# Distribution Systems Hardening against Natural Disasters

Yushi Tan, *Student Member, IEEE* Arindam K. Das, *Member, IEEE*, Payman Arabshahi, *Senior Member, IEEE*, and Daniel S. Kirschen, *Fellow, IEEE*

***Abstract*—Distribution systems are often crippled by catastrophic damage caused by a natural disaster. Well-designed hardening can significantly improve the performance of post-disaster restoration operations. Such performance is quantified by a resilience measure associated with the operability trajectory. The distribution system hardening problem can be formulated as a two-stage stochastic problem, where the inner operational problem addresses the proper scheduling of post-disaster repairs and the outer problem the judicious selection of components to harden. We propose a deterministic single crew approximation with two solution methods, an MILP formulation and a heu approach. We provide computational evidence on various test feeders which illustrates that the heuristic approach pro near-optimal hardening solutions efficiently.**

## I. Introduction

NATURAL disasters have caused major damage to tricity distribution networks and deprived homes businesses of electricity for prolonged periods, for exa Hurricane Sandy in November 2012 [1], the Christcl Earthquake in February 2011 [2] and the June 2012 Atlantic and Midwest Derecho [3]. Estimates of the a cost of power outages caused by severe weather between and 2012 range from $18 billion to $33 billion on average [4]. Physical damage to grid components must be repaired before power can be restored [1], [5]. On the operational side, approaches have been proposed for scheduling the available repair crews in order to minimize the cumulative duration of customer interruption, which reduces the harm done to the affected community [6]–[8]. On the planning side, Kwasinski et al. [2] reported facilities that had been upgraded or hardened in Christchurch, at a cost of $5 million, remained serviceable immediately after the September 2010 earthquake and saved approximately $30 to $50 million in subsequent repairs. Hardening minimizes the potential damages caused by disruptions, thereby facilitating restoration and recovery efforts, and the time it takes for the infrastructure system to resume operation [9]. However, as indicated in [10], the difficulty of hardening does not lie in the design or construction of a hardened system, rather in the ability to quantify the expected performance improvement so that rational decisions can be made regarding increased cost versus potential future benefit.

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Y. Tan, P. Arabshahi and D. S. Kirschen are with the Department of Electrical Engineering, University of Washington, Seattle, WA, 98195-2500 USA. (Email: ystan@uw.edu)

A. K. Das is with the Department of Electrical Engineering, Eastern Washington University, Cheney, WA, 99004-2493, USA.

### A. Concept and quantification of resilience

Resilience in infrastructure systems under natural disasters is an important current area of research. While several definitions of resilience have been proposed [11]–[13], infrastructure resilience is typically defined as the ability to anticipate, prepare for, adapt to changing climate conditions and withstand, respond to, and recover rapidly from disruptions [14]. Resilience usually addresses the following

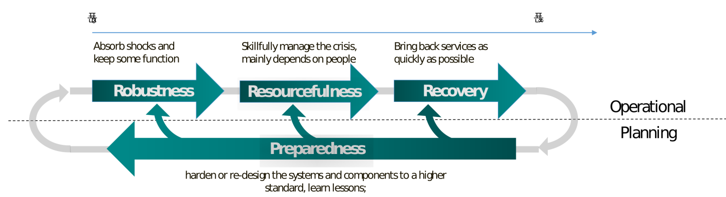



Fig. 1: Interactions between the four aspects of resilience

In the civil engineering context, resilience can be illustrated using the "operability trajectory", $Q(t)$, as shown in Figure 2, adopted from [16]. The trajectory shows the increase in infrastructure functionality over time and is an effective visual indicator of the 'goodness of the restoration process'. Robustness is quantified by the depth of functionality drop at time zero (without any loss of generality, we assume that the the disaster occurs at time $t = 0$ and the restoration process commences immediately afterward), while the quality of the recovery process is quantified by the ramp up time of the operability trajectory to full/satisfactory functionality, post time zero. Obviously, we desire that an infrastructure system exhibit a relatively small drop in functionality at time zero and a quick ramp up time to full/satisfactory functionality, post time zero. Consequently, the ideal operability trajectory is defined by $Q_{ideal}(t) = 1$, $\forall t \geq 0$, assuming that operability is measured in fractional units instead of percentages. These two metrics can naturally be combined into an unifying measure of resilience [12]. Letting $T$ be the restoration time horizon, a resilience measure, $R$, can be defined as follows [16]:

$$R = \int_0^T Q(t)dt, \tag{1}$$

The closer $Q(t)$ is to $Q_{ideal}(t)$, the greater is the area under $Q(t)$, and therefore the greater is the resilience measure.

Instead of maximizing the resilience measure defined in eqn. 1, we could choose to minimize the quantity

$\int_0^T Q_{ideal}(t)dt - \int_0^T Q(t)dt$, which is the area over the $Q(t)$ curve, bounded from above by $Q_{ideal}(t)$. This area, informally, the 'other side of resilience', can be interpreted as a measure of 'aggregate harm'. In a power system, it can be shown using the Lebesgue integral that minimizing this area is equivalent to minimizing the quantity $\sum_n w_n T_n$, where $w_n$ can be interpreted as the contribution of node $n$ to the overall loss in functionality of the system or the importance of node $n$ and $T_n$ is the time to restore node $n$. *Therefore, the objective for operational problems is to minimize the measure* $\sum_n w_n T_n$*, given a specific disaster scenario, while the objective for planning problems is to minimize* $\sum_n w_n T_n$ *in an expected sense, where the expectation is over all possible disaster scenarios.*

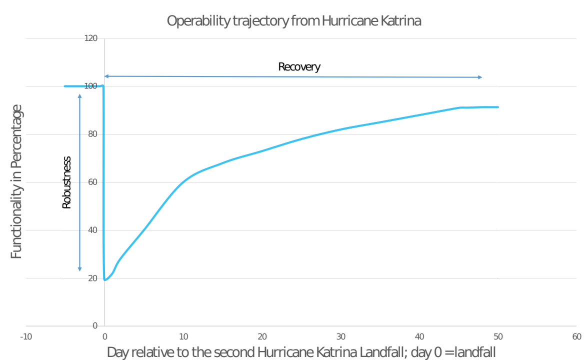



Fig. 2: Operability trajectory after Hurricane Katrina [16].

### B. Literature review

Defending critical infrastructures at the transmission level has been a major research focus over the past decade [17]–[19]. In general, this research adopted the setting of Stackelberg game and formulated the problem with a tri-level defender-attacker-defender model. Such a model assumes that the attacker has perfect knowledge of how the defender will optimally operate the system after the attack and the attacker manipulates the system to its best advantage.

In recent years, several researchers have investigated different methods for distribution systems hardening, but most focus solely on the robustness, i.e., worst-case load shedding at the onset of disaster. Of note, a resilient distribution network planning problem (RDNP) was proposed in [20] to coordinate the hardening and distributed generation resource allocation. A tri-level defender-attacker-defender model is studied, in which the defender (hardening planner) selects a network hardening plan in the first stage, the attacker (natural disaster) disrupts the system with an interdiction budget, and finally, the defender (the distribution system operator) reacts by controlling DGs and switches in order to minimize the shed load. This model is improved in [21] by considering the investment cost and by eliminating the assumption that enhanced components should remain intact during any disaster scenario. Another direction of research enforces chance constraints on the loss of critical loads and normal loads respectively [22], [23]. A two-stage stochastic program and heuristic solution of hardening strategy were proposed in [24], specifically for earthquake hazards, under the assumption that the repair times for similar types of components follow an uniform distribution, which simplifies the problem to a certain extent.

### C. Our approach

To the best of our knowledge, this paper is the first to consider the restoration process in conjunction with hardening. Our approach can be seen as a two-stage stochastic problem. The first stage selects from the set of potential hardening choices and determines the extent of hardening to maximize the expected resilience measure $R$, while the second stage solves the operational problem in each possible scenario by optimizing the sequence of repairs given the hardening results. In the operational problem, we consider scheduling post-disaster repairs in distribution network with parallel repair crews. This issue will be discussed in more detail in Section VI. Since an ideal formulation of the problem is hard to solve and also turns out impractical, we developed a deterministic single crew approximation with a heuristic approach to solve the hardening problem. Wang et al. [25] make a distinction between hardening activities and resiliency activities which are focused on the effectiveness of humans post-disaster. By using only one repair crew in the operational problem, we can also focus on the effects of network structure and components, and reduce the reliance on resourcefulness (i.e. the number of repair crews available).

The rest of the paper is organized as follows. In Section II, we briefly review the operational problem of scheduling post-disaster repairs in distribution networks with multiple repair crews and discuss an algorithm that converts an optimal single crew repair schedule to an arbitrary $m$-crew schedule with a proven performance bound. An MILP model for solving the single crew repair sequencing problem is also discussed in this section. In Section III, we formulate the problem of distribution system hardening against natural disasters and model it as a stochastic optimization problem, followed by a deterministic reformulation (Section IV) and single crew approximation (Section V). In Section VI, we motivate why we believe it is important to consider the restoration process (operational phase) in the hardening problem (planning phase) and develop the so called 'restoration process aware hardening problem'. Two solution methods, an MILP formulation and an iterative heuristic algorithm, are also discussed in this section. The performance of these methods is validated by various case studies on various standard IEEE test feeders in Section VII.

## II. Scheduling Post-Disaster Repairs in Distribution Networks

In this section, we briefly review the operational problem of scheduling post-disaster repairs in distribution networks with multiple repair crews. Further details can be found in [6].

### A. Distribution networks modeling

A distribution network can be modeled by a graph $G$ with a set of nodes $N$ and a set of edges $L$. Let $S \subset N$ represent the set of source nodes which are initially energized and $D = N \setminus S$ represent the set of sink nodes where consumers are located. An edge in $G$ represents a distribution feeder or some other connecting component. We assume that the network topology $G$ is radial, which is a valid assumption for many electricity distribution networks. Instead of a rigorous power flow model, we model network connectivity using a simple network flow model, i.e., as long as a sink node is connected to the source, we assume that all the loads connectd to this node can be supplied without violating any security constraint. For simplicity, we treat the three-phase distribution network as if it were a single-phase system. Our analysis could be extended to a three-phase system using a multi-commodity flow model, as in [26].

### B. Damage modeling

Let $L^D$ denote the set of damaged edges. Without loss of generality, we assume that there is only one source node in $G$. If an edge is damaged, all downstream nodes lose power due to lack of electrical connectivity. Each damaged edge $l \in L^D$ has a (potentially) unique repair time $p_l$. At the operational stage, we assume perfect knowledge of the set $L^D$ and the corresponding repair times, while, at the planning stage, the repair times are modeled as random variables following some probability distribution.

### C. Scheduling post-disaster repairs

Let $w_n$ be a nonnegative quantity that captures the importance of the load at node $n$. The importance of a node can depend on multiple factors, including but not limited to, the amount of load connected to it, the type of load served, and interdependency with other critical infrastructures. For example, re-energizing a node supplying a major hospital should receive a higher priority than a node supplying a similar amount of residential load. Similarly, it is likely that a node that provides electricity to a water sanitation plant would be assigned a higher priority. These priority factors would need to be assigned by the utility and their determination is outside the scope of this paper. In this paper We assume that the $w_n$'s are known.

Based on conversations with an industry expert, we also make the assumption that crew travel times *in a typical distribution network* are small compared with the time required for each repair. These travel times are therefore neglected as a first order approximation. Additionally, since hardening decisions are made at the planning stage, it is unrealistic to expect an accurate prediction what travel times might be when a disaster occurs. Within this framework, the operational goal is therefore to find a schedule by which the damaged lines should be repaired such that the aggregate harm, $\sum_{n \in N} w_n T_n$, is minimized.

We construct two simplified directed radial graphs to model the effect that the topology of the distribution network has on scheduling. The first graph, $G'$, is called the 'damaged component graph'. All nodes in $G$ that are connected by intact edges are contracted into a supernode in $G'$. The set of edges in $G'$ is the set of damaged lines in $G$, $L^D$. The directions to these edges follow trivially from the network topology. The second graph, $P$, is called a 'soft precedence constraint graph', and is constructed as follows. The nodes in this graph are the damaged lines in $G$ and an edge exists between two nodes in this graph if they share the same node in $G'$. Such a graph enables us to consider the hierarchal relationships between damaged lines, which we define as *soft precedence constraints*. See Fig. 4 for examples of $P$ and $G'$.

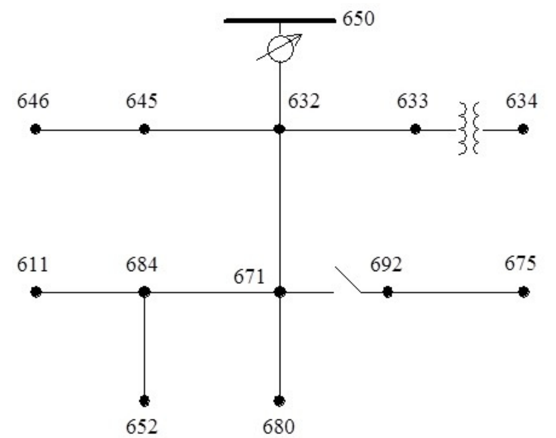
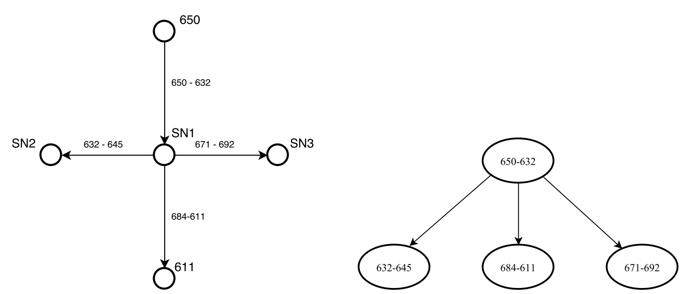



Fig. 3: IEEE 13 Node Test Feeder




(a) $G'$ graph (b) $P$ graph

Fig. 4: (a) The damaged component graph, $G'$, obtained from Fig. 3, assuming that the damaged edges are $650-632$, $632-645$, $684-611$ and $671-692$. (b) The corresponding soft precedence graph, $P$.

Two different time vectors are of interest in the operational problem: a vector of completion times of line repairs, denoted by $C_l$'s, and a vector of energization times of nodes, denoted by $E_n$'s. While we have so far associated the term 'energization time' with nodes in a given network topology, $G$, it is also possible to define energization times for the lines. Given a directed edge $l \in G'$, let $h(l)$ and $t(l)$ denote its head and tail nodes, i.e., $l = h(l) \to t(l)$. Then, $E_l = E_{t(l)}$, where $E_l$ is the energization time of line $l$ and $E_{t(l)}$ is the energization time of the node $t(l)$ in $G$. Analogously, the weight of node $t(l)$, $w_{t(l)}$, can be interpreted as a weight on the line $l$, $w_l$. The *soft precedence constraints*, $i \prec_S j$, therefore implies that line $j$ cannot be energized unless line $i$ is energized, or equivalently, $E_j \geq E_i$.

### *D. Solution methods for optimal single crew repair sequencing*

We begin with the following lemma, which connects the problem defined above to a well studied problem in scheduling theory, for which a polynomial time optimal algorithm exists [27]. A proof of this lemma can be found in [6].

**Lemma 1.** *Single crew repair and restoration scheduling in distribution networks is equivalent to* $1 \mid outtree \mid \sum w_j C_j$*, where the outtree precedences are given in the soft precedence constraint graph* $P$.

The problem of $1 \mid outtree \mid \sum w_j C_j$ can be solved in polynomial time. Algorithm 1 shown below is based on the algorithm in Chapter 4 of [28], which solves this problem in $O(n \log n)$ time (see Theorem 4.8 in [28]). Additional details and an example can be found in [6].

**Algorithm 1** Optimal algorithm for single crew restoration in distribution networks. The input to this algorithm is the precedence graph $P$. The notation $\text{pred}(n)$ denotes the predecessor of any node $n \in P$.

```
w(1) ← −∞;    pred(1) ← 0;
for n = 1 to |N(P)| do
    A(n) ← n;    B_n ← {n};    q(n) ← w(n)/p(n);
end for
for n = 2 to |N(P)| do
    pred(n) ← parent of n in P;
end for
nodeSet ← {1, 2, ··· , |N(P)|};
while nodeSet ≠ {1} do
    Find j ∈ nodeSet such that q(j) is largest; % ties can be broken arbitrarily
    Find i such that pred(j) ∈ B_i, i = 1, 2, ... |N(P)|;
    w(i) ← w(i) + w(j);
    p(i) ← p(i) + p(j);
    q(i) ← w(i)/p(i);
    pred(j) ← A(i);
    A(i) ← A(j);
    B_i ← {B_i, B_j};  % ',' denotes concatenation
    nodeSet ← nodeSet \ {j};
end while
```

We also present an MILP formulation for determining the optimal repair sequence. With only one repair crew, the damaged components must be repaired one by one, so there can be $L^D$ decisions to make, one at each time stage. The duration of each stage depends on the repair time of the component. We use two sets of binary decision variables. The first set of decision variables is denoted by $\{x_l^t\}$, where $x_l^t = 1$ if edge $l$ is repaired at time stage $t$ and is equal to 0 otherwise. The second set of decision variables is denoted by $\{u_i^t\}$, where $u_i^t = 1$ if node $i$ is energized at the end of time stage $t$ and is equal to 0 otherwise. Let $T$ denote the restoration time horizon and $h^t$ denote the harm till time stage $t$. The MILP model for minimizing the aggregate harm is shown below:

$$\min_{x,u} \quad \sum_{t=1}^{T} h^t \tag{2a}$$

$$\text{s.t.} \quad u_i^0 = 1,\ \forall i \in S \tag{2b}$$

$$\sum_{t=1}^{T} u_i^t = 1,\ \forall i \in D \tag{2c}$$

$$\sum_{l \in L^D} x_l^t = 1,\ \forall t \in [1,T] \tag{2d}$$

$$\sum_{\tau=0}^{t-1} u_i^\tau + u_{t(l)}^t - 2x_l^t \geq 0, l \in L^D, i \in Ne(t(l)), \forall t \tag{2e}$$

$$d^0 = 0 \tag{2f}$$

$$d^t \geq d^{t-1} + p_l \times x_l^t,\ \forall t \in [1,T],\ \forall l \in L^D \tag{2g}$$

$$h^t \geq w_j u_j^t d^t,\ \forall j \in D,\ \forall t \in [1,T] \tag{2h}$$

The first set of constraints binds the two sets of decision variables. Constraints (2b) and (2c) specify that all source nodes be energized initially and all sink nodes be energized by time $T$. Constraint (2d) requires that only one damaged edge be chosen for repair at any time stage. Constraint (2e) requires that when an edge $l \in L^D$ is chosen for repair at time stage $t$, i.e., $x_l^t = 1$, both $u_{t(l)}^t$ and $\sum_{\tau=0}^{t-1} u_i^\tau$ must be equal to 1. In other words, if the tail node of edge $l$, $t(l)$, is to be energized at time stage $t$, at least one of its neighbors in the damaged component graph $G'$, denoted by $Ne(t(l))$, must have been energized at some previous time stage. This constraint follows directly from the outtree precedences in Lemma 1.

The second set of constraints connects the aggregate harm with the decision variables. The intermediate variable $d^t$ models the aggregate restoration time just prior to time stage $t$. Constraint (2f) initializes the aggregate restoration time to 0 while constraint (2g) requires that the difference $d^t - d^{t-1}$, for some $t$, be at least the repair time of the edge being repaired at time $t$. Finally, constraint (2h) models the $t^{th}$ stage harm if $l$ is the edge being repaired at time stage $t$, whose tail node is $j$. Note that constraint (2h) can be easily linearized using the big-$M$ method, details of which are omitted.

### *E. An algorithm for converting the optimal single crew sequence to an m-crew sequence*

A greedy procedure for converting the optimal single crew sequence to a multiple crew schedule is given in Algorithm 2. Let $H^{1,*}$, $H^{m,*}$ and $H^\infty$ denote the optimal harms when the number of repair crews is 1, some arbitrary $m$ ($2 \leq m < \infty$), and $\infty$ respectively. Note that the case with infinite many repair teams is trivial so we drop the superscript $*$ for simplicity. Then, the following results hold:

$$H^{m,*} \geq \frac{1}{m} H^{1,*} \tag{3}$$

$$H^{m,*} \geq H^\infty \tag{4}$$

Due to space limitations, we omit additional details and instead refer the reader to the proofs of Propositions 4 and 5 in Section 5.3 of [6]. Furthermore:

**Algorithm 2** Algorithm for converting the optimal single crew schedule to an $m$-crew schedule

*Treat the optimal single crew repair sequence as a priority list, and, whenever a crew is free, assign to it the next job from the list. The first $m$ jobs in the single crew repair sequence are assigned arbitrarily to the $m$ crews.*

**Proposition 1.** *Let $H^m$ be the harm corresponding to an $m$-crew schedule, obtained by application of Algorithm 2 on an optimal single crew sequence (which can be computed using Algorithm 1). Then:*

$$H^m \leq \frac{1}{m} H^{1,*} + \frac{m-1}{m} H^{\infty} \tag{5}$$

*Proof.* See proof of Theorem 4 in Section 5.3 of [6]. □

Consequently,

**Theorem 1.** *Algorithm 2 is a $\left(2 - \frac{1}{m}\right)$ approximation. That is:*

$$H^m \leq \left(2 - \frac{1}{m}\right) H^{m,*}$$

*Proof.* Follows straightforwardly from eqns (3) ∼ (5). □

## III. The Hardening Problem: Formulation

### A. Damage modeling

As mentioned above, damages are modeled by repair time vectors associated with network components. Since no *a priori* exact information about the damages is available at the planning stage, we model the repair times as a random vector $\vec{\mathcal{P}}$. The uncertainties are twofold: all possible natural disasters that planners want to take into account and the uncertain damages to components caused by a specific disaster. The distribution of $\vec{\mathcal{P}}$ can be a mixture of a Bernoulli distribution which represents the probability of damage and a (possibly) continuous distribution of repair time, such as the exponential [29] or log-normal distribution [30]. Mixed distributions, usually do not admit a closed-form expression of their distribution functions. In our work, we do not assume any knowledge of the distribution function, except for knowledge of the first moment $\mathbb{E}[\vec{\mathcal{P}}]$.

Some planners tend to use the sample average approximation (SAA) methods by considering a limited set of component damage scenarios, which are either defined by users or drawn from a probabilistic model, as in [22], [23]. It is known that SAA methods converge to the optimal solution as the sample size goes to infinity. However, SAA methods require that the selected scenarios be typical and right on target, or the sample averaging needs to be performed over a large number of cases.

### B. Hardening options and costs

In practice, multiple hardening actions are usually available for each network component. For example, hardening an edge can involve some combination of vegetation management, pole reinforcement, undergrounding, enhanced pole guying, [21]. Typically, the goal of hardening a component is to lower the probability of its failure in the event of a disaster. However, since we are interested in maximizing the resilience of the system, or equivalently, minimizing the aggregate harm, simply lowering the probability of failure of a component is not sufficient. Since the aggregate harm is a function of the restoration times of the nodes, which in turn depend on the repair times of the damaged components (and the repair schedule), hardening a component can only be beneficial if it leads to a corresponding reduction in the repair time of that component.

In this paper, we assume that there is a finite set of hardening strategies for each edge $l$, which we denote by $K_l$. Each such strategy can be some combination of several disjoint hardening actions. We require that the hardening process select one strategy from the set $K_l$. Let $\vec{p} = \{p_l\}$, where $p_l$ is the 'expected repair time' of component $l$ before hardening, $\Delta\vec{p} = \{\Delta p_{lk}\}$, where $\Delta p_{lk}$ is the 'expected reduction in the repair time' of component $l$ due to hardening strategy $k \in K_l$, and $c_{lk}$ be the cost of implementing hardening strategy $k$ on edge $l$. We make the following assumption on the relationship between $c_{lk}$ and $\Delta p_{lk}$:

**Assumption 1.** *For any two hardening strategies $(k_1, k_2) \in K_l$, if $\Delta p_{lk_1} < \Delta p_{lk_2}$, then $c_{lk_1} < c_{lk_2}$ and vice versa.*

Generally, the more a component is hardened, the greater is the cost of hardening, but so is the reduction in repair times. The reasoning behind Assumption 1 is similar to that of Proposition 1 in [31]. If there exists two hardening strategies $(k_1, k_2) \in K_l$ which violate the assumption, i.e., $\Delta p_{lk_1} > \Delta p_{lk_2}$ is true while $c_{lk_1} < c_{lk_2}$, strategy $k_2$ cannot be part of the optimal hardening solution.

### C. A stochastic programming model

**Definition 1.** *Given a repair time vector $\vec{p}$, the min-harm (or equivalently, max-resilience) function, denoted by $f^m(\cdot)$, is the mapping $\vec{p} \overset{f^m(\cdot)}{\longmapsto} H^{m,*}$, where $H^{m,*}$ is the harm when repairs are scheduled optimally with $m$ repair crews.*

Let $C$ denote the capital budget available for hardening, $\mathcal{P}$ denote the repair time after hardening (modeled as a random vector to account for different disaster scenarios), and $y_{lk}$ be a binary variable which is equal to 1 if hardening strategy $k$ is chosen for edge $l$ and 0 otherwise. A stochastic optimization model for minimizing the expected aggregate harm assuming $m$ repair crews is shown below:

$$\min_{\{\Delta p_l\},\{y_{lk}\}} \quad \mathbb{E}[f^m(\vec{\mathcal{P}})] \tag{6a}$$

$$\text{s.t.} \quad \vec{p} - \Delta\vec{p} = \mathbb{E}[\vec{\mathcal{P}}] \tag{6b}$$

$$\sum_{k \in K_l} y_{lk} \leq 1, \ \forall l \in L^D \tag{6c}$$

$$\sum_{l \in L^D} \sum_{k \in K_l} c_{lk} y_{lk} \leq C \tag{6d}$$

$$\Delta p_l = \sum_{k \in K_l} \Delta p_{lk} y_{lk}, \ \forall l \in L^D \tag{6e}$$

$$y_{lk} \in \{0,1\}, \ \forall l \in L^D, \ \forall k \in K_l \tag{6f}$$

where the expectation in eqn (6a) is over all disaster scenarios as explained in Section III-A. While the notation $L^D$ denotes the set of actual damaged edges in the context of the post-disaster scheduling problem (operational phase), we interpret it as the set of all edges which could potentially be damaged in the event of a disaster, the worst case operational scenario, in the context of the hardening problem. Eqn. (6b) is the mean-enforcing constraint (which requires that we have knowledge of the first moment of $\vec{\mathcal{P}}$), eqns. (6c) and (6f) force at most one hardening strategy to be chosen per edge from the set $K_l$, eqn. (6d) enforces the budget constraint, and eqn. (6e) models the (possible) reduction in repair time of each edge $l$ due to hardening. Observe that the set of constraints (6c), (6d) and (6f) mimics a 0-1 knapsack constraint since we are essentially choosing a subset of hardening strategies from the set of all hardening strategies over all edges, subject to a budget constraint.

## IV. Deterministic robust reformulation by Jensen's inequality

Unfortunately, the aforementioned stochastic program is extremely difficult to solve, even with perfect knowledge of the statistical distribution of $\vec{\mathcal{P}}$. This is due to two reasons. First, it is almost impossible to know beforehand the explicit form of $f^m(\cdot)$, even when the operational problem is solvable in polynomial time for $m = 1$. Second, evaluation of the objective function requires knowledge of the distribution function of $\vec{\mathcal{P}}$, while at the same time, this distribution function depends upon the decision variable (see eqns. 6a and 6b). This circular dependency effectively rules out the applicability of SAA methods. While metaheuristics such as simulated annealing could be used to solve the above problem to (near) optimality, doing so might require an inordinate amount of computation time. We therefore propose a deterministic robust reformulation in Section IV which is more computationally tractable. We begin this section by showing that the min-harm function $f^m(\vec{p})$ is concave.

**Theorem 2.** *The min-harm function $f^m(\vec{p})$ is concave.*

*Proof.* Let $\vec{p_i}$ and $\vec{p_j}$ be two different repair time vectors and $f^m_{\vec{p_i}}(\vec{p_j})$ denote the harm evaluated by the optimal schedule corresponding to $\vec{p_i}$ when the actual repair time vector is $\vec{p_j}$. Obviously, $f^m(\vec{p_j}) = f^m_{\vec{p_j}}(\vec{p_j})$. For some $\vec{p_0} \neq \vec{p}$, we have:

$$f^m_{\vec{p_0}}(\vec{p}) - f^m_{\vec{p_0}}(\vec{p} - \Delta\vec{p}) = \sum_{l \in L^D} \Delta p_l \sum_{j \in R_l} w_j, \tag{7}$$

where $R_l$ denotes the set of jobs assigned to the same crew as $l$, scheduled no earlier than $l$ in the optimal schedule corresponding to $\vec{p_0}$. This shows that $f^m_{\vec{p_0}}(\vec{p})$ is a linear function of the $p_l$'s in $\vec{p_0}$. And since $f^m(\vec{p})$ is the optimal schedule,

$$f^m(\vec{p}) = \min_{\vec{p_0}} \; f^m_{\vec{p_0}}(\vec{p}), \tag{8}$$

which implies that $f^m(\vec{p})$ is the point-wise minimum of a set of affine functions and is therefore concave. ∎

Since $f^m(\vec{p})$ is concave, Jensen's inequality [32] holds and the objective function (6a) can be naturally upper bounded as follows:

$$\mathbb{E}[f^m(\vec{\mathcal{P}})] \leq f^m(\mathbb{E}[\vec{\mathcal{P}}]) \tag{9}$$

The preceding discussion motivates the following deterministic robust reformulation (note that constraint (6b) has been wrapped into the objective function):

$$\begin{aligned} \min_{\{\Delta p_l\}, \{y_{lk}\}} \quad & f^m(\vec{p} - \Delta\vec{p}) \\ \text{s.t.} \quad & (6c) \sim (6f) \end{aligned} \tag{10}$$

As will be apparent from Section VI, the above model is a key development which allows for an integrated treatment of the restoration process and the hardening problem.

We conclude this section with a note on the worst case impact on the objective function caused by the upper bounding by Jensen's inequality. Assume that the support of $\vec{\mathcal{P}}$ is bounded, i.e., $\vec{\mathcal{P}} \in [\vec{0}, \vec{p}_{max}]$. Then, it follows from Theorem 1 in [33] that:

$$f^m\left(\mathbb{E}[\vec{\mathcal{P}}]\right) - \mathbb{E}\left[f^m(\vec{\mathcal{P}})\right] \leq f^m\left(\vec{p}_{max}\right) - 2f^m\left(\frac{\vec{p}_{max}}{2}\right) \tag{11}$$

## V. Single crew approximation

While the stochastic model and its deterministic reformulation discussed above are applicable for any value of $m$, for the rest of the paper, we make the assumption that $m = 1$. That is, the hardening decisions, which are made at the planning stage, are based on an assumption of single crew repair sequencing at the operational stage. The main motivation for making the single crew assumption is that it is practically impossible to know at the planning stage the actual number of repair crews that will be available in the event of a disaster. While hardening decisions based on an assumption of $m_1$ repair crews are most likely not the optimal decisions if the number of crews is actually $m_2$, a single crew assumption allows us to factor in the restoration process in these hardening decisions, without requiring a precise *a priori* knowledge of $m$ or a joint probability distribution on the type/magnitude/scale of the disaster event and $m$.

We now provide a theoretical upper bound on the aggregate harm during the operational stage, applicable for any arbitrary value of $m$, even when hardening decisions have been made based on $m = 1$. Consider two hardening strategies, $A$ and $B$, with corresponding expected reduction in repair time vectors, $\Delta\vec{p}_A$ and $\Delta\vec{p}_B$. Suppose strategy $A$ is obtained from the minimization of the objective function (10) with $m = 1$ and $B$ is an arbitrary hardening strategy. For a strategy $S$, we also define $H^{m,*}_S := f^m(\vec{p} - \Delta\vec{p}_S)$, the deterministic optimal harm (i.e., the objective function function in eqn. 10) at the operational stage with $m$ repair crews and $H^m_S$ denote the harm for an $m$-crew schedule computed using Algorithm 2 with repair time vector $\vec{p} - \Delta\vec{p}_S$. Then:

$$H_B^{m,*} \geq \frac{1}{m} H_B^{1,*} \quad (12)$$
$$\geq \frac{1}{m} H_A^{1,*} \quad (13)$$
$$\geq H_A^m - \left(\frac{m-1}{m}\right) H_A^{\infty} \quad (14)$$
$$\geq H_A^m - \left(\frac{m-1}{m}\right) H^{\infty} \quad (15)$$

where the first inequality follows from eqn. 3, the second inequality follows from the fact that hardening strategy $A$ is by definition optimal when $m = 1$, the third inequality follows from Proposition 1, and the fourth inequality follows from the fact that the aggregate harm defined by any $m$ after hardening is upper bounded by the aggregate harm before hardening. Rearranging terms, we have:

$$H_A^m \leq H_B^{m,*} + \left(\frac{m-1}{m}\right) H^{\infty} \quad (16)$$

Since $B$ represents any hardening strategy,

$$H_A^m \leq \min_B \left\{ H_B^{m,*} + \left(\frac{m-1}{m}\right) H^{\infty} \right\}$$
$$= H_{OPT}^{m,*} + \left(\frac{m-1}{m}\right) H^{\infty}, \quad (17)$$

where $H_{OPT}^{m,*}$ represents the deterministic optimal harm when a network has been hardened by minimizing objective function (10), with perfect knowledge of $m$.

The implication of eqn. (17) is that, while hardening strategy $A$ may not be optimal for some chosen $m > 1$, the approximation gap between the harm when an $m$-crew schedule (obtained by applying Algorithm 2) is used during the operational stage and the harm corresponding to an optimal hardening strategy for that specific value of $m$ is at most $\left(\frac{m-1}{m}\right) H^{\infty}$. In practice, the value of $H^{\infty}$ can be determined straightforwardly during the planning stage. Clearly, the smaller $H^{\infty}$ is, the better the single crew approximation is and an exact or probabilistic *a priori* knowledge of $m$ corresponding to different disaster events may not even be necessary if $H^{\infty}$ is small enough. Note that the $H^{\infty}$ term on the r.h.s of eqn. (17) could be way smaller than $H_{OPT}^{m,*}$. This is likely to be so when the hardening budget is limited since the benefits of an infinite number of repair crews will outweigh the benefits of hardening.

We wish to emphasize that our single crew approximation during the planning stage does not prevent the network operator from deploying multiple crews during the operational stage for post-disaster restoration. In fact, a network which has been designed/hardened with an eye on the restoration process, albeit with one repair crew, will ensure a smaller aggregate harm (or improved resilience) during the restoration process post-disaster when additional repair crews might be available, as opposed to a network which has been designed/hardened with no consideration given to the restoration process. Simulation results discussed in Section VII-C confirm this.

## VI. Restoration Process Aware Hardening Problem

Usually, the restoration problem and the hardening problem are treated separately because the former is an operational problem while the latter is a planning problem. However, we argue that the two problems should not be treated in isolation because hardening can affect the repair times, which in turn, can influence the restoration times through the sequencing process and thereby the aggregate harm or resilience. The model that we formulate is similar to single machine scheduling with controllable processing times, which dates back to the 1980s [34]. See Section 2 in [35] for a review of recent advances. Our problem is more complicated in the sense that the effect of hardening decisions (analogous to ‘costs of compression amount’ in the context of single machine scheduling with controllable processing times) are not just linear, instead they are embedded in the sequencing problem.

In this section, we discuss two solution approaches for the so-called ‘restoration process aware hardening problem’ (RPAHP), first an MILP formulation, followed by a heuristic algorithm framework inspired by a continuous convex relaxation, considering $m = 1$.

### A. MILP formulation

In Section II-D, we developed an MILP model for optimizing the repair schedule with one repair crew, while in Section V, we developed a deterministic single crew approximation of the hardening problem , both with the same objective, minimization of the aggregate harm. These two models can be easily incorporated into an integrated MILP formulation, as shown below:

$$\min_{\vec{x},\vec{u},\Delta\vec{p},\vec{y}} \quad \sum_{t=1}^{T} h^t \quad (18)$$
$$\text{s.t} \quad (2b) \sim (2f)$$
$$d^t \geq d^{t-1} + (p_l - \Delta p_l) \times x_l^t, \forall t \in [1, T], \forall l \in L^D \quad (19)$$
$$(6c) \sim (6f)$$

Observe that the impact of hardening, $\Delta p_l$, is incorporated into constraint (19). The product of $\Delta p_l$ and $x_l^t$ on the r.h.s of eqn. (19) can be easily linearized using the big-$M$ method, details of which are omitted.

### B. A continuous convex relaxation

For notational brevity, we define $f(\cdot) := f^1(\cdot)$. As stated previously, the min-harm function $f^m(\cdot)$ is concave piecewise affine and so is $f(\cdot)$. In general, concave minimization problems are $\mathcal{NP}$-hard [36]. In our case, there are at most $n!$ affine pieces, corresponding to $n!$ number of affine possible sequences, where $n = |L^D|$ is the number of damaged edges.

The RPAHP involves two types of decision variables, the sequencing variables (the $x$’s and $u$’s) and the hardening variables (the $\Delta p$’s). Given the hardening variables, it is straightforward to see that the joint optimization problem reduces to the single crew sequencing problem, which can be solved optimally in polynomial time as stated previously in Section II-D.

Now let us consider the case where the sequencing variables are fixed. Let $\Omega_l := \sum_{j \in R_l} w_j$, where $R_l$ is the set of some edges $l \in L^D$ and all its successors in the given sequence. The quantity $\Omega_l$ represents the reduction in aggregate harm per unit decrease in $p_l$. The objective function for the hardening problem can now be recast as $f(\vec{p}) = \sum_{l \in L^D} \Omega_l \, p_l$, which implies:

$$f(\vec{p} - \Delta\vec{p}) = \sum_{l \in L^D} \Omega_l p_l - \sum_{l \in L^D} \Omega_l \Delta p_l \tag{20}$$

Since the first term on the r.h.s of eqn. (20) is a constant, instead of minimizing $f(\vec{p} - \Delta\vec{p})$, an equivalent formulation is:

$$\max_{\vec{y}} \quad \sum_{l \in L^D} \sum_{k \in K_l} \Omega_l \Delta p_{lk} y_{lk} \tag{21a}$$

$$\text{s.t.} \quad \sum_{k \in K_l} y_{lk} \leq 1 \ \ , \forall l \in L^D \tag{21b}$$

$$\sum_{l \in L^D} \sum_{k \in K_l} c_{lk} y_{lk} \leq C \tag{21c}$$

$$y_{lk} \in \{0, 1\}, \ \ \forall l \in L^D, \ \ \forall k \in K_l \tag{21d}$$

This model is similar to that of the multiple choice knapsack problem [31], where $\Omega_l \Delta p_{lk}$'s are the value coefficients and $c_{lk}$'s are the cost coefficients. Since the multiple choice knapsack is known to be $\mathcal{NP}$-hard, we propose an algorithm based on convex envelopes and LP relaxation, similar to [37].

**Definition 2** (Convex envelope [38])**.** *Let $M \subset \mathcal{R}^n$ be convex and compact and let $g : M \rightarrow \mathcal{R}$ be lower continuous on $M$. A function $\hat{g} : M \rightarrow \mathcal{R}$ is called the convex envelope of $f$ on $M$ if it satisfies:*

- *$\hat{g}(x)$ is convex on $M$,*
- *$\hat{g}(x) \leq g(x)$ for all $x \in M$,*
- *there is no function $h : M \rightarrow \mathcal{R}$ satisfying (1), (2) and $g(x_0) < h(x_0)$ for some point $x_0 \in M$.*

Intuitively, the convex envelope is the best underestimating convex function of the original function. Details of a polynomial time algorithm for computing the convex envelope of a piecewise linear function can be found in [37].

Given a discrete function of $c_{lk}$ vs. $\Delta p_{lk}$ for some edge $l$ and a set of all hardening actions $k \in K_l$, we first connect the neighboring points, starting from the origin, to construct a continuous piecewise linear cost function $C_l(\Delta p_l)$, where $\Delta p_l$ is the relaxed continuous decision variable. It follows from Assumption 1 that $C_l$ is a strictly increasing function. Let $\hat{C}_l$ denote the convex envelope of $C_l$ and $\hat{K}_l = \{1, 2, \cdots, |\hat{K}_l|\}$ denote the set of breakpoints/knots on the convex envelope (excluding the origin) corresponding to the hardening strategies in consideration, indexed in ascending order of $\Delta p_{lk}$. The linear relaxation of (21) based on the convex envelope approximations, which we denote as **(LP)**, can then be formulated as:

$$\max_{\Delta\vec{p}} \quad \sum_{l \in L^D} \Omega_l \Delta p_l \tag{22a}$$

$$\text{s.t.} \quad Q_l \geq \max_{k \in \hat{K}_l} \left[\mu_{lk} \left(\Delta p_l - \alpha_{lk}\right) + b_{lk}\right], \forall l \in L^D \tag{22b}$$

$$\sum_{l \in L^D} Q_l \leq C \tag{22c}$$

$$0 \leq \Delta p_l \leq \Delta p_{l,|\hat{K}_l|}, \forall l \in L^D \tag{22d}$$

where $\mu_{lk}$ and $b_{lk}$ are the slope and intercept of the $k^{th}$ piece of $\hat{C}_l$, $\alpha_{lk}$ is the lower breakpoint of the $k^{th}$ piece of $\hat{C}_l$, and $Q_l$ is an intermediate decision variable which accounts for the budget spent on edge $l$. This formulation is similar to the conventional continuous knapsack problem, and it turns out that the optimal values of $\Delta p_l$ are always from the set $\{0, \text{some } \beta_{lk}, (\alpha_{lk}, \beta_{lk})\}$, where $\beta_{lk}$ is the upper breakpoint of the $k^{th}$ piece of $\hat{C}_l$. Furthermore, at most one $\Delta p_l$ can have an intermediate value in the range $(\alpha_{lk}, \beta_{lk})$ in the optimal solution. For some $l$ and $k > 1$, the intercept parameter, $b_{lk}$, is: $b_{lk} = \hat{C}_l\left(\alpha_{lk}\right) = \hat{C}_l\left(\beta_{l(k-1)}\right)$. For $k = 1$, $\alpha_{lk} = \hat{C}_l\left(\alpha_{lk}\right) = b_{lk} = 0$.

The preceding LP relaxation (22) can also be solved optimally using a greedy algorithm by first sorting the ratios $\left\{\frac{\Omega_l}{\mu_{lk}}\right\}$ in a descending order, and then choosing the components (and the degree of hardening) based on that sorted list iteratively, until the budget is exhausted. Ties, if any, during the selection process, are broken arbitrarily. We use a $\Delta p$ variable for each edge $l$ and each segment $k$ of $\hat{C}_l$. All these $\Delta p_{lk}$ variables are initialized to 0. Once a selection is made from the sorted list at any iteration $T$, say $(l_T, k_T)$, we set $\Delta p_{l_T k_T}$ equal to the maximum value possible within the range $[\alpha_{l_T k_T}, \beta_{l_T k_T}]$ such that the 'cumulative budget' at the end of iteration $T$ does not exceed $C$. Typically, this maximum value will be at the upper breakpoint $\beta_{l_T k_T}$, unless, doing so results in a budget violation. In that case, a proper value within the range $(\alpha_{l_T k_T}, \beta_{l_T k_T})$ is chosen such that the budget is met exactly. At the end of every iteration, we evaluate the expression of budget spent:

$$\Lambda = \sum_{l \in L^D} \hat{C}_l\left(\max_{k \in \hat{K}_l} \{\Delta p_{lk}\}\right), \tag{23}$$

which represents the cumulative budget consumed till the current iteration. The algorithm terminates when $\Lambda = C$. Upon termination, the optimal $\Delta p_l$ values can be obtained from the $\Delta p_{lk}$ values as follows:

$$\Delta p_l = \max_{k \in \hat{K}_l} \{\Delta p_{lk}\}. \tag{24}$$

The optimality of the greedy algorithm for solving the LP relaxation (22) using the convex envelopes of the hardening cost functions is proven in the appendix of [39].

### *C. An iterative heuristic algorithm*

We now discuss an iterative heuristic algorithm for solving the RPAHP. First, we note that the solutions obtained from the greedy algorithm used to solve the convex relaxation formulation (22) may need to be *rounded down* to the nearest lower breakpoints on the convex envelopes so that the hardening strategy is feasible for each edge. However, after completion

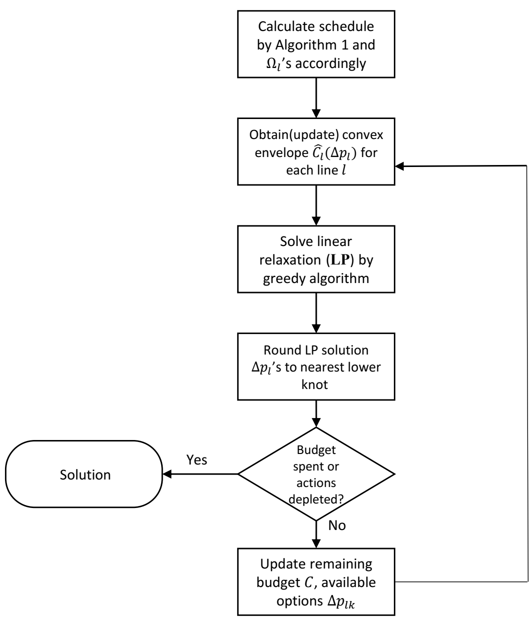



Fig. 5: Flowchart of an iterative heuristic algorithm for solving the RPAHP.

## VII. Case studies

### A. IEEE 13 node test feeder

We first tested the MILP and the heuristic approach discussed in the previous section on the IEEE 13 node test feeder with randomly generated $C_l$'s and two different budgets. Values of $\mathbb{E}[f(\cdot)]$ in this section were computed using Monte Carlo simulations assuming an independent geometric distribution for each $p_l$. With a budget of $C = 20$, hardening actions did not result in different repair schedules and both the MILP and heuristic approaches yielded identical results. With a budget of $C = 60$, even though the hardening actions suggested by the MILP and heuristic approaches differ for two edges, as shown in Table I, the objective values obtained from the greedy algorithm, both for $\mathbb{E}[f(\cdot)]$ and its upper bound $f(\mathbb{E}[\cdot])$, are very close to those provided by the MILP formulation. In fact, the ratios of the $f(\mathbb{E}[\cdot])$ measure from the greedy algorithm to the $\mathbb{E}[f(\cdot)]$ measure from the MILP algorithm are both approximately 1.04 for $C = 20$ and $C = 60$ (note that this ratio captures the worst case performance loss, including the effect of upper bounding the true objective function using Jensen's inequality).

TABLE I: Comparison of hardening results on the IEEE 13 node test feeder with a budget of $C = 60$.

| Edge $l$ | $\Delta p_l$ by MILP | $\Delta p_l$ by heuristic |
|---|---|---|
| 671-684 | 0.2 | 0.4 |
| 645-646 | 0 | 0.4 |
| 632-645 | 0.5 | 0.8 |
| 632-671 | 5.3 | 3.5 |
| $f(\mathbb{E}[\cdot])$ | 14.411 | 14.520 |
| $\mathbb{E}[f(\cdot)]$ | 13.987 | 14.146 |

We then varied the hardening budget from 0 to 50. Fig. 6 shows the aggregate harm as a function of the budget for $m = 1$. The MILP and the heuristic produced almost identical results when using the $f(\mathbb{E}[\cdot])$ measure so that their plots almost overlap. The plots corresponding to the $\mathbb{E}[f(\cdot)]$ measure are also very close, considering the errors introduced by Monte Carlo simulations. The gap between the true objective and the deterministic objective, $\mathbb{E}[f(\cdot)] - f(\mathbb{E}[\cdot])$, is fairly constant for both the MILP and the heuristic. As expected, the aggregate harm decreases (resilience increases) as the hardening budget increases.

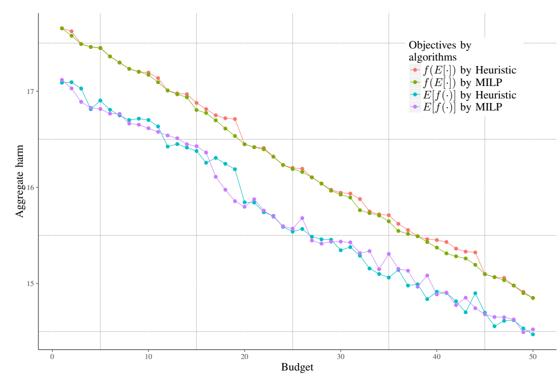
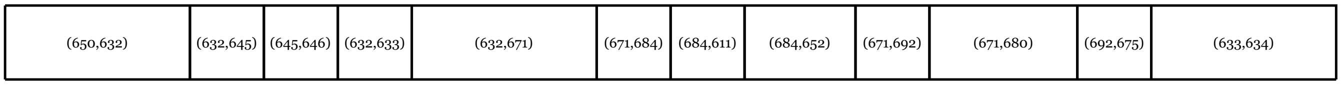



Fig. 6: Aggregate harm vs. hardening budget for the IEEE 13 node test feeder.

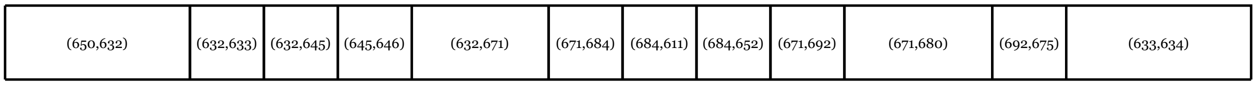
Finally, in Fig. 7, we compare the pre-hardening sequencing decisions with the post-hardening decisions calculated using the MILP for the same case study as in Table I. In these Gantt charts, each 'box' represents the repair of a line and the width of each 'box' is proportional to the repair time, appropriately scaled for better visualization. Observe that the two sequences differ by the relative locations of lines (632, 645), (645, 646) and (632, 633). This confirms the interaction between sequencing and hardening decisions. Finally, we want to emphasize that these sequencing decisions are abstract constructs that only serve to model operational decisions at the planning stage.

### B. IEEE 37 node test feeder

Next, we ran our algorithms on one instance of the IEEE 37 node test feeder [40]. Since the running time of the MILP




(a) Gantt chart of sequencing decisions before hardening




(b) Gantt chart of sequencing decisions after hardening (MILP)

Fig. 7: Comparison of optimal single crew repair schedules before and after hardening.

formulation increases exponentially with network size, we allocated a time budget of 10 hours. In contrast, the heuristic algorithm yielded a solution within seconds. Table II shows the edges for which the MILP and heuristic approach produced different hardening results, along with the objective function values.

TABLE II: Comparison of hardening results, $\Delta p_l$'s, on the IEEE 37 node test feeder with $C = 200$.

| Edge $l$ | $\Delta p_l$ by MILP(10 hours) | $\Delta p_l$ by heuristic |
|---|---|---|
| 744-729 | 0.8 | 0 |
| 708-733 | 0 | 0.3 |
| 702-705 | 2.1 | 0.4 |
| 708-732 | 0 | 0.2 |
| 734-710 | 0.1 | 1.4 |
| $f(\mathbb{E}[\cdot])$ | 843.08 | 842.04 |
| $\mathbb{E}[f(\cdot)]$ | 672.21 | 667.35 |

Analogous to Fig. 6, Fig. 8 shows a plot of the aggregate harm vs. the hardening budget for $m = 1$. Due to the inordinate amount of time required to solve the MILP, we show results only for the heuristic. Unlike the 13 node feeder, the aggregate harm in this case exhibits a steep drop initially before gradually tapering off. This tapering off reflects the fact that our cost functions were so chosen that no line could be hardened enough to reduce its repair time to zero; i.e., for every line $l$, we ensured that $p_l - \Delta p_l > 0$.

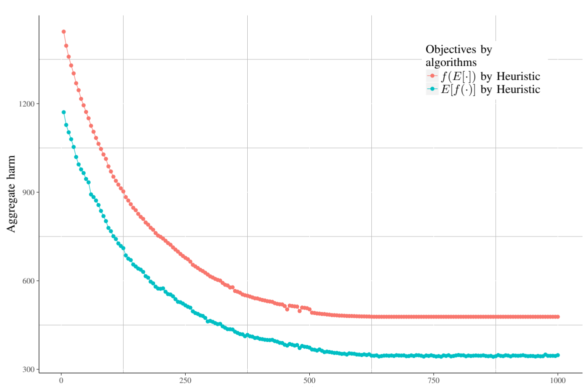



Fig. 8: Aggregate harm vs. hardening budget for the IEEE 37 node test feeder.

## C. IEEE 8500 node test feeder

Finally, we tested the performance of the heuristic algorithm on one instance of the IEEE 8500 node test feeder medium voltage subsystem [41] containing roughly 2500 edges. We did not attempt to solve the ILP model in this case, but the heuristic algorithm took just 9.36 seconds to solve this problem.

Analogous to Figs. 6 and 8, Fig. 9 shows a plot of the aggregate harm vs. the hardening budget for $m = 1$. Due to issues with computational time, we chose to plot the $f(\mathbb{E}[\cdot])$ measure only as a function of the budget using the heuristic algorithm.

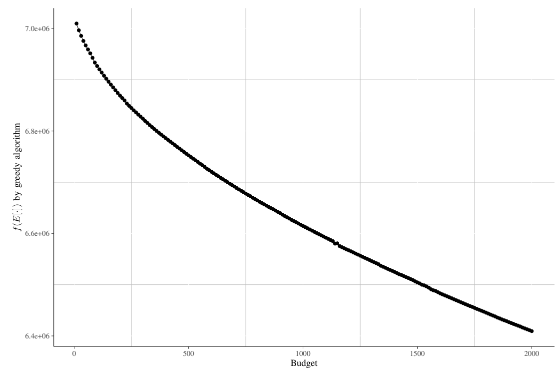



Fig. 9: Aggregate harm vs. hardening budget for the IEEE 8500 node test feeder.

Even if hardening decisions at the planning stage are made based on single crew operational scheduling, the resilience of the system would still improve if multiple crews are deployed at the operational stage. To emphasize this aspect, Fig. 10 shows the normalized improvement in harm,

$$\beta := \frac{H^m - H_A^m}{H^m} \tag{25}$$

for different values of $m$. The reduction in the repair time vector due to hardening, $\Delta\vec{p}_A$, was obtained using the iterative heuristic algorithm with a budget of $C = 2000$. For $m > 1$, the aggregate harms before and after hardening, $H^m$ and $H_A^m$, were determined from $m$-crew schedules obtained using Algorithm 2. The normalized improvement in harm shows a slight decrease (note the scale on the $y$-axis). This generally decreasing trend is understandable since the improvement in system resilience due to the availability of an increasing number of repair crews will gradually outweigh the improvement in resilience due to hardening with a limited budget. Nevertheless, even for $m = 50$, we can observe that the normalized improvement in harm due to hardening remains above 8%, even though the hardening decisions were made considering $m = 1$.


Fig. 10: Normalized improvement in harm, $\beta$ (see eqn. 25), as a function of the number of repair crews, $m$, for the IEEE 8500 node test feeder.

## VIII. Conclusions

In this paper, we investigated the problem of strategically hardening a distribution network to be resilient against natural disasters. Motivated by research on resilient infrastructure systems in civil engineering, we proposed an equivalent definition of resilience with a clear physical interpretation. This allows us to integrate the post disaster restoration process and the planning stage component hardening decision process into one problem, which, we argued, is necessary since both aspects ultimately contribute to system resilience. This is a major departure from most current research where the two aspects of resilience are treated separately. We first modeled the restoration problem as an MILP and the hardening problem as a stochastic program, which was reformulated using Jensen's inequality and approximated by single crew for computational tractability. Finally, we unified the sequencing and hardening aspects and proposed an integrated MILP model as well as an iterative heuristic algorithm. The expected component repair times are used to generate an optimal single crew repair sequence, based on which hardening decisions are made sequentially in a greedy manner. Simulations on IEEE standard test feeders show that the heuristic approach provides near-optimal solutions efficiently even for large networks.

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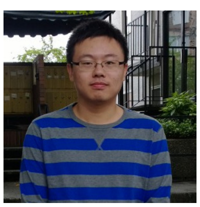

**Yushi Tan** received the B.S. degree from the Electrical Engineering Department of Tsinghua University in China, in 2013. He is currently pursuing Ph.D. degree in the University of Washington, Seattle, WA, USA. His research interests include power system resilience under natural disasters.

**Arindam K. Das** received his M.S and Ph.D in Electrical Engineering from the Univ. of Washington, Seattle, in 1999 and 2003 respectively. From 2003 to 2007, he was a post doctoral research associate jointly with the Dept. of Electrical Eng. and the Dept. of Aeronautics and Astronautics, Univ. of Washington, Seattle. From 2007 t0 2012, he worked as a Research Scientist at the Applied Physics Laboratory, Univ. of Washington, Seattle. He is currently an Assistant Professor in Electrical Engineering at Eastern Washington University, Cheney, WA and an Affiliate Assistant Professor in Electrical Engineering at the Univ. of Washington, Seattle. He is also an independent consultant to several technology and biotech companies. His current research interests are broadly in the areas of optimization theory and applications, machine learning, meta-heuristic optimization, and wireless networking.

**Payman Arabshahi** received his M.S. and PhD in Electrical Engineering from the University of Washington in 1990 and 1994 respectively. He is currently Associate Professor, and Associate Chair for Advancement at the University of Washington's Department of Electrical Engineering; Faculty Lead of Innovation Training at UWâĂŹs Collaborative Innovation Hub (CoMotion); and a principal research scientist with the UW Applied Physics Laboratory. From 1994-1996 he served on the faculty of the Electrical and Computer Engineering Department at the University of Alabama in Huntsville. From 1997-2006 he was on the senior technical staff of NASAâĂŹs Jet Propulsion Laboratory, in the Communications Architectures and Research Section, working on Earth orbit and deep space communication networks and protocols, and design of planetary exploration missions. While at JPL he also served as affiliate graduate faculty at the Department of Electrical Engineering at Caltech, where he taught the three-course graduate sequence on digital communications. He has been a guest editor of the IEEE Transactions on Neural Networks; and most recently, General Co-Chair of the 11th ACM International Conference on Underwater Networks & Systems in Shanghai. He has been the co-founder of, or advisor to, a number of technology startups. His interests are in entrepreneurship education, underwater and space communications, wireless networks, data mining and search, and signal processing.

**Daniel Krischen** (M'86-SM'91-FâĂŹ07) is the Donald W. and Ruth Mary Close Professor of Electrical Engineering at the University of Washington. Prior to joining the University of Washington, he taught for 16 years at The University of Manchester (UK). Before becoming an academic, he worked for Control Data and Siemens on the development of application software for utility control centers. He holds a PhD from the University of Wisconsin-Madison and an Electro-Mechanical Engineering degree from the Free University of Brussels (Belgium).