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Manual of BlackMax. A black-hole event generator with rotation, recoil, split branes, and brane tension. Version 2.02 ☆

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ABSTRACT

This is the users manual of the black-hole event generator BlackMax (Dai et al., 2008), which simulates the experimental signatures of microscopic and Planckian black-hole production and evolution at proton–proton, proton–antiproton and electron–positron colliders in the context of brane world models with low-scale quantum gravity. The generator is based on phenomenologically realistic models free of serious problems that plague low-scale gravity. It includes all of the black-hole gray-body factors known to date and incorporates the effects of black-hole rotation, splitting between the fermions, non-zero brane tension and black-hole recoil due to Hawking radiation (although not all simultaneously). The main code can be downloaded from Dai et al. (0000).

Program summary

Program title: BlackMax
Program Files doi: http://dx.doi.org/10.17632/p9jg9dypcg.1
Licensing provisions: GNU General Public License version 3
Programming language: C (with *Fortran* subroutines)
Nature of problem: In the class of models with low scale quantum gravity (known as the "TeV scale gravity models") collisions of particles at the particle accelerators may lead to novel phenomena, in particular mini black hole production. In order to confirm or exclude this class of models, one needs to calculate the probability of the black hole production in collisions of particles, properties of the formed black holes (mass, spin, charge, momentum), and the signature of the black hole decay (Hawking radiation).
Solution method: BlackMax calculates the probability of the black hole production by utilizing the so-called "geometric cross section" for black hole production. From the energy and quantum numbers of the colliding particles BlackMax calculates the mass, spin, charge, and momentum of the formed black holes. In the next step, BlackMax utilizes the greybody factors that characterize Hawking radiation and calculates the final output. The produced particles are then supposed to leave the signature in particle detectors.
References: Phys. Rev. D **77**, 076007 (2008)

Theoretical background summary

Models with TeV-scale quantum gravity offer very rich collider phenomenology. Most of them assume the existence of a three-plus-one-dimensional hypersurface, which is referred as "the brane," where Standard-Model particles are confined, while only gravity and possibly other particles that carry no gauge quantum numbers, such as right handed neutrinos, can propagate in the full space, the so-called "bulk". Under certain assumptions, this setup allows the fundamental quantum gravity energy scale, M_*, to be close to the electroweak scale. The observed weakness of gravity compared to other forces on the brane (i.e. in the laboratory) is a consequence of the large volume of the bulk which dilutes the strength of gravity. In the context of these models of TeV-scale quantum gravity, probably the most exciting new

☆ This paper and its associated computer program are available via the Computer Physics Communication homepage on ScienceDirect (http://www.sciencedirect.com/science/journal/00104655).

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physics is the production of micro-black-holes in near-future accelerators like the Large Hadron Collider (LHC). According to the "hoop conjecture", if the impact parameter of two colliding particles is less than two times the gravitational radius, rh, corresponding to their center-of-mass energy (ECM), a black-hole with a mass of the order of ECM and horizon radius, r_h, will form. Typically, this gravitational radius is approximately ECM/M_*^2. Thus, when particles collide at center-of-mass energies above M_*, the probability of black-hole formation is high.

Once a black-hole is formed, it is believed to decay via Hawking radiation. This Hawking radiation will consist of two parts: radiation of Standard-Model particles into the brane and radiation of gravitons and any other bulk modes into the bulk. The relative probability for the emission of each particle type is given by the gray-body factor for that mode. This gray-body factor depends on the properties of the particle (charge, spin, mass, momentum), of the black-hole (mass, spin, charge) and, in the context of TeV-scale quantum gravity, on environmental properties such as the number of extra dimensions, the location of the black-hole relative to the brane (or branes), etc. In order to properly describe the experimental signatures of black-hole production and decay one must therefore calculate the gray-body factors for all of the relevant degrees of freedom.

Since a black hole can emit particles like quarks and gluons which cannot freely propagate long distance, one has to simulate the process of hadronization. The generator can be interfaced with hadronization generators Herwig and Pythia to obtain the final signature measurable in particle detectors.

1. Main features

BlackMax is a very versatile semi classical and quantum black-hole generator which simulates a number of different extra dimension models and black-hole evolution scenarios. It also gives the user the possibility to set many of the parameters which influence the formation and decay of black holes. The manual intends to explain the various parameters the user can set. We also give an extensive reference list, reflecting the great interest in the topic, [1–133], which however is but by no means complete due to the fast evolution of this field.

The main features of BlackMax version 2.00 are listed below and will be discussed in the upcoming sections in more detail.

1. BlackMax is able to simulate black-hole production in proton–proton, proton–anti-proton and electron–positron collisions.
2. The user is able to choose among different Planck scale conventions or use his/her own convention.
3. Graviton emission can be simulated.
4. The user can define if baryon, lepton numbers or flavor should be conserved or not. **In case the user does not conserve baryon numbers BlackMax events cannot be hadronized with Pythia or HERWIG.**
5. *BlackMax can be interfaced to the parton density functions of the LHAPDF package [3].*
6. The user can choose to use the Yoshino–Rychkov cross section enhancement factors and simulate the energy loss before the formation of the event horizon as described in [4]. These corrections to the cross section and the energy loss are only applicable for non-tension models and BlackMax will not apply any corrections or modifications to the energy loss if the user turns the Yoshino–Rychkov corrections on for a positive tension brane scenario.
7. The default BlackMax outputs are MaxLHArecord.txt [5] and output.txt. Details about these two files can be found in Section 8.
8. Graviton and photon emission is now also included into the balding phase before the black-hole is formed. If the user has turned on graviton emission and chooses to use the Yoshino–Rychkov suppression factors or sets the mass loss factor to a non-zero value, BlackMax sheds this energy/mass by emitting two gravitons before the formation of the black-hole. If the user has disabled graviton emission BlackMax will shed this energy by emitting two photons.
9. If the user simulates the split-fermion scenario BlackMax takes into account the position of the black-hole remnant when it calculates which particles it should decay into during the final burst phase.
10. Users should note that the output format has changed with respect to the description in [1] and is described in Section 8.2.

2. Installation

The most up-to-date source code and *tarball* can be downloaded from:

http://projects.hepforge.org/blackmax/

Having downloaded the zipped tar file you will need to unzip it, then extract the files and make the executable:

```
gunzip BlackMax-2.02.0.tar.gz
tar -xvf BlackMax-2.02.0.tar
```

Before compilation you will need to check the compiler version of gcc you are using. The gcc compiler version (4.1.2) has changed the names of some system libraries needed to compile Fortran with C code. The download is configured to use gcc version 4. If you have an older gcc version e.g. 3.4.6 then you will need to modify the BlackMax Makefile. Versions higher than 4 should not have this problem. You can check your compiler version by doing the following:

```
gcc --version
```

Which will generate output like this:

```
gcc (GCC) 3.4.6 20060404 (Red Hat 3.4.6--10)
```

```
Copyright (C) 2006 Free Software Foundation, Inc.
...
```

You will need to change the Fortran system library names in the Makefile in case your compiler version is an older one. Do this by uncommenting the following lines in the Makefile

```
F77LIB =gfortran
F77COMP=gfortran
```

You are now ready to compile BlackMax.

There are three different ways to run BlackMax:

1. in standalone mode — no additional libraries required
2. accessing PDFs from LHAPDF
3. accessing PDFs from LHAPDF and simultaneous hadronisation from Pythia

To produce the executable for each method simply requires a different compilation/linking step and is described below. In all three options the default format of the event output is the Les Houches Accord format [5]. This text file can be used as input to HERWIG/Pythia to hadronize the BlackMax events at a later date[1]

2.1. To run in standalone mode

In this version the proton parton densities are taken from CTEQ6m which are packaged with BlackMax. After unpacking simply do:

```
gmake BlackMaxOnly
```

modify parameter.txt to pick one of the 41 CTEQ6m PDF sets that has been bundled with BlackMax, for example:

```
choose_a_pdf_file(200_to_240_cteq6)Or_>10000_for_LHAPDF
200
```

Then run the executable

```
BlackMax
```

2.2. To run with LHAPDF

This version of the executable uses the proton parton densities from the LHAPDF library which you will need to download and install from here:

http://projects.hepforge.org/lhapdf/

For LHAPDF please ensure you install the package in a directory where you have write permission. You can do this by specifying an installation directory — for more info see the LHAPDF manual. Edit the BlackMax Makefile and insert the library locations. Ensure that your `LD_LIBRARY_PATH` environment variable includes the location of your newly built LHAPDF library:

```
export LD_LIBRARY_PATH = $LD_LIBRARY_PATH : /data/rizvi/atlas/lhapdf - 5.3.0/lhapdf/lib
export LHAPATH = /data/rizvi/atlas/lhapdf - 5.3.0/lhapdf/share/lhapdf/PDFsets
```

BlackMax can be built against LHAPDF6. The only modification to the procedure in the README is to replace LHAPATH with `LHAPATH_DATA_PATH`. Ensure you have chosen a valid PDF set in parameter.txt, for example to choose the LHAPDF partons from the H1 PDF2000 fit of HERA data:

```
choose_a_pdf_file(200_to_240_cteq6)Or_>10000_for_LHAPDF
70050
```

After unpacking the source files do:

```
gmake BlackMax
```

Once the executable is created you can run the program:

```
BlackMax
```

2.3. To run with simultaneous pythia hadronisation

In order the hadronize the events during the generation job BlackMax comes with an interface to Pythia. The user can only run BlackMax simultaneously with Pythia if s/he is conserving baryon numbers (see Section 8.1, bullet item 29). In order to generate fully hadronized events you will need to download and install the latest versions of LHAPDF and PYTHIA. They are available at:

[1] Only if baryon numbers are conserved, please see Section 8.1, bullet item 29.

http://www.hepforge.org/downloads/pythia6
http://www.hepforge.org/downloads/lhapdf

BlackMax has been tested with Pythia 6.4.10 and LHAPDF 5.3.0. Install Pythia and LHAPDF according to the instructions. With Pythia, create the libraries, but remove the following dummy routines from Pythia:

upinit.f
upevnt.f
pdfset.f
structm.f

You will also need to remove the mention of the pdfset.f routine from the Pythia Makefile. The four routines above are all dummy routines which actually exist in LHAPDF. Edit the BlackMax Makefile and insert the library locations. Ensure that your `LD_LIBRARY_PATH` environment variable includes the location of your newly built Pythia and LHAPDF libraries:

export `LD_LIBRARY_PATH = $LD_LIBRARY_PATH : /data/rizvi/atlas/lhapdf − 5.3.0/lhapdf/lib`
export `LHAPATH = /data/rizvi/atlas/lhapdf − 5.3.0/lhapdf/share/lhapdf/PDFsets`

Then create the BlackMax executable using the target "all" which will link to the Pythia and LHAPDF libraries:

```
gmake all
```

Ensure you have chosen a valid PDF set in parameter.txt, for example:

```
choose_a_pdf_file(200_to_240_cteq6)Or_>10000_for_LHAPDF
10050
```

then run the executable:

```
BlackMax
```

3. Black-Hole production

We assume that the fundamental quantum-gravity energy scale M_* is not too far above the electroweak scale. Consider two particles colliding with a center-of-mass energy E_{CM}. They will also have an angular momentum J in their center-of-mass (CM) frame. By the hoop conjecture [6], if the impact parameter, b, between the two colliding particles is smaller than the diameter of the horizon of a $(d+1)$-dimensional black-hole (where d is the total number of space-like dimensions) of mass $M = E_{CM}$ and angular momentum J,

$$b < 2r_h(d, M, J), \tag{1}$$

then a black-hole with r_h will be formed. The cross section for this process is approximately equal to the interaction area $\pi(2r_h)^2$.

We consider a black hole with only one angular momentum. A new parameter a is introduced to describe the angular momentum (Eq. (6)). In Boyer–Lindquist coordinates, the metric for a $(d+1)$-dimensional rotating black-hole (with angular momentum parallel to the $\hat{\omega}$ in the rest frame of the black-hole) is:

$$\begin{aligned} ds^2 = &\left(1 - \frac{\mu r^{4-d}}{\Sigma(r,\theta)}\right) dt^2 \\ &- \sin^2\theta \left(r^2 + a^2\left(1 + \sin^2\theta \frac{\mu r^{4-d}}{\Sigma(r,\theta)}\right)\right) d\phi^2 \\ &+ 2a\sin^2\theta \frac{\mu r^{4-d}}{\Sigma(r,\theta)} dt d\phi - \frac{\Sigma(r,\theta)}{\Delta} dr^2 \\ &- \Sigma(r,\theta) d\theta^2 - r^2\cos^2\theta d^{d-3}\Omega \end{aligned} \tag{2}$$

where μ is a parameter related to mass of the black-hole (Eq. (5)), while

$$\Sigma = r^2 + a^2 cos^2\theta \tag{3}$$

and

$$\Delta = r^2 + a^2 - \mu r^{4-d}. \tag{4}$$

The mass of the black-hole is

$$M = \frac{(d-1)A_{d-1}}{16\pi G_d}\mu, \tag{5}$$

and

$$J = \frac{2Ma}{d-1} \tag{6}$$

is its angular momentum. Here,

$$A_{d-1} = \frac{2\pi^{d/2}}{\Gamma(d/2)} \tag{7}$$

is the hyper-surface area of a $(d-1)$-dimensional unit sphere. The higher-dimensional gravitational constant G_d is defined as

$$G_d = \frac{(2\pi)^{d-4}}{4M_\star^{d-1}}. \tag{8}$$

M_* is the higher dimensional Planck mass (the user is able to choose other Planck scale conventions, different than the one in Eq. (8), see Section 8). The horizon occurs when $\Delta = 0$. That is at a radius given implicitly by

$$r_h^{(d)} = \left[\frac{\mu}{1+(a/r_h^{(d)})^2}\right]^{\frac{1}{d-2}} = \frac{r_s^{(d)}}{\left[1+(a/r_h^{(d)})^2\right]^{\frac{1}{d-2}}}. \tag{9}$$

Here

$$r_s^{(d)} \equiv \mu^{1/(d-2)} \tag{10}$$

is the Schwarzschild radius of a $(d+1)$-dimensional black-hole, *i.e.* the horizon radius of a non-rotating black-hole. Eq. (10) can be rewritten as:

$$r_s^{(d)}(E_{CM}, d, M_*) = k(d)M_*^{-1}[E_{CM}/M_*]^{1/(d-2)}, \tag{11}$$

where

$$k(d) \equiv \left[2^{d-3}\pi^{(d-6)/2}\frac{\Gamma[d/2]}{d-1}\right]^{1/(d-2)}. \tag{12}$$

The Hawking temperature of a black-hole is

$$T_H = \frac{d-2}{4\pi r_h}. \tag{13}$$

If two highly relativistic particles collide with center-of-mass energy E_{CM}, and impact parameter b, then their angular momentum in the center-of-mass frame before the collision is $L_{in} = bE_{CM}/2$. Suppose for now that the black-hole that is formed retains all this energy and angular momentum. Then the mass and angular momentum of the black-hole will be $M_{in} = E_{CM}$ and $J_{in} = L_{in}$. *A black-hole will be formed if:*

$$b < b_{\max} \equiv 2r_h^{(d)}(E_{CM}, b_{max}E_{CM}/2). \tag{14}$$

We see that b_{max} is a function of both E_{CM} and the number of extra dimensions.

We can rewrite condition (14) as (see [7])

$$b_{max}(E_{CM}; d) = 2\frac{r_s^{(d)}(E_{CM})}{\left[1+\left(\frac{d-1}{2}\right)^2\right]^{\frac{1}{d-2}}}. \tag{15}$$

There is one exception to this condition. In the case where we are including the effects of the brane tension, the metric (and hence gray-body factors) for a rotating black-hole are not known. Gray-body factors are the absorption coefficients for the black hole potential barrier. They define the probability for a particle to penetrate the potential barrier and propagate out. In this case we consider only non-rotating black-holes. For the model with non-zero tension brane, the radius of the black-hole is defined as

$$r_h = \frac{r_s}{B^{1/3}}, \tag{16}$$

with B the deficit-angle parameter which is inversely proportional to the tension of the brane $B = 1 - \frac{\lambda}{2\pi M_d^{d-2}}$ (see Eq. (8) in [8]). Therefore, for branes with tension

$$b_{max}^{tension}(E_{CM}, d) = 2r_h^{(d)}(E_{CM}). \tag{17}$$

Also, for branes with positive tension only the $d = 5$ metric is known.

4. Simulated scenarios in BlackMax

BlackMax is able to simulate several different scenarios.

- *Non-rotating black-hole on a tensionless brane:* For a non-rotating black-hole, we used previously known gray-body factors for spin 0, $\frac{1}{2}$ and 1 fields in the brane, and for spin 2 fields (*i.e.* gravitons) in the bulk.
- *Rotating black-hole on a tensionless brane:* For rotating black-holes, we used known gray-body factors for spin 0, $\frac{1}{2}$ and 1 fields on the brane. The correct emission spectrum for spin 2 bulk fields is not yet known for rotating black-holes, we currently do not allow for the emission of bulk gravitons from rotating black-holes.
- *Non-rotating black-holes on a tensionless brane with fermion brane splitting:* In the split-fermion models, gauge fields can propagate through the bulk as well as on the brane, so we have calculated gray-body factors for spin 0 and 1 fields propagating through the bulk, but only for a non-rotating black-hole for the split-fermion model [9].
- *Non-rotating black-holes on a non-zero tension brane:* The bulk gray-body factors for a brane with non-zero tension are affected by non-zero tension because of the modified bulk geometry (deficit angle). We have calculated gray-body factors for spin 0, 1 and 2 fields propagating through the bulk, again only for the non-rotating black-hole for a brane with non-zero tension and $d = 5$.

- *Two particle final states:* We use the same gray-body factors as a non-rotating black-hole to calculate the cross section of two-particle final states (excluding gravitons according to [10]).
- *String ball:* We use the same gray-body factors as for non-rotating black-holes to calculate the emission in the string ball case [11].

5. The black-hole formation

The formation of the black-hole is a very non-linear and complicated process. BlackMax gives the user the possibility to set parameters which affect the formation phase of the black-hole by hand or use corrections calculated by Yoshino and Rychkov [4].

If the user would like to set the formation parameters her/himself BlackMax assumes that, before settling down to a stationary phase, a black-hole loses some fraction of its energy, linear and angular momentum. *These losses are parameterized by three parameters:* $1 - f_E$, $1 - f_P$ *and* $1 - f_L$, *which are* `Mass\_loss\_factor`, `momentum\_loss\_factor` and `Angular\_momentum\_loss_factor` *in the parameter.txt file correspondingly*. Thus the black-hole initial state that we actually evolve is characterized by

$$\begin{aligned} E &= E_{\rm in} f_{\rm E}; \\ P_z &= P_{\rm z,in} f_{\rm P}; \\ J' &= L_{\rm in} f_{\rm L}; \end{aligned} \tag{18}$$

where $E_{\rm in}$, $P_{\rm z,in}$ and $L_{\rm in}$ are initial energy, momentum and angular momentum of colliding partons, while $f_{\rm E}$, $f_{\rm P}$ and $f_{\rm L}$ are the fractions of the initial energy, momentum and angular momentum that are retained by the stationary black-hole.

6. Black-Hole evolution in BlackMax

The Hawking radiation spectra are calculated for the black-hole at rest in the center-of-mass frame of the colliding partons. The spectra are then transformed to the laboratory frame as needed. In all cases we have not (yet) taken the charge of the black-hole into account in calculating the emission spectrum, but have included phenomenological factors to account for it as explained below.

6.1. Electric and color charge suppression

A charged and highly rotating black-hole will tend to shed its charge and angular momentum. Thus, emission of particles with charges of the same sign as that of the black-hole and angular momentum parallel to the black-hole's will be preferred. Emission of particles that increase the black-hole's charge or angular momentum should be suppressed. *Therefore a charged black hole prefers to release particles with the same charges and a rotating black hole prefers to release particles which take away angular momentum. These preference emissions will keep going until the black hole becomes neutral.* The precise calculation of these effects has not as yet been accomplished. Therefore, to account for these effects we allow optional phenomenological suppression factors for both charge and angular momentum.

The following charge-suppression factors can currently be used by setting parameter `charge_suppression` (cf. Section 8) equal to 2.

$$F^{\rm Q} = \exp(\zeta_Q Q^{bh} Q^{em}) \tag{19}$$

$$F^3_a = \exp(\zeta_3 c^{bh}_a c^{em}_a) \qquad {\rm a} = {\rm r, b, g}. \tag{20}$$

$F^{\rm Q}$ and F^3 *are electric charge and color charge suppressed factor respectively.* Q^{bh} is the electromagnetic charge of the black-hole, Q^{em} is the charge of the emitted particle; c^{bh}_a, is the color value for the color a, with a = r, b, g, of the black-hole, and c^{em}_a, is the color value for the color a, with a = r, b, g, of the emitted particle. ζ_Q and ζ_3 are phenomenological suppression parameters that are set as input parameters of the generator.

We assume $\zeta_Q = \mathcal{O}(\alpha_{em})$ *and* $\zeta_3 = \mathcal{O}(\alpha_s)$, *where* α_{em} *and* α_s *are the values of the electromagnetic and strong couplings at the Hawking temperature of the black-hole.* Note that we currently neglect the possible restoration of the electroweak symmetry in the vicinity of the black-hole when its Hawking temperature is above the electroweak scale. Clearly, since $\alpha_{em} \simeq 10^{-2}$ we do not expect electromagnetic (or more correctly) electroweak charge suppression to be a significant effect. However, since $\alpha_s(1\,{\rm TeV}) \simeq 0.1$, color suppression may well play a role in the evolution of the black-hole.

Once we have determined the type of particle to be emitted by the black-hole, we draw a random number N_r between 0 and 1 from a uniform distribution. If $N_r > F^{\rm Q}$ then the emission process is allowed to occur, if $N_r < F^{\rm Q}$ then the emission process is aborted. We repeat the same procedure for color suppression factor, F^3_a. Thus, particle emission which decreases the magnitude of the charge or color of the black-hole is unsuppressed; this suppression prevents the black-hole from acquiring a large charge/color, and gives preference to particle emission which reduces the charge/color of the black-hole.

6.2. Angular momentum suppression

Since the TeV black-holes are quantum black-holes, the gray-body factors should really depend on both the initial and final black-hole parameters. The calculation of the gray-body spectra on a fixed background can cause some problems. In particular, in the current case, the angular momentum of the emitted particle (as indeed the energy) may well be comparable to that of the black-hole itself. There should be a suppression of particle emission processes in which the black-hole final state is very different from the initial state. We therefore introduce a new phenomenological suppression factor, parameter `L_suppression`, to reduce the probability of emission in which the angular momentum of the black-hole changes by a large amount.

If parameter `L_suppression` is equal to 1 (cf. Section 8), BlackMax does suppress the increase of the angular momentum of the black-holes. There are three angular momentum suppression models BlackMax is able to simulate.

1. ΔArea suppression:
   If the user sets `L_suppression` equal to 2 the ΔArea suppression model is used where the suppression factor is defined as

$$F^L = \exp(\zeta_L(r_h^{bh}(t+\Delta t)^2/r_h^{bh}(t)^2 - 1)), \tag{21}$$

   here Δt is the next time step in the simulation.
2. J_{bh} suppression: By setting `L_suppression` equal to 3 the J_{bh} suppression model is used and the angular suppression factor is defined as

$$F^L = \exp(-\zeta_L |J^{bh}(t+\Delta t)|). \tag{22}$$

3. ΔJ_{bh} suppression: For the ΔJ_{bh} suppression model `L_suppression` has to be equal to 4 and

$$F^L = \exp(-\zeta_L |J^{bh}(t+\Delta t) - J^{bh}(t)|). \tag{23}$$

We might expect $\zeta_L \sim 1$, however there is no detailed theory to support this; as indeed there is no detailed theory to choose among these three phenomenological suppression scenarios.

7. Final burst of black-holes in BlackMax

In the absence of a self-consistent theory of quantum gravity, the last stage of the evaporation cannot be described accurately. Once the mass of black-hole becomes close to the fundamental scale M_*, the classical black-hole solution can certainly not be used anymore. We adopt a scenario in which the final stage of evaporation is a burst of particles which conserves energy, momentum[2] and all of the gauge quantum numbers. For definiteness, we assume the remaining black-hole will decay into the lowest number of Standard-Model particles that conserve all quantum number, momentum and energy.

Black-holes do not conserve global quantum numbers, like flavor, baryon or lepton numbers [12]. If the user would like to conserve global quantum numbers s/he can set this in the parameter file (see Section 8.1).

8. Input and output

8.1. Input

The input parameters for the generator are read from the file parameter.txt, see Fig. 1. In the following bulleted list an explanation is given for each of the input parameters in parameter.txt.

1. `Number_of_simulations`: sets the total number of black-hole events to be simulated;
2. `Center_of_mass_energy_of_protons`: sets the center-of-mass energy of the colliding protons in GeV;
3. `M_ph`: sets the value of the fundamental quantum-gravity scale (M_*) in GeV;
4. `definition_of_M_pl:(1:M_D_2:M_p_3:M_DL_4:put_in_by_hand)`: this sets the definition of the used fundamental quantum-gravity scale;
   - 1 = Convention according to [13] and first reference of [14] (PDG definition).
   - 2 = Convention according [15], which is useful in quoting experimental bounds (Giddings and Thomas).
   - 3 = Convention given by Dimopoulos and Landsberg; $M_{DL}^{D-2} = 1/G_D$, with D the total number of dimensions and G_D the D-dimensional Newton gravity constant.
   - 4 = User defined convention. In this case BlackMax will read the value of the next parameter `if_definition==4` to set the scale. The value of `if_definition==4` is $k(d)$ in Eq. (11). Note that $k(d)$ depends on the number of total space dimensions and the user has to make sure to that the value of this parameter is appropriately updated when the user simulates black holes in scenarios with different number of extra dimensions.
5. `Choose_a_case`: defines the extra dimension model to be simulated:
   - 1 = non-rotating black-holes on a tensionless brane with possibility of fermion splitting,
   - 2 = non-rotating black-holes on a brane with non-zero positive tension,
   - 3 = rotating black-holes on a tensionless brane with $d = 5$,
   - 4 = two-particle final-state scenario;
6. `number_of_extra_dimensions`: sets the number of extra dimensions; this must equal 2 for branes with non-zero positive tension (`Choose_a_case=2`);
7. `number_of_splitting_dimensions`: sets the number of extra split-fermion dimensions (`Choose_a_case=1`);
8. `extradimension_size`: sets the size of the mini-bulk[3] in units of 1/TeV (`Choose_a_case=1`); Please also refer to the discussion under item number 28.
9. `tension`: sets the deficit-angle parameter B [8,16]; typical values for branes with tension are from 1 to 0.9. (`Choose_a_case=2`);

[2] Black holes are not elementary particles and do not conserve spin and angular momentum separately. Hence spin is not separately conserved in BlackMax, only the total angular momentum is.

[3] This is the distance between fermion branes where only gauge bosons and Higgs field can propagate in split-fermion brane scenario.

```
Number_of_simulations
10
incoming_particle(1:pp_2:ppbar_3:ee+)
2
Center_of_mass_energy_of_incoming_particle
14000
M_ph(GeV)
1000.
definition_of_M_pl:(1:M_D_2:M_p_3:M_DL_4:put_in_by_hand)
1
if_definition==4
1.
Choose_a_case:
(1:tensionless_nonrotating_2:tension_nonrotating_3:rotating_nonsplit_
4:Lisa_two_particles_final_states)
1
number_of_extra_dimensions
4
number_of_splitting_dimensions
1
extradimension_size(1/Mpl)
10.
tension(parameter_of_deficit_angle:1_to_0)
1.0
choose_a_pdf_file(200_to_240_cteq6)Or_>100_for_LHAPDF
10050
Minimum_mass(GeV)
5000.
Maxmum_mass(GeV)
15000.
fix_time_step(1:fix_2:no)
2
time_step(1/GeV)
1.e-5
do_yo_shino(1:do_0:no)
0
Mass_loss_factor(0~1.0)
0.00
momentum_loss_factor(0~1.0)
0.0
Angular_momentum_loss_factor(0~1.0)
0.0
turn_on_graviton(0:off_1:on)
1
Seed
123589341
Write_LHA_Output_Record?_0=NO__1=Yes
1
L_suppression(1:none_2:delta_area_3:anular_momentum_4:delta_angular_momentum)
1
angular_momentum_suppression_factor
1
charge_suppression(1:none_2:do)
1
charge_suppression_factor
1
color_suppression_factor
20
split_fermion_width(1/Mpl)_and_location(from-15to15)(up_to_9extradimensions)
u_quark_Right(Note:do_not_insert_blank_spaces)
1.0
10.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0,0.0
u_quark_Left(Note:do_not_insert_blank_spaces)
1.0
       .
       .
       .

number_of_conservation
2
d,s,b,u,c,t,e,mu,tau,nu_e,nu_mu,nu_tau
1,1,1,1,1,1,0,0,0,0,0,0,
0,0,0,0,0,0,1,1,1,1,1,1
0,0,0,0,0,0,0,0,0,0,0,0
0,0,0,0,0,0,0,0,0,0,0,0
0,0,0,0,0,0,0,0,0,0,0,0
0,0,0,0,0,0,0,0,0,0,0,0
```

Fig. 1. parameter.txt is the input file containing the parameters that the user can set.

10. `choose_a_pdf_file(200_to_240_cteq6)Or_>100_for_LHAPDF`: The user can choose to run BlackMax with the bundled CTEQ6.1m PDFs [17,18] which come with the BlackMax release (see subdirectory cteq_pdf) or to run BlackMax with the PDFs of the LHAPDF package. In case the user is running BlackMax in standalone mode s/he has to set this variable in parameter.txt between 200 and 240 to choose one of the 41 CTEQ6.1M PDF sets. It is recommended to use 200 — the others correspond to PDF uncertainties. If the user wants to use the PDFs of the LHAPDF package the user must link the BlackMax executable to the LHAPDF library (see Section 2.2). The input parameter to be given here then is the chosen LHAPDF ID number. The definition of the LHAPDF ID numbers can be found in [3].
11. `Minimum_mass`: sets the minimum mass M_{min} in GeV of the initial black-holes;
12. `Maxmum_mass`: sets the maximum mass M_{max} in GeV of the initial black-holes;
13. `Include_string_ball:(1:no_2:yes)`: Decides whether the code includes string balls in a suitable energy range.

- 1 = does not include string balls
- 2 = does include string balls. This case can only be executed under non-rotating black hole case and cannot include the Rychkov cross section enhancement factors.

14. `String_scale(M_s)(GeV)`: Sets the energy scale of a string ball.
15. `string_coupling(g_s)`: Sets the string coupling constant. It is generally smaller than 1.
16. `The_minimum_mass_of_a_string_ball_or_black_hole(in_unit_Mpl)`: Sets the mass for a black hole or a string ball to execute the final burst stage. It is generally set equal to the Planck mass.
17. `fix_time_step`:
    - In case the user has chosen to simulate black-holes in a split fermion scenario BlackMax will simulate the production and evaporation of the black-holes in time steps. This variable affects the way the size of the time steps is calculated in the split fermion scenario. If the set equals 1, the code uses the parameter `time_step` to determine the time interval between events; if the set equals 2, then code tries to optimize the time step, keeping the probability of emitting a particle in any given time step below 10%.
    - In case the user has chosen to simulate black-holes in a non-split fermion scenario BlackMax will simulate the production and evaporation of black-holes in fix time steps if it is set to equal 1 using the input of `time_step` to set the size of the time steps. If the user would like to save computation time s/he can turn this off by setting this parameter to equal 2. In this case the code will not calculate the location of the black-hole for each time step, which speeds up the event generation.
18. `time_step`: defines the time interval Δt in GeV^{-1} which the generator will use for the black-hole evolution if `fix_time_step` = 1;
19. `other_definition_of_cross_section(0:no_1:yoshino_2:pi*r2̂_3:4pi*r2̂)`: This setting lets user choose other possible formulas for the black hole production cross section.
    - 0 : the default setting. The cross section is calculated according to what is described in the manual.
    - 1 : including Yoshino–Rychkov cross section enhancement factors. If the user would like to include the Yoshino–Rychkov cross section enhancement factors and the energy loss before the event horizon formation [4] this parameter needs to be set equal to 1. In case it is set to 1 the `Mass_loss_factor`, `momentum_loss_factor` and `Angular_momentum_loss_factor` will be ignored by BlackMax.
    - 2 : $\sigma = \pi r_s^2$
    - 3 : $\sigma = 4\pi r_s^2$
20. `Mass_loss_factor`: sets the energy loss factor $0 \geq f_E \geq 1$ as defined in Eq. (18); Recommended values are between 10% to 15%. This depends also on the minimum black-hole mass the user wished to simulate. If the energy loss factor is too big the probability to create a black-hole will be to low and BlackMax would stop the generation.
21. `momentum_loss_factor`: defines the loss factor $0 \leq f_p \leq 1$ for the momentum of initial black-holes as defined in Eq. (18); Recommended values are 10%–15%.
22. `Angular_momentum_loss_factor`: sets the loss factor $0 \leq f_L \leq 1$ for the angular momentum of initial black-holes s defined in Eq. (18); Recommended values are 10%–15%.
23. `turn_on_graviton(0:off_1:on)`: If this parameter is set to equal 1 BlackMax produces gravitons. Otherwise, BlackMax does not produce any gravitons.
24. `Seed`: This sets the seed for the random-number generator (9 digit positive integer);
25. `Write_LHA_Output_Record(0=NO, 1=Yes, 2=more detailed output)`: If the user sets this variable to 1 BlackMaxLHA.txt is written containing the full event info and output.txt will contain input parameters and cross section information only. If the user sets this variable equal to 2 BlackMaxLHA.txt is written containing full event information and output.txt will contain information about the chosen input parameters, the cross section, the particles which were emitted and the black-holes produced (see Section 8.2).
26. `L_suppression`: This chooses the model for suppressing the accumulation of large black-hole angular momenta during the evolution phase of the black-holes (cf. discussion surrounding Eqs. (21)–(23));
    - 1 = no suppression;
    - 2 = Δ Area suppression;
    - 3 = J_{bh} suppression;
    - 4 = ΔJ suppression;
27. `angular_momentum_suppression_factor`: defines the phenomenological angular-momentum suppression factor, ζ_L (cf. discussion surrounding Eqs. (21)–(23)); Recommended value is 0.2.
28. `charge_suppression`: turns the suppression of accumulation of large black-hole electromagnetic and color charge during the black-hole evolution process on or off (cf. discussion surrounding Eq. (19))
    - 1 = charge suppression turned off;
    - 2 = charge suppression turned on;
29. `charge_suppression_factor`: sets the electromagnetic charge suppression factor, ζ_Q, in Eq. (19); Recommended value is 0.2.
30. `color_suppression_factor`: sets the color charge suppression factor, ζ_3 in Eq. (19); Recommended value is 0.2.
31. `split_fermion_width(1/Mpl)_and_location(from −15to15)(up_to_9extradimensions)` In the first line after the name of the fermion, e.g. `u_quark_Right`, the user can set the width of fermion wave functions (in M_*^{-1} units). In the next line the user can set the centers of fermion wave functions (in M_*^{-1} units) in split-brane models, represented as 9-dimensional vectors (for non-split models, set all entries to 0).

To satisfy the suppression of proton decay, the quark brane and lepton brane must be separated by at least $10W$ (with W the width of fermion wavefunction in extra dimension) for one extra dimension. For higher extra dimensions, the separation can be smaller. It can be less than $10W/n$ with n the number of extra dimension ($n = d - 3$).
Gravitationally induced interactions may be able to convert neutron into anti-neutron (n-nbar oscillation). This process will introduce instability in ordinary matter made of neutrons. This decay causes matter unstable. To suppress n-nbar oscillation, the quarks have to be separated. The recommended value needs to be at least $3W/n$. to $5W/n$, with n the number of extra dimensions ($n = d - 3$).
There is no way to satisfy all the constraints at the same time for the split-fermion case. But there are several papers in which the possible position for each fermion from the CP violation constraint is calculated. [19,20] which we would like to refer the user to.

32. `number_of_conservation`: During the balding and evaporation phase, the black hole will emit a certain number of quarks and leptons and acquire global quantum numbers [12].[4] These quantum numbers are recorded by BlackMax as it reaches the final burst step, at which point it will generate further particle emissions so that the whole process, from beginning to end, will obey certain user-specified global fermion number conservation rules.

The `number_of_conservation` parameter specifies the number of conservation rules which are to follow the subsequent `d,s,b,u,c,t,e,mu,tau,nu_e,nu_mu,nu_tau` line in the parameter file. The individual conservation rules are specified as an ordered set of integral coefficients $\{a_f\}$ (conservation matrix) such that the quantity $\sum_f a_f N_f$ is a constant, where N_f is the number of particular fermion flavor.

For example, baryon number B can be conserved in BlackMax by making sure that in the final burst stage, the sum of the number of emitted u and d-type quarks compensates for the number u and d-type quarks emitted in previous stages. The conservation rule is thus

$$N_u + N_d + N_c + N_s + N_t + N_b = 3B \tag{24}$$

(recalling that quarks have a baryon number of 1/3). If this is the only additional rule, then the following lines would be entered into the parameter file:

```
number_of_conservation
1
d,s,b,u,c,t,e,mu,tau,nu_e,nu_mu,nu_tau
1,1,1,1,1,1,0,0,0,0,0,0
```

The zeros for the leptons indicate that they play no role in baryon number conservation. If, on the other hand, the user wishes to conserve lepton number L, the quark and lepton coefficients are swapped:

```
number_of_conservation
1
d,s,b,u,c,t,e,mu,tau,nu_e,nu_mu,nu_tau
0,0,0,0,0,0,1,1,1,1,1,1
```

The two rules can be combined in the parameter file to conserve B and L separately:

```
number_of_conservation
2
d,s,b,u,c,t,e,mu,tau,nu_e,nu_mu,nu_tau
1,1,1,1,1,1,0,0,0,0,0,0
0,0,0,0,0,0,1,1,1,1,1,1
```

Conservation of B − L takes the following form:

$$\frac{1}{3}(N_u + N_d + N_c + N_s + N_t + N_b) - (N_e + N_{\nu_e} + N_\mu + N_{\nu_\mu} + N_\tau + N_{\nu_\tau}) = B - L \tag{25}$$

from which parameters follow:

```
number_of_conservation
1
d,s,b,u,c,t,e,mu,tau,nu_e,nu_mu,nu_tau
1,1,1,1,1,1,-3,-3,-3,-3,-3,-3
```

Finally, if all fermion flavors are to be conserved individually, twelve equations need to be specified:

```
number_of_conservation
12
d,s,b,u,c,t,e,mu,tau,nu_e,nu_mu,nu_tau
1,0,0,0,0,0,0,0,0,0,0,0
0,1,0,0,0,0,0,0,0,0,0,0
0,0,1,0,0,0,0,0,0,0,0,0
```

4 It will also acquire angular momentum and electric and color charges, but this is not relevant for this discussion.

```
0,0,0,1,0,0,0,0,0,0,0,0
0,0,0,0,1,0,0,0,0,0,0,0
0,0,0,0,0,1,0,0,0,0,0,0
0,0,0,0,0,0,1,0,0,0,0,0
0,0,0,0,0,0,0,1,0,0,0,0
0,0,0,0,0,0,0,0,1,0,0,0
0,0,0,0,0,0,0,0,0,1,0,0
0,0,0,0,0,0,0,0,0,0,1,0
0,0,0,0,0,0,0,0,0,0,0,1
```

It should be noted that not all the possibilities the user can enter as rules are realized in BlackMax. If the user specifies a rule which BlackMax cannot implement, it will print an error message and halt.

Fig. 1 shows an example where `number_of_conservation` is set to 2 and there are 6 lines in the conservation matrix. Only the first two lines of the conservation matrix in Fig. 1 will be read and the rest will be ignored.

8.2. Output

The BlackMax code produces 3 types of output: basic information printed to screen, the BlackMaxLHArecord.txt file, and the output.txt file shown in Figs. 2–4. This is controlled by the parameter `Write_LHA_Output_Record` which can be set to 0, 1, or 2 for increasingly detailed output (see Fig. 9).

The screen dump provides basic version and input settings, cross section and timing information and can be redirected to a file. This information is always given. For parameter value 0 output.txt is written and includes a complete list of all input parameters and the calculated cross section only. For parameter value 1 (default) the BlackMaxLHArecord.txt file is additionally created containing all input parameters, followed by an event-by-event record in LHA format [5]. This file may be used as input to any other LHA compliant MC e.g. for further hadronization by Pythia or HERWIG. The modern hadronisation generators (Pythia and Herwig, and any other which accepts LHEF output as input) are supported because BlackMax does output in LHEF format. Finally for parameter value 2 the output.txt file then also contains much more detailed event-by-event information for each emission step in the black-hole decay. The information is tagged by an ID word (Begin, Parent, Pbh, trace, Pem, Pemc or Elast). Note that the energy in the output.txt file is the bulk energy of the particles where as the energy in BlackmaxLHArecord.txt is the observed energy as defined in equation 57 in [1].

- **Parent**: identifies the partons whose collision resulted in the formation of the initial black-hole (see Fig. 5).
  - column 1: identifies the black-hole;
  - column 2: PDGID code of the parton;
  - column 3: electric charge of parent parton in $3Q$;
  - column 4–6: color-charge vector components of the parent parton;
  - column 7: energy of the parton in GeV;
  - columns 8–10: brane momenta of the parton in GeV;
  - column:11: mass of the parton in GeV.
- **Pbh**: contains the evolution of the charge, color, momentum and energy of the black-holes, and, for rotating black-holes, their angular momentum (cf. Fig. 6).
  - column 1: identifies the black-hole;
  - column 2: time at which the black-hole emitted a particle;
  - column 3: PDGID code of a black-hole;
  - column 4: three times the electromagnetic charge of the black-hole;
  - columns 5–7: color-charge vector components of the black-hole;
  - columns 8: energy of the black-hole in the laboratory frame;
  - columns 9–11: brane components of the black-hole momentum in the laboratory frame;
  - columns 12 to $(8 + d)$: bulk components of the black-hole momentum;
  - column $(9 + d)$: mass of the black-hole;
  - column $(10 + d)$: angular momentum of the black-hole, in the case of rotating black-holes; empty otherwise.
- **trace**: contains the evolution history of the black-holes' positions (cf. Fig. 7):
  - column 1: identifies the black-hole;
  - column 2: the times at which the black-hole emitted a particle;
  - columns 3–5 are the brane components of the black-hole position vector when the black-hole emitted a particle;
  - columns 6 to $(2 + d)$: the bulk components of the black-hole position vector, when the black-hole emitted a particle.
- **Pem**: contains a list of particles the black-holes have emitted during its evaporation phase (cf. Fig. 8):
  - column 1: identifies the black-hole;
  - column 2: the times at which the black-hole emitted a particle;
  - column 3: PDGID code of the emitted particle;
  - column 4: three times the charge of the emitted particle;
  - columns 5–7: color-vector components of the emitted particle;
  - columns 8: energy of the emitted particle in the laboratory frame in GeV;

```
*****************************************************************
*****************************************************************
***                                                           ***
***   Welcome to:        BLACKMAX                             ***
***                                                           ***
***   A TeV-Scale-Gravity Final-State Generator               ***
***                                                           ***
***   De-Chang Dai, Glenn Starkman, Dejan Stojkovic,          ***
***   Cigdem Issever, Eram Rizvi, Jeff Tseng                  ***
***                                                           ***
***          De-Chang Dai   :  de-chang.dai@case.edu          ***
***          Cigdem Issever:  c.issever1@physics.ox.ac.uk     ***
***          Eram Rizvi     :  e.rizvi@qmul.ac.uk             ***
***                                                           ***
***   See arXiv:0711.3012 [hep-ph]                            ***
***                                                           ***
***   Release:                                                ***
***    $Name: REL_1_07 $
***    $Id: BlackMax.c,v 1.59 2008/11/03 15:56:58 rizvi Exp $
***    $Date: 2008/11/03 15:56:58 $                           ***
***                                                           ***
*****************************************************************
*****************************************************************

***** Initialising .....

***** Reading Parameters .....

*****  Nevents                  = 10
*****  Beam Particles           = PP
*****  Centre of Mass Energy    = 14000.000000 GeV
*****  M_ph                     = 1000.000000 GeV
*****  Model Type               = No Tension, Non-rotating BlackHole
*****  Number Extra Dimensions  = 4
*****  Number Split Dimensions  = 1
*****  Size   Extra Dimensions  = 0.010000
*****  Tension Parameter        = 1.000000
*****  Input PDFs               = 200
*****  Minimum BH Mass          = 5000.000000
*****  Maximum BH Mass          = 15000.000000
*****  Time Step Fixed?         = No
*****  Time Step                = 1.000000e-05 (1/GeV)
*****  Do YoShino suppression?  = No
*****  Mass Loss Factor         = 0.100000
*****  Momentum Loss Factor     = 0.000000
*****  Ang.Momentum Loss Factor = 0.000000
*****  Gravitons in FinalBurst? = Yes


 Table file = cteq_pdf/ctq61.00.tbl
 InitLHA: Finished ipdf= 200
Done initialising LHAPDF with set =     200

***** Calculating Cross Section .....

*****
***** BlackMax cross section =    5.71466e-11   +/-    5.36096e-13 barn
*****

***** BlackMax will write output to LHA compatible text files
***** Finished writing BlackMax LHA Header information - keep file open for event info...

***** Starting Event Generation

      Event Number      1
      Event Number      2
      Event Number      3
      Event Number      4
      Event Number      5
      Event Number      6
      Event Number      7
      Event Number      8
      Event Number      9
      Event Number     10
*****
***** Closed LHA event file
*****

*****
***** BlackMax clock timing info:
*****
***** Initialisation time = 12.740000 seconds
*****     Event Loop time = 1.220000 seconds for 10 events
*****     Time per event  = 0.122000 seconds
*****
```

Fig. 2. This is the basic information which is printed to screen and which can be redirected to a file. It contains information about version numbers, dates, input parameters and run time.

  - columns 9–11: brane components of the momentum of the emitted particle, in the laboratory frame in GeV;
  - columns 12 to $(8 + d)$: bulk components of the momentum of the emitted particle in GeV;
  - column $9 + d$: mass of the emitted particle in GeV.

- **Pemc**: contains the same information as Pem, but in the center-of-mass frame of the collision.
- **Begin**: contains the same information as Pem, but for the particles which are emitted before the black-hole is formed. This happens if the user chooses to use the Yoshion and Rykhov factors or sets the mass-loss-factor to a non-zero value.

```
Printout of choosen input parameters:

Number_of_simulations
10
incoming_particle(1:pp_2:ppbar_3:ee+)
1
Center_of_mass_energy_of_incoming_particle
14000.000000
      .
      .
      .
Information about cross section:

total cross section = 6.745298e-08 (GeV^-2) = 2.623921e-11 b
cross section error = 6.402274e-10 (GeV^-2) = 2.490485e-13 b
      .
      .
      .
Information about generated black holes and emitted particles:

trace     1   0.00000e+00                    -9.48767e-05  -3.13192e-04   0.00000e+00  -2.37725e-04
Pbh       1   0.00000e+00  40   4   0   1  1  6.01785e+03   0.00000e+00   0.00000e+00  -2.79088e+03   0.00000e+00   5.33156e+03
0.00000e+00
Parent    1                 4   2   0   0  1  1.61349e+03   0.00000e+00   0.00000e+00   1.61349e+03   1.20000e+00
Parent    1                 2   2   0   1  0  4.40436e+03   0.00000e+00   0.00000e+00  -4.40436e+03   3.20000e-03
trace     1   0.00000e+00                    -9.48767e-05  -3.13192e-04   0.00000e+00  -2.37725e-04
Pbh       1   0.00000e+00  40   3   1   1  1  5.76609e+03  -1.17708e+02   3.60077e+01  -2.57130e+03   0.00000e+00   5.15956e+03
0.00000e+00
Pem       1   0.00000e+00  -5   1  -1   0  0  2.51757e+02   1.17708e+02  -3.60077e+01  -2.19573e+02   0.00000e+00   4.20000e+00
Pemc      1   0.00000e+00  -5   1  -1   0  0  1.69226e+02   1.17708e+02  -3.60077e+01  -1.16051e+02   0.00000e+00   4.20000e+00

      .
      .
      .

Pem      10   0.00000e+00   5  -1   0   1  0  1.24224e+02   1.82547e+01  -1.16090e+02   4.00465e+01   0.00000e+00   4.20000e+00
Pemc     10   0.00000e+00   5  -1   0   1  0  2.61338e+02  -1.04661e+01  -2.28924e+02   1.25557e+02   0.00000e+00   4.20000e+00
Elast    10   0.00000e+00  11  -3   0   0  0  9.97744e+02  -2.46146e+02   8.86896e+02  -3.85124e+02   0.00000e+00   5.10000e-04
Elast    10   0.00000e+00   5  -1   0   0  1  5.46214e+02   4.45826e+02   8.53813e+01  -3.03778e+02   0.00000e+00   4.20000e+00
```

Fig. 3. output.txt: There are three parts to this file. The first part is a copy of parameter.txt followed by the information on the black-hole production cross section as inferred from the events in this generator run and is produced for `Write_LHA_Output_Record` equal to 0 and 1. The third part includes information about the black-hole and the emitted particles and is produced for `Write_LHA_Output_Record` equal to 2. The first column identifies what type of information each row is supplying: lines starting with "Begin" contain information about the emitted particles before the formation of the black-hole; lines beginning with "parent" have information about the two incoming partons; lines beginning "Pbh" contain information on the energy and momenta of the produced black-holes; lines starting with "trace" describe the location of the black-holes; rows beginning with "Pem" contain information about the emitted particles in the lab frame; lines headed by "Pemc" have the information about the emitted particles in the center-of-mass frame; rows starting with "Elast" describe the final burst.

```
********************************
***   Welcome to:       BLACKMAX
***   Release tag : $Name: REL_1_07 $
***   Release ID  :   $Id: BlackMax.c,v 1.59 2008/11/03 15:56:58 rizvi Exp $
***   Release date: $Date: 2008/11/03 15:56:58 $
***   Writing input parameter set...
***
Number_of_simulations
10
incoming_particle(1:pp_2:ppbar_3:ee+)
1
        .
        .
        .
        .
***   Finished writing input parameter set
********************************
  2212    2212    7.0000000e+03    7.0000000e+03       4       4      41      41       3       1
  5.7146597e+01    5.3609564e-01    0.0000000e+00       1
    18       1    1.0000000e+00    1.3994101e+04   -1.0000000e+00   -1.0000000e+00
     2      -1       0       0     506       0    0.0000000e+00    0.0000000e+00    2.4876139e+03    2.4876139e+03    3.1999412e-03    0.0000000e+00    0.0000000e+00
     2      -1       0       0     507       0    0.0000000e+00    0.0000000e+00   -3.5576648e+03    3.5576648e+03    3.2002308e-03    0.0000000e+00    0.0000000e+00
    39       1       1       2       0       0   -4.5124750e+01    2.1139359e+02   -6.6818111e+01    2.2624799e+02    3.0462581e-06    0.0000000e+00    0.0000000e+00
    39       1       1       2       0       0    4.5124750e+01   -2.1139359e+02    6.6818111e+01    2.2624799e+02    3.0462581e-06    0.0000000e+00    0.0000000e+00
    -4       1       1       2       0     501    1.5431276e+02   -1.8526639e+00    1.3976068e+01    1.5496010e+02    1.2000000e+00    0.0000000e+00    0.0000000e+00
    -3       1       1       2       0     502   -2.1118507e+02    6.3289592e+02    3.8860434e+01    6.6833114e+02    7.0000000e-02    0.0000000e+00    0.0000000e+00
    24       1       1       2       0       0   -5.4745365e+01    5.0680883e+01    1.1459160e+02    1.5863487e+02    8.0425000e+01    0.0000000e+00    0.0000000e+00
     5       1       1       2     501       0    1.9032106e+01   -1.4407840e+01    1.3500616e+02    1.3716454e+02    4.2000000e+00    0.0000000e+00    0.0000000e+00
    -2       1       1       2       0     503   -2.2311343e+02    3.3135010e+02   -4.4979714e+02    6.0157290e+02    3.2000132e-03    0.0000000e+00    0.0000000e+00
     3       1       1       2     504       0    3.6006468e+02   -4.5258870e+02   -3.3742488e+02    6.6958096e+02    7.0000000e-02    0.0000000e+00    0.0000000e+00
    -5       1       1       2       0     504   -2.0739246e+02    2.0443502e+01   -4.3278013e+02    4.8036012e+02    4.2000000e+00    0.0000000e+00    0.0000000e+00
    -5       1       1       2       0     505    4.8735597e+02    1.0669136e+02   -2.6101426e+01    4.9959765e+02    4.2000000e+00    0.0000000e+00    0.0000000e+00
     5       1       1       2     503       0   -2.8014190e+02    3.1337342e+02   -6.7788477e+01    4.2578787e+02    4.2000000e+00    0.0000000e+00    0.0000000e+00
     5       1       1       2     502       0    2.1565688e+02   -1.0426138e+02   -2.6021028e+01    2.4098353e+02    4.2000000e+00    0.0000000e+00    0.0000000e+00
    24       1       1       2       0       0   -2.0297186e+00   -4.5754642e+01   -8.5747871e+01    1.2616848e+02    8.0425000e+01    0.0000000e+00    0.0000000e+00
     6       1       1       2     505       0   -3.5278679e+02   -5.7815418e+02    3.8622763e+02    7.9975757e+02    1.7810000e+02    0.0000000e+00    0.0000000e+00
     4       1       1       2     506       0    7.0120765e+01   -1.8985992e+02   -2.6320670e+02    3.3202848e+02    1.2000000e+00    0.0000000e+00    0.0000000e+00
     3       1       1       2     507       0    2.4851565e+01   -6.8555863e+01   -6.9845149e+01    1.0097453e+02    7.0000000e-02    0.0000000e+00    0.0000000e+00
        .
        .
        .
        .
        .
        .
```

Fig. 4. BlackMaxLHArecord.txt: This file has the output of BlackMax in LHA format.

	BH ID	PDGID	3Q	Colours			Energy	Px	Py	Pz	Mass
Parent	1	4	2	0	0	1	1.61349e+03	0.00000e+00	0.00000e+00	1.61349e+03	1.20000e+00
Parent	1	2	2	0	1	0	4.40436e+03	0.00000e+00	0.00000e+00	−4.40436e+03	3.20000e−03
Parent	2	21	0	1	0	−1	2.58804e+03	0.00000e+00	0.00000e+00	2.58804e+03	0.00000e+00
Parent	2	21	0	−1	1	0	2.73896e+03	0.00000e+00	0.00000e+00	−2.73896e+03	0.00000e+00
Parent	3	2	2	0	0	1	5.17716e+03	0.00000e+00	0.00000e+00	5.17716e+03	3.20000e−03

Fig. 5. Lines in the output file headed by the ID = `Parent` contain information about the initial partons which formed the black-hole.

```
        BH ID    Time       PDGID  3Q   Colors      Energy        Px           Py           Pz          Pextra..      Mass         Jbh

Pbh       1  0.00000e+00  40   4   0   1   1  6.01785e+03  0.00000e+00  0.00000e+00 -2.79088e+03  0.00000e+00  5.33156e+03  0.00000e+00
Pbh       1  0.00000e+00  40   3   1   1   1  5.76609e+03 -1.17708e+02  3.60077e+01 -2.57130e+03  0.00000e+00  5.15956e+03  0.00000e+00
Pbh       1  0.00000e+00  40   1   0   1   1  5.23160e+03 -5.43908e+01  1.80610e+02 -2.09272e+03  0.00000e+00  4.79110e+03  0.00000e+00
Pbh       1  0.00000e+00  40   1   0   1   1  4.66423e+03 -3.43247e+02 -3.07262e+02 -2.07124e+03  0.00000e+00  4.15364e+03  0.00000e+00
Pbh       1  0.00000e+00  40   3   1   1   1  4.40733e+03 -2.68425e+02 -1.68412e+02 -1.97431e+03  0.00000e+00  3.92763e+03  0.00000e+00
```

Fig. 6. Lines in the output file headed by the ID = `Pbh` contain the energies and momenta of the black-holes for each emission step. In case of rotating black-holes, the last column in the line is the angular momentum.

```
        BH ID   Time                          X             Y             Z          Xextra...

trace     1   0.00000e+00              -9.48767e-05  -3.13192e-04   0.00000e+00  -2.37725e-04
trace     1   0.00000e+00              -9.48767e-05  -3.13192e-04   0.00000e+00  -2.37725e-04
trace     1   0.00000e+00              -9.48767e-05  -3.13192e-04   0.00000e+00  -2.37725e-04
trace     1   0.00000e+00              -9.48767e-05  -3.13192e-04   0.00000e+00  -2.37725e-04
trace     1   0.00000e+00              -9.48767e-05  -3.13192e-04   0.00000e+00  -2.37725e-04
```

Fig. 7. Lines in the output file headed by the ID = `trace` contain the location of the black-hole to reach emission step.

```
        BH ID    Time       PDGID 3Q    Colors     Energy        Px           Py           Pz          Pextra...     Mass

Pem       1  0.00000e+00  -5   1  -1   0   0  2.51757e+02  1.17708e+02 -3.60077e+01 -2.19573e+02  0.00000e+00  4.20000e+00
Pem       1  0.00000e+00   6   2   1   0   0  5.34489e+02 -6.33172e+01 -1.44602e+02 -4.78582e+02  0.00000e+00  1.78100e+02
Pem       1  0.00000e+00 -16   0   0   0   0  5.67379e+02  2.88857e+02  4.87872e+02 -2.14790e+01  0.00000e+00  0.00000e+00
Pem       1  0.00000e+00  -6  -2  -1   0   0  2.56892e+02 -7.48229e+01 -1.38850e+02 -9.69335e+01  0.00000e+00  1.78100e+02
Pem       1  0.00000e+00  -1   1  -1   0   0  8.26391e+02  1.19497e+02 -1.63989e+02 -8.01093e+02  0.00000e+00  3.20000e-03
```

Fig. 8. Lines in the output file headed by the ID = `Pem` contain the types of the emitted particles, their energies and momenta in the lab frame and the times of their emission.

```
        BH ID    Time      PDGID  3Q   Colors      Energy        Px           Py           Pz          Pextra...     Mass

Elast     1  0.00000e+00   2   2   1   0   0  1.69854e+02  1.49973e+02  5.22441e+01 -6.02406e+01  0.00000e+00  3.20000e-03
Elast     1  0.00000e+00   4   2   1   0   0  1.73812e+02  1.32172e+02  4.97772e+01 -1.01301e+02  0.00000e+00  1.20000e+00
Elast     1  0.00000e+00   1  -1   0   1   0  8.97411e+02  1.86142e+02  4.63555e+01 -8.76669e+02  0.00000e+00  3.20000e-03
Elast     1  0.00000e+00   5  -1   0   0   1  2.06777e+02 -1.58071e+01  1.36863e+02 -1.54136e+02  0.00000e+00  4.20000e+00
Elast     2  0.00000e+00  21   0   0   1  -1  2.09713e+02 -2.09011e+02 -1.42072e+01  9.58863e+00  0.00000e+00  0.00000e+00
```

Fig. 9. Lines in the output file headed by the ID = `Elast` contain the types, energies and momenta of particles of the final burst.

Table 1
$M_D = 1000$ GeV, $M_{bh} > 5000$ GeV, and D is the total number of dimensions (space + time).

D	σ_{ch} [pb]	σ_{bm} [pb]	σ_{bm} with $L_{extra} = 0$ [pb]	σ_{bm}/σ_{ch}	σ_{bm}/σ_{ch} with $L_{extra} = 0$	b_d^2
6	75.20 ± 0.6968	90.69 ± 0.8407	99.70 ± 0.9128	1.21	1.32	1.36
7	122.0 ± 1.126	161.9 ± 1.502	177.0 ± 1.638	1.32	1.45	1.48
8	172.6 ± 1.590	247.6 ± 2.304	266.2 ± 2.449	1.43	1.54	1.59
9	225.7 ± 2.076	352.7 ± 3.149	369.0 ± 3.285	1.56	1.63	1.69
10	280.7 ± 2.579	455.2 ± 4.182	484.8 ± 4.419	1.62	1.72	1.78

- **Elast**: contains the same information as Pem for the particles emitted in the final decay burst of the black-hole but column 12 is the mass of the particle. There is no information on the bulk momentum since these particles have no bulk momentum.

Appendix. Comparison between blackmax and charybdis

In this section we compare the cross sections of BlackMax and Charybdis [21] which is a very commonly used black hole generator.[5] There are several differences between BlackMax and Charybdis which need to be considered.

For a direct comparison one has to make sure that both generators use the same convention for the definition of the Planck Mass. In Charybdis the user can choose between the different conventions by setting the parameter `MSSDEF`. In BlackMax the user can switch between the different conventions with the help of the parameter `definition_of_M_pl`.

- **PDG definition,** M_D:
  `definition_of_M_pl` = 1 corresponds to `MSSDEF` = 3;
- **Giddings and Thomas,** M_p:
  `definition_of_M_pl` = 2 corresponds to `MSSDEF` = 1;
- **Dimopoulos and Landsberg,** M_{DL}:
  `definition_of_M_pl` = 3 corresponds to `MSSDEF` = 2.

One important difference between the two generators is the definition of cross sections. Charybdis uses the cross section definition for a non-rotating black-hole

$$\sigma_{ch} = \pi r_s^2. \tag{26}$$

Here, r_s is the Schwarzschild radius of the black-hole.

5 For the comparison in this section MSRT98LO was used as the input PDF.

Table 2
$M_p = 1000$ GeV, $M_{bh} > 5000$ GeV, and D is total number of dimensions (space + time).

D	σ_{ch} [pb]	σ_{bm} [pb]	σ_{bm} with $L_{extra} = 0$ [pb]	σ_{bm}/σ_{ch}	σ_{bm}/σ_{ch} with $L_{extra} = 0$	b_d^2
6	119.4 ± 1.106	149.1 ± 1.375	158.3 ± 1.449	1.25	1.33	1.36
7	172.6 ± 1.593	234.5 ± 2.158	250.4 ± 2.316	1.36	1.45	1.48
8	227.8 ± 2.098	334.2 ± 3.090	351.2 ± 3.231	1.47	1.50	1.59
9	284.4 ± 2.626	450.2 ± 4.010	464.8 ± 4.139	1.58	1.63	1.69
10	342.1 ± 3.144	561.5 ± 5.186	591.0 ± 5.387	1.64	1.73	1.78

Table 3
$M_{DL} = 1000$ GeV, $M_{bh} > 5000$ GeV, and D is the total number of dimensions (space + time).

D	σ_{ch} [pb]	σ_{bm} [pb]	σ_{bm} with $L_{extra} = 0$ [pb]	σ_{bm}/σ_{ch}	σ_{bm}/σ_{ch} with $L_{extra} = 0$	b_d^2
6	55.65 ± 0.5156	64.82 ± 0.6047	73.78 ± 0.6755	1.16	1.33	1.36
7	38.85 ± 0.3586	41.78 ± 0.3894	56.35 ± 0.5212	1.08	1.45	1.48
8	33.12 ± 0.3050	32.86 ± 0.3083	51.07 ± 0.4698	0.992	1.54	1.59
9	30.90 ± 0.2842	27.81 ± 0.2590	50.51 ± 0.4498	0.900	1.63	1.69
10	30.20 ± 0.2775	24.70 ± 0.2353	52.16 ± 0.4755	0.818	1.73	1.78

But BlackMax uses

$$\sigma_{bm} = b_d^2 \pi r_s^2 \tag{27}$$

$$b_d = \frac{2}{(1 + (\frac{d-1}{2})^2)^{\frac{1}{d-2}}} \tag{28}$$

which is the definition of the cross section for rotating black-holes (see also Eq. (15)), where d is number of space dimensions. The variable b_d is in general larger than one. Therefore one expects that the cross section of BlackMax is larger than the cross section of Charybdis. σ_{bm}/σ_{ch} is expected to be equal to b_d^2. In Tables 1–3 the value of σ_{bm}/σ_{ch} is in general smaller than b_d^2, and in some cases it is even smaller than 1. This difference comes from the fact that BlackMax assumes that the width of the brane is M_{pl}^{-1} (even in the single brane scenario), and Charybdis assumes that the width of the brane is 0. Because of the finite width of the brane, the cross section of BlackMax is further reduced compared to Charybdis.

To make sure that the two generators indeed output the same cross section in the same situation, we manually put the width of the brane to zero in BlackMax for the purpose of this comparison. The results are shown in the sixth column of the Tables 1–3 and they agree with the expected value well and there is a small difference between b_d^2 and σ_{bm}/σ_{ch} of 3%.

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