituting (15) in (16), a new expression for η_2 can be given:

$$\eta_2 = \frac{\gamma_1 \gamma_2 \eta_0 - \Phi_1 \eta_3}{\Phi_1 \Phi_2 - \gamma_2}. \tag{17}$$

The following value for the elevation can be obtained in the same way, resulting in

$$\eta_3 = \frac{-\gamma_1 \gamma_2 \gamma_3 \eta_0 - (\Phi_1 \Phi_2 - \gamma_2) \eta_4}{\Phi_3 (\Phi_2 \Phi_1 - \gamma_2) - \gamma_3 \Phi_1}. \tag{18}$$

Therefore, we can generalize the calculation of the solution for the tridiagonal system (14) to find the recurrent expression for the elevation, which is given by

$$\eta_i = \frac{(-1)^i (\prod_{j=1}^{i} \gamma_j) \eta_0 - C_{i-1} \eta_{i+1}}{C_i}, \tag{19}$$

with $i = 1 \quad to \quad N - 2$, and where we have used the coefficients $C_0 = 1$, $C_1 = \Phi_1$, $C_i = \Phi_i C_{i-1} - \gamma_i C_{i-2}$ with $i = 2 \quad to \quad N - 2$.

The recursive formula (19) serves to calculate the values of the elevation along the entire mesh, except for the last point before the boundary, with $i = N - 1$. To calculate the value η_{N-1}, we substitute i by $N - 2$ in (19) to obtain

$$\eta_{N-2} = \frac{(-1)^{N-2}(\prod_{j=1}^{N-2} \gamma_j)\eta_0 - C_{N-3}\eta_{N-1}}{C_{N-2}}. \tag{20}$$

We calculate the value of η_{N-2} by solving the last equation of the system (14) and using the right boundary condition, obtaining

$$\eta_{N-1} = \frac{-\gamma_{N-1}\eta_{N-2}}{\phi_{N-1} + 1}. \tag{21}$$

Now, we calculate the value of η_{N-2} in the above equation and substitute it in (20) to obtain the following expression for η_{N-1}, which depends only on the value of η_0:

$$\eta_{N-1} = \frac{(-1)^{N-1}(\prod_{j=1}^{N-1} \gamma_j)\eta_0}{C_{N-2}(\phi_{N-1} + 1) - C_{N-3}\gamma_{N-1}}. \tag{22}$$

Thus, with equation (22) we can get the elevation value at a point immediately prior to the closed boundary. To solve for unknown elevations in the rest of the mesh points, we replace the values of the elevations backwards, using equation (19), from $i = N - 2 \quad to \quad i = 1$.

4. Numerical solution: velocity

We are now interested in solving the equation (5), numerically proposing a tridiagonal system as we did in the case of elevation. To do this, we substitute $u_i \equiv \hat{}(x_i)$ in equation (5) and obtain the following discrete equation using the finite differences method with the mesh of figure 2:

$$\begin{aligned} &\frac{H_i}{\Delta x^2}(u_{i+1} - 2u_i + u_{i-1}) + 2\frac{(H_{i+1} - H_i)(u_{i+1} - u_i)}{\Delta x^2} \\ &+ \frac{u_i}{\Delta x^2}(H_{i+1} - 2H_i + H_{i-1}) + \frac{\omega^2}{g}u_i = 0. \end{aligned} \tag{23}$$

This can be rewritten as the tridiagonal system

$$u_{i+1} + \phi_i u_i + \gamma_i u_{i-1} = 0, \tag{24}$$

where

$$\gamma_i = \frac{H_i}{-H_i + 2H_{i+1}}, \tag{25}$$

and

$$\phi_i = \frac{-H_{i+1} - 2H_i + H_{i-1} + \frac{\omega^2 \Delta x^2}{g}}{-H_i + 2H_{i+1}}. \tag{26}$$

As boundary conditions, we chose the following relationship between u_0 and η_0 to exemplify the velocity at the channel entrance [24]:

$$u_0 = \eta_0 \sqrt{\frac{g}{H_0}}, \tag{27}$$

and we use the Dirichlet condition for the closed boundary

$$u_N = 0. \tag{28}$$

Thus, the tridiagonal system (24) can be rearranged:

$$\begin{bmatrix} \Phi_1 & 1 & & & & & \\ \gamma_2 & \Phi_2 & 1 & & & & \\ & \gamma_3 & \Phi_3 & 1 & & & \\ & & \cdots & \cdots & \cdots & \cdots & \\ & & & & \gamma_{N-2} & \Phi_{N-2} & 1 \\ & & & & & \gamma_{N-1} & \Phi_{N-1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ \vdots \\ u_{N-2} \\ u_{N-1} \end{bmatrix} = \begin{bmatrix} -\gamma_1 u_0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \end{bmatrix}. \tag{29}$$

From the system (23) we can obtain the recursive formula for the velocity, following the same procedure as for the case of elevation, which is given by

$$u_i = \frac{(-1)^i (\prod_{j=1}^{i} \gamma_j) u_0 - C_{i-1} u_{i+1}}{C_i}, \tag{30}$$

with $i = 1$ to $N - 2$, and where we have used the same coefficients as in the case of the elevation, $C_0 = 1$, $C_1 = \Phi_1$ and $C_i = \Phi_i C_{i-1} - \gamma_i C_{i-2}$, with $i = 2$ to $i = N - 2$.

The velocity in the $i = N - 2$ mesh point, u_{N-2}, depends on the velocity at the entrance of the channel, u_0, and the velocity u_{N-1} by the expression

$$u_{N-2} = \frac{(-1)^{N-2} (\prod_{j=1}^{N-2} \gamma_j) u_0 - C_{N-3} u_{N-1}}{C_{N-2}}. \tag{31}$$

Using the latest equation of the tridiagonal system (29) and the boundary conditions (27) and (28), we can obtain an expression for calculating the value of the velocity at the point immediately before the closed border, which is

$$u_{N-1} = \frac{(-1)^{N-1} (\prod_{j=1}^{N-1} \gamma_j) u_0}{C_{N-2} \phi_{N-1} - C_{N-3} \gamma_{N-1}}. \tag{32}$$

Therefore, we can calculate the unknown velocities in the rest of the mesh points, replacing the values of the velocities backwards, using equation (29), from $i = N - 2$ to $i = 1$.

5. Numerical examples

In this section, two numerical examples are presented in order to show the effectiveness of the algorithm to solve the shallow water equations in 1D (1). In the first example we assume a flat bottom bathymetry. In this case, the analytical solution of the problem is known. In the second case, the bathymetry has been modelled by a Gaussian function.

5.1. Flat bottom case

Suppose we have a channel with a flat bottom bathymetry, i.e. a constant depth $H_i = H$, as it is shown in figure 3.

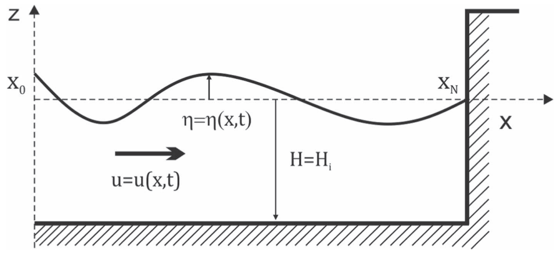

Figure 3. Model with flat bathymetry.

This reduces the system (1) to a Helmholtz equation, given by the following equations:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\omega^2}{gH} u = 0, \tag{33}$$

$$\frac{\partial^2 \eta}{\partial x^2} + \frac{\omega^2}{gH} \eta = 0. \tag{34}$$

The wave number $k = 2\pi/L$, where L is a characteristic wavelength, is defined in this setting by the relation

$$k^2 = \frac{\omega^2}{gH}.$$

The above formula is motivated by the well-known dispersion relation $c = \sqrt{gH}$ for shallow water waves; for linear waves, the wave phasespeed is obtained by way of the relation $c = \omega/k$ [1, 2].

We consider the boundary conditions (12), (27) and (28); the analytical solution to the equations (33) and (34) is

$$u(x) = u_0(\cos(kx) - \cot(k)\sin(kx)), \tag{35}$$

$$\eta(x) = \eta_0(\cos(kx) + \tan(k)\sin(kx)). \tag{36}$$

To verify that the result of the numerical method, presented in this paper, fits the analytical result, we use the equations (23) to (32), in section 4, to calculate the velocity u_i considering that H is constant. Given this, the parameters used in the method must be modified to comply with this condition. Thus, equations (25) and (26) become

$$\gamma = 1,$$

$$\phi = -2 + \frac{\omega^2 \Delta x^2}{gH},$$

and the coefficients C_i are, in this case, $C_1 = 1$, $C_2 = \phi$, and $C_i = \phi C_{i-1} - C_{i-2}$, with $i = 2$ to $i = N - 2$. Thus, the equation (30) becomes

$$u_i = \frac{(-1)^i u_0 - C_{i-1} u_{i+1}}{C_i}. \tag{37}$$

Taking into account the above conditions and following the procedure used in section 4, we can generalize the solution to find the value of u_i for the case in which we consider bathymetry with a flat bottom:

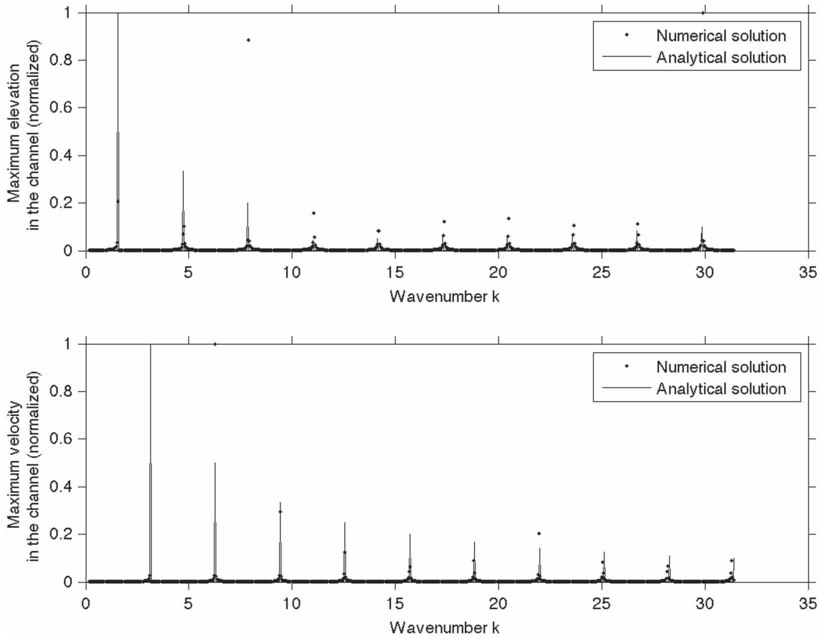

Figure 4. Comparison between analytical and numerical results for elevation (a) and velocity (b) for the flat bottom case.

$$u_{N-1} = \frac{(-1)^{N-1}u_0}{\Phi C_{N-2} - C_{N-3}}. \tag{38}$$

Hence, we can obtain the rest of the unknowns by back-substitution using the equation (37), with $i = N - 2$ to $i = 1$. The same procedure is used to calculate the elevation η_i numerically, with constant depth H_i. Figure 4 shows the comparison between the numerical and analytical results for elevation and velocity. The simulation was performed for different lengths of the incident wave to visualize the points where resonance occurs.

As has been shown, both the analytical solution and the numerical one coincide in the same points where resonance occurs, so we can say that the model works properly for bathymetry with a flat bottom.

5.2. Gaussian modelled bottom case

In this section we consider that the bathymetry presents an elevation like a dome, which can be modelled using a Gaussian function (see figure 5) given by the following equation:

$$H(x) = H_o - (A * \exp(-((x - x_m)/\sigma)^2)), \tag{39}$$

where the value H_0 represents the maximum depth of the channel; A is the maximum amplitude of the Gaussian with respect to the channel bottom, which models the bathymetric high; x_m is the central position of the channel; and σ corresponds to half the width of the Gaussian function and hence to half the width of the bathymetric high.

To calculate numerically the elevation η_i and the velocity u_i for this case, we use the equations presented in sections 3 and 4. We can compare the resulting maximum resonance frequencies with those obtained in the case of flat bottom bathymetry, as shown in figure 6.

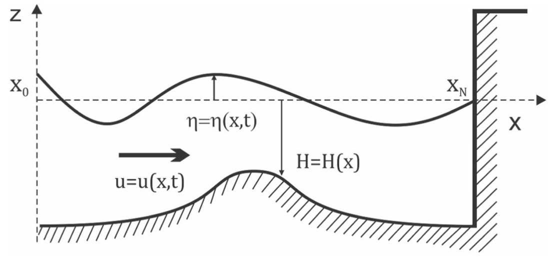

Figure 5. Bathymetry with the presence of a dome, modelled as a Gaussian function.

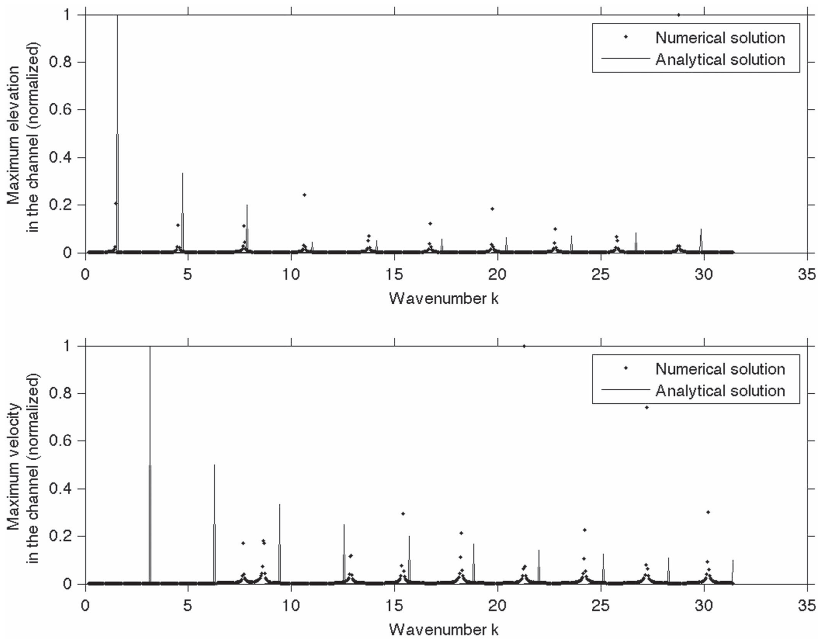

Figure 6. Comparison between the maximum resonance frequencies for the constant depth case and the variable depth modelled by a Gaussian function, for elevation (a) and velocity (b), where the solid line represents the analytic solution for the flat bottom case, and the dotted line represents the bottom solution with the Gaussian model.

We can see that the maximum frequencies do not match because there is a change in the resonance frequencies within the channel due to the existence of the high.

6. Conclusions

In this paper, we have proposed an efficient numerical method to solve tridiagonal systems using a recursive form, based on the ANA method [23], with little memory requirement. This method works adequately for the problem of searching the resonant modes in an open channel with variable depth. We found that the variations in the depth of the channel can produce changes in the position of the expected resonances with respect to the wave number. This becomes more evident in the case of the fluid velocity.

This new method is easy to use by undergraduate students. As a didactic exercise, they can deduce the recurrence equation. This will give them the opportunity to experiment with a simple numerical method and compare the results with the traditional techniques used for this type of problem.

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