

A New Encounter Between Leader-Follower Tracking and Observer-Based Control: Towards Enhancing Robustness against Disturbances

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Abstract

This paper studies robust tracking control for a leader-follower multi-agent system (MAS) subject to disturbances. A challenging problem is considered here, which differs from those in the literature in two aspects. First, we consider the case when all the leader and follower agents are affected by disturbances, while the existing studies assume only the followers to suffer disturbances. Second, we assume the disturbances to be bounded only in rates of change rather than magnitude as in the literature. To address this new problem, we propose a novel observer-based distributed tracking control design. As a distinguishing feature, the followers can cooperatively estimate the disturbance affecting the leader to adjust their maneuvers accordingly, which is enabled by the design of the first-of-its-kind distributed disturbance observers. We build specific tracking control approaches for both first- and second-order MASs and prove that they can lead to bounded-error tracking, despite the challenges due to the relaxed assumptions about disturbances. We further perform simulation to validate the proposed approaches.

Keywords:

Multi-agent system, observer-based control, distributed control, unknown disturbances.

1. Introduction

A leader-follower multi-agent system (MAS) refers to an MAS in which a group of follower agents perform distributed control while interchanging information with their neighbors to collectively track the state of a leader agent. A large body of work has been developed recently to deal with the tracking control design under diverse challenging situations, e.g., complex dynamics, communication delays, noisy measurements, switching topologies, and limited energy budget, see [1–18] and the references therein. However, a problem that has received inadequate attention to date is the case when the agents are subjected to disturbances. In a real world, disturbances can result from unmodeled dynamics, change in ambient conditions, inherent variability of the dynamic process, and sensor noises. They can cause degradation and even failure of tracking control if not well addressed.

A lead is taken in [3] with the study of disturbance-robust leader-follower tracking. It presents a distributed control design that achieves tracking with a bounded error when magnitude-bounded disturbances affect the followers. This notion is extended in [19] to make the followers affected by disturbances enter a bounded region centered around the leader in finite time. Another finite-time tracking control approach is offered in [20], where the sliding mode control technique is used to suppress the effects of disturbances. It is noted that, while the control designs in these works yield robustness, they are based on upper bounds of the disturbances. By contrast, a different way is to capture the disturbances by designing some observers and then adjust the control run based on the disturbance estimation. Obtaining an explicit knowledge of disturbances, this approach can advantageously reduce conservatism in control and thus enhance the tracking performance further. In [21, 22], disturbance observers are developed and integrated into tracking controllers such that a follower can estimate and offset the local disturbance interfering with its dynamics

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during tracking. The results in both studies point to the effectiveness of disturbance observers for improving tracking accuracy — for instance, the tracking errors can approach zero despite non-zero disturbances under certain conditions. However, other than these two, there are no more studies on this subject to the best of our knowledge. This leaves many problems still open. Meanwhile, the potential of the disturbance-observer-based approach is still far from being fully explored. It is noteworthy that observer-based tracking control has been investigated in a few works, e.g., [3, 19, 23–25], but observers in these studies are meant to infer various state or input variables rather than disturbances.

In this study, we uniquely focus on an open problem: can we enable distributed tracking control when not only the followers but also the leader are affected by unknown disturbances and when only the rates of change of the disturbances are bounded? The state of the art, e.g., [3, 19–22], generally considers that disturbances plague just the followers and that they are bounded in magnitude or approach fixed values as time goes by. The leader’s dynamics, however, can also involve disturbances from a practical viewpoint. For example, consider an MAS composed of a few mobile ground robots, the changes in the slope of the road act as disturbances on every robot including the leader. The same can be said for the wind affecting a group of unmanned aerial vehicles. Such disturbances are more difficult to be rejected because the leader cannot measure them and share the information with any of the followers. Therefore, the tracking performance may be damaged when this occurs. Furthermore, it is usually desirable to relax the assumptions about disturbances to enhance the practical robustness of the control design. In [19–22], the disturbances are assumed to be bounded in magnitude. However, we wish to require the disturbances to be bounded in rates of change. This relaxation will be realistically beneficial for dealing with large disturbances but also present more complexity to capture and suppress the disturbances. It must be pointed out that the observer designs in [3, 19–25] cannot be extended to deal with the considered problem, due to the more challenging presence and nature of the disturbances. Hence, a solution is still absent from the literature.

To address the above problem, we develop a novel observer-based distributed tracking control framework, which is the main contribution of this paper. Different from the previous studies, this framework builds on the notion that a follower can gain a real-time awareness of not only its own but also the leader’s dynamics through distributed estimation. We hence design a set of new observers and particularly, distributed disturbance observers that, for the first time, can enable the followers to collectively infer the disturbance affecting the leader. We perform the observer-based tracking control design for both first- and second-order MASs. We then conduct theoretical analysis of the proposed approaches. We show that, even though disturbances are imposed on all the agents, the tracking errors are still upper bounded (bounded-error tracking) as long as the rates of change of the disturbances are bounded. Further, the tracking errors will approach zero (zero-error tracking) if the disturbances converge to certain fixed points. We finally present simulations to validate the proposed approaches.

The rest of this paper is organized as follows. Section 2 introduces some preliminaries. Section 3 considers a leader-follower MAS with first-order dynamics, develops an observer-based distributed tracking control approach, and analyzes its performance rigorously. Section 4 proceeds to study the second-order MASs and develops a more sophisticated tracking control approach. Numerical simulation is offered in Section 5 to illustrate the effectiveness of the proposed results. Finally, Section 6 gathers our concluding remarks.

2. Preliminaries

This section introduces notation and basic concepts about graph theory and nonsmooth analysis.

2.1. Notation

The notation used throughout this paper is standard. The n -dimensional Euclidean space is denoted as $\mathbb{R}^n$. For a vector, $\|\cdot\|_1$ denotes the 1-norm, and $\|\cdot\|$ stands for the 2-norm. The notation $\mathbf{1}$ represents a column vector of ones. We let $\text{diag}(\dots)$ and $\det(\cdot)$ represent a block-diagonal matrix and the determinant of a matrix, respectively. The eigenvalues of an $N \times N$ matrix are $\lambda_i(\cdot)$ for $i = 1, 2, \dots, N$. The minimum and maximum eigenvalues of a real, symmetric matrix are denoted as $\underline{\lambda}(\cdot)$ and $\bar{\lambda}(\cdot)$. Matrices are assumed to be compatible for algebraic operations if their dimensions are not explicitly stated. A C^k function is a function with k continuous derivatives.

2.2. Graph Theory

We use a graph to describe the information exchange topology for a leader-follower MAS. First, consider a network composed of N independent followers, and model the interaction topology as an undirected graph. The follower graph then is expressed as $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \dots, N\}$ is the vertex set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set containing unordered pairs of vertices. A path is a sequence of connected edges in a graph. The follower graph is connected if there is a path between every pair of vertices. The neighbor set of agent i is denoted as $\mathcal{N}_i$, which includes all the agents in communication with it. The adjacency matrix of $\mathcal{G}$ is $A = [a_{ij}] \in \mathbb{R}^{N \times N}$, which has non-negative elements. The element $a_{ij} > 0$ if and only if $(i, j) \in \mathcal{E}$, and moreover, $a_{ii} = 0$ for all $i \in \mathcal{V}$. For the Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N \times N}$, $l_{ij} = -a_{ij}$ if $i \neq j$ and $l_{ii} = \sum_{k \in \mathcal{N}_i} a_{ik}$. The leader is numbered as vertex 0 and can send information to its neighboring followers. Then, we have a graph $\tilde{\mathcal{G}}$, which consists of graph $\mathcal{G}$, vertex 0 and edges from the leader to its neighbors. The leader is globally reachable in $\tilde{\mathcal{G}}$ if there is a path in graph $\tilde{\mathcal{G}}$ from every vertex i to vertex 0. To express the graph $\tilde{\mathcal{G}}$ more precisely, we denote the leader adjacency matrix associated with $\tilde{\mathcal{G}}$ by $B = \text{diag}(b_1, \dots, b_N)$, where $b_i > 0$ if the leader is a neighbor of agent i and $b_i = 0$ if it is not. The following lemmas will be useful.

Lemma 1. [26, Lemma 1.1] *The Laplacian matrix $L(\mathcal{G})$ has at least one zero eigenvalue, and all the nonzero eigenvalues are positive. Furthermore, $L(\mathcal{G})$ has a simple zero eigenvalue and all the other eigenvalues are positive if and only if $\mathcal{G}$ is connected.*

Lemma 2. [27, Lemma 4] *The matrix $H = L + B$ is positive stable if and only if vertex 0 is globally reachable in $\tilde{\mathcal{G}}$.*

2.3. Nonsmooth Analysis

Consider the following discontinuous dynamical system

$$\dot{x} = f(x), \quad x \in \mathbb{R}^n, \quad x(0) = x_0 \in \mathbb{R}^n, \quad (1)$$

where $f(x) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is defined almost everywhere (a.e.). In other words, it is defined everywhere for $x \in \mathbb{R}^n \setminus W$, where W is a subset of $\mathbb{R}^n$ of Lebesgue measure zero. Moreover, $f(x)$ is Lebesgue measurable in an open region and locally bounded. A vector variable $x(\cdot) \in \mathbb{R}^n$ is a Filippov solution of (1) on $[t_0, t_1]$ if $x(\cdot)$ is absolutely continuous on $[t_0, t_1]$ while for almost all $t \in [t_0, t_1]$, satisfying the following differential inclusion:

$$\dot{x} \in \mathcal{K}[f](x) \triangleq \bigcap_{\delta > 0} \bigcap_{\mu(M)=0} \text{co}\{f(B(x, \delta) \setminus M)\}, \quad (2)$$

where $\bigcap_{\mu(M)=0}$ represents the intersection over all sets M of Lebesgue measure zero, $\text{co}(\cdot)$ denotes the closure of a convex hull and $B(x, \delta)$ denotes an open ball of radius δ centered at x . Let $V(x) : \mathbb{R}^n \rightarrow \mathbb{R}$ be a locally Lipschitz continuous function. Its Clarke's generalized gradient is given by

$$\partial V(x) \triangleq \text{co} \left\{ \lim_{i \rightarrow \infty} \nabla V(x_i) \mid x_i \rightarrow x, x_i \notin \Omega_V \cup M \right\},$$

where $\nabla V(x)$ is the conventional gradient, and Ω_V denotes a set of Lebesgue measure zero which includes all points where $\nabla V(x)$ does not exist. Moreover, the set value of the derivative of V associated with (1) is defined as

$$\mathcal{L}\dot{V}(x) = \{a \in \mathbb{R} \mid \exists v \in \mathcal{K}[f](x) \text{ such that } \zeta \cdot v = a, \forall \zeta \in \partial V(x)\}.$$

The following lemma will be used later.

Lemma 3. [28, Page 32] *Let $x(t) : [t_0, t_1] \rightarrow \mathbb{R}^n$ be a Filippov solution of (2). Let $V(x)$ be a locally Lipschitz and regular function. Then $d/dt(V(x(t)))$ exists a.e. and $dV(x(t))/dt \in \mathcal{L}\dot{V}(x)$ a.e.*

2.4. Assumptions

Throughout this paper, we consider a leader-follower MAS with $N + 1$ agents. The agents are numbered sequentially. The one numbered as 0 serves as the leader, and the other agents are followers. Each agent is driven by an input u_i and simultaneously affected by an external disturbance f_i for $i = 0, 1, \dots, N$. We make the following assumptions.

Assumption 1. *The input $u_0 \in C^1$ has a bounded first-order derivative, satisfying $|\dot{u}_0| \leq w$, where w is unknown.*

Assumption 2. *The external disturbance f_i for $i = 0, 1, \dots, N$ has a bounded first-order derivative, i.e., $\|\dot{f}_0 \mathbf{1}_{N \times 1}\| \leq q_0$ and $\left\| \begin{bmatrix} \dot{f}_1 & \dot{f}_2 & \dots & \dot{f}_N \end{bmatrix}^T \right\| \leq q_1$, where $q_0, q_1 \geq 0$.*

3. First-Order Leader-Follower Tracking

This section studies first-order leader-follower tracking with disturbances. We develop an observer-based control approach, pivoting the design on a set of observers to make a follower aware of the leader's and its own disturbances. We further analyze the closed-loop stability of the proposed approach.

3.1. Problem Formulation

Consider an MAS with $N + 1$ agents, in which agent 0 is the leader and the others are followers. An agent's dynamics is given by

$$\dot{x}_i = u_i + f_i, \quad x_i, u_i, f_i \in \mathbb{R}, \quad i = 0, 1, \dots, N, \quad (3)$$

where x_i is the position, u_i the control input equivalent to the velocity maneuver, and f_i the unknown disturbance. Suppose that Assumptions 1-2 hold. Here, the objective is to design a distributed control law for u_i such that each follower can control its dynamics to track the leader's trajectory via exchanging information with its neighbors.

Remark 1. Compared with previous studies, the problem setting here is more generic and applicable to a wide range of practical scenarios. Below, we outline a comparison with [3, 19–22], which are the main references about tracking control with disturbances and henceforth referred to as the existing literature. First, this work considers an input-driven leader, while the leader is usually assumed to be input-free in the literature. Assumption 1 only requires the leader's input to be bounded in rate of change (with the bound unknown), which can be easily satisfied since practical actuators only allow limited ramp-ups. Second, Assumption 2 imposes disturbances on all the leader and follower agents, while the literature assumes only followers to be affected by disturbances. Note that the case when a disturbance is inflicted on the leader is nontrivial. This is because the leader's disturbance is very difficult to be determined by the followers, especially in a distributed network where many followers cannot directly interchange information with the leader. Further, the disturbances are assumed to have only bounded rates of change rather than bounded magnitude as required in the literature. This can be greatly useful for dealing with very large disturbances. From the comparison, we conclude that the considered problem is less restrictive than the predecessors, which still remains an open challenge. •

3.2. Proposed Algorithm

Given the above problem setting, we propose an observer-based tracking control approach. The development begins with the design of a distributed linear continuous controller for a follower (say, follower i). It crucially incorporates the estimation of three unknown variables, u_0 , f_0 and f_i , enabling follower i to maneuver through simultaneously emulating the input and disturbance driving the leader and offsetting the local disturbance. We subsequently construct three observers to achieve the estimation to be integrated with the controller.

Considering follower i , we propose to design its controller as follows:

$$u_i = -k \left[\sum_{j \in \mathcal{N}_i} a_{ij}(x_i - x_j) + b_i(x_i - x_0) \right] + \hat{u}_{0,i} + \hat{f}_{0,i} - \hat{f}_i, \quad (4)$$

where $k > 0$ is the control gain, $\hat{f}_{0,i}$ and $\hat{u}_{0,i}$ are follower i 's respective estimates of the leader's disturbance f_0 and input u_0 , and $\hat{f}_i$ is follower i 's estimate of its own disturbance f_i . Note that $b_i > 0$ if the leader is agent i 's neighbor and $b_i = 0$ if it is not. In (4), the term $-\sum_{j \in \mathcal{N}_i} a_{ij}(x_i - x_j) - b_i(x_i - x_0)$ is employed to drive follower i approaching the leader; the term $\hat{u}_{0,i} + \hat{f}_{0,i}$ ensures that follower i applies maneuvers consistent with the leader's input and disturbance; the term $-\hat{f}_i$ is used to cancel the local disturbance. For this controller, we build a series of observers as shown below to estimate u_0 , f_0 and f_i , respectively.

To begin with we propose an observer as follows to obtain $\hat{u}_{0,i}$, which is based on the design in [16]:

$$\dot{\hat{u}}_{0,i} = - \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) - b_i(\hat{u}_{0,i} - u_0) - d_i \cdot \text{sgn} \left[\sum_{j \in \mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + b_i(\hat{u}_{0,i} - u_0) \right], \quad (5a)$$

$$\dot{\hat{d}}_i = \tau_i \left| \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + b_i(\hat{u}_{0,i} - u_0) \right|, \quad (5b)$$

for $i = 1, 2, \dots, N$, where d_i is the observer gain and $\tau_i > 0$ a scalar coefficient. For (5a), the leading term on the right-hand side, $-\sum_{j \in N_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) - b_i(\hat{u}_{0,i} - u_0)$, is used to make $\hat{u}_{0,i}$ approach u_0 ; the signum function $\text{sgn}(\cdot)$ is aimed to overcome the effects of u_0 's first-order dynamics, i.e., $\dot{u}_0$, and ensure the convergence of $\hat{u}_{0,i}$ to u_0 . Note that the observer gain, d_i , is adaptively adjusted through (5b). As such, a reasonable gain can be determined even if the upper bound of $\dot{u}_0$, w , is unknown (see Assumption 1).

The following disturbance observer is proposed for follower i to estimate f_0 :

$$\dot{z}_{f0,i} = -b_i z_i - b_i^2 x_0 - \sum_{j \in N_i} a_{ij}(\hat{f}_{0,i} - \hat{f}_{0,j}) - b_i u_0, \quad (6a)$$

$$\hat{f}_{0,i} = z_{f0,i} + b_i x_0, \quad (6b)$$

where $z_{f0,i}$ is the internal state. The design of (6) is inspired by [29], in which a centralized disturbance observer is developed for a single plant. Here, transforming the original design, we build the distributed observer as above such that follower i can estimate f_0 in a distributed manner.

The last observer, designed as follows, enables follower i able to infer the disturbance f_i inherent in its own dynamics:

$$\dot{z}_{f,i} = -l z_{f,i} - l^2 x_i + u_i, \quad (7a)$$

$$\hat{f}_i = z_{f,i} + l x_i. \quad (7b)$$

Here, $l > 0$ is the observer gain, and $z_{f,i}$ is this observer's internal state.

Combining (4)-(7), we obtain a complete description of an observer-based distributed tracking controller. Next, we will analyze its closed-loop stability.

3.3. Stability Analysis

Define $e_{u,i} = \hat{u}_{0,i} - u_0$, which is the input estimation error. According to (5), the closed-loop dynamics of $e_{u,i}$ can be written as

$$\dot{e}_{u,i} = -b_i e_{u,i} - \sum_{j \in N_i} a_{ij}(e_{u,i} - e_{u,j}) - d_i \cdot \text{sgn} \left[\sum_{j \in N_i} a_{ij}(e_{u,i} - e_{u,j}) + b_i e_{u,i} \right] - \dot{u}_0.$$

Further, let us concatenate $e_{u,i}$ for $i = 1, 2, \dots, N$ and define $e_u = [e_{u,1} \quad e_{u,2} \quad \dots \quad e_{u,N}]^\top$. The dynamics of e_u can be expressed as

$$\dot{e}_u = -H e_u - D \cdot \text{sgn}(H e_u) - \dot{u}_0 \mathbf{1}, \quad (8)$$

where $H = B + L$ and $D = \text{diag}(d_1, d_2, \dots, d_N)$. It is noted that the signum-function-based term at the right-hand side of (8) is discontinuous, measurable and locally bounded. Hence, there exists a Filippov solution to (8), which is represented by a differential inclusion as follows:

$$\dot{e}_u \in \mathcal{K}[-H e_u - D \cdot \text{sgn}(H e_u) - \dot{u}_0 \mathbf{1}].$$

The following lemma characterizes the convergence of e_u .

Lemma 4. *If Assumption 1 holds, then*

$$\lim_{t \rightarrow \infty} |e_{u,i}| = 0, \quad (9)$$

for $i = 1, 2, \dots, N$.

Proof: By Lemmas 1 and 2, H is positive definite. For (8), consider

$$\bar{V}_1(e_u) = \frac{1}{2} e_u^\top H e_u, \quad \tilde{V}_1 = \sum_{i=1}^N \frac{(d_i - \beta)^2}{2\tau_i},$$

where $\beta \geq w$, as noted. We then take $V_1 = \bar{V}_1(e_u) + \tilde{V}_1$ as a Lyapunov functional candidate. For the set-valued Lie derivative of $\bar{V}_1(e_u)$, we have

$$\begin{aligned}\mathcal{L}\dot{\bar{V}}_1 &= \mathcal{K} \left[-e_u^\top H^2 e_u - e_u^\top H D \cdot \text{sgn}(H e_u) - e_u^\top H \dot{u}_0 \mathbf{1} \right] \\ &= \mathcal{K} \left[- \sum_{i=1}^N d_i \left(\sum_{j \in N_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + b_i(\hat{u}_{0,i} - u_0) \right)^\top \right. \\ &\quad \cdot \text{sgn} \left(\sum_{j \in N_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + b_i(\hat{u}_{0,i} - u_0) \right) - e_u^\top H^2 e_u - e_u^\top H \dot{u}_0 \mathbf{1} \left. \right] \\ &\leq - \sum_{i=1}^N d_i \left| \sum_{j \in N_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + b_i(\hat{u}_{0,i} - u_0) \right| - e_u^\top H^2 e_u + w \|H e_u\|_1\end{aligned}$$

by the fact that $\mathcal{K}[f] = \{f\}$ if f is continuous. Invoking Lemma 3, we obtain that $\dot{\bar{V}}_1 \in \mathcal{L}\dot{\bar{V}}_1$. Then, the derivative of V_1 is given by

$$\begin{aligned}\dot{V}_1 &= \dot{\bar{V}}_1 + \dot{\tilde{V}}_1 = \dot{\bar{V}}_1 + \sum_{i=1}^N \frac{(d_i - \beta)\dot{d}_i}{\tau_i} \\ &\leq - \sum_{i=1}^N d_i \left| \sum_{j \in N_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + b_i(\hat{u}_{0,i} - u_0) \right| - e_u^\top H^2 e_u + w \|H e_u\|_1 \\ &\quad + \sum_{i=1}^N (d_i - \beta) \left| \sum_{j \in N_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + b_i(\hat{u}_{0,i} - u_0) \right| \\ &= -e_u^\top H^2 e_u - (\beta - w) \|H e_u\|_1.\end{aligned}$$

It is noted that $e_u^\top H^2 e_u \geq 0$. This, in addition to the fact that there always exists a β such that $\beta \geq w$ by Assumption 1, ensures $\dot{V}_1 \leq 0$. As a result, $V_1(e_u)$ is nonincreasing, which implies that e_u and d_i are bounded. From (5b), it follows that d_i is monotonically increasing, indicating that d_i should converge to some finite value. In the meantime, since V_1 is nonincreasing and lower-bounded by zero, it should approach a finite limit. Defining $s(t) = \int_0^t e_u^\top(\tau) H^2 e_u(\tau) d\tau$, we have $s(t) \leq V_1(0) - V_1(t)$ by integrating $\dot{V}_1(e_u) \leq -e_u^\top H^2 e_u$. Hence, $s(t)$ will also approach a finite limit. Due to the boundedness of e_u and $\dot{e}_u$, $\dot{s}$ is also bounded. This implies that $\dot{s}$ is uniformly continuous. By Barbalat's Lemma [30], $\dot{s}(t) \rightarrow 0$ as $t \rightarrow \infty$, indicating that $e_u \rightarrow 0$ as $t \rightarrow \infty$. •

Now, consider the distributed observer for f_0 . Define $e_{0f,i} = \hat{f}_{0,i} - f_0$, which is follower i 's estimation error for f_0 . Using (6), the dynamics of $e_{0f,i}$ is given by

$$\dot{e}_{0f,i} = -b_i e_{0f,i} - \sum_{j \in N_i} a_{ij}(\hat{f}_{0,i} - \hat{f}_{0,j}) - \dot{f}_0.$$

Then, defining $e_{0f} = [e_{0f,1} \quad e_{0f,2} \quad \cdots \quad e_{0f,N}]^\top$, we have

$$\dot{e}_{0f} = -H e_{0f} - \dot{f}_0 \mathbf{1}. \quad (10)$$

The following lemma reveals the upper boundedness of e_{0f} under Assumption 2.

Lemma 5. *If Assumption 2 holds, then*

$$\|e_{0f}(t)\| \leq \|e_{0f}(0)\| + \frac{q_0}{\underline{\lambda}(H)}, \quad t > 0, \quad (11a)$$

$$\lim_{t \rightarrow \infty} \|e_{0f}(t)\| \leq \frac{q_0}{\underline{\lambda}(H)}. \quad (11b)$$

Proof: Consider the Lyapunov function candidate $V_2(e_{0f}) = \frac{1}{2}e_{0f}^\top e_{0f}$ for (10). According to Assumption 2, we have

$$\dot{V}_2(e_{0f}) = -e_{0f}^\top H e_{0f} - e_{0f}^\top \dot{f}_0 \mathbf{1} \leq -\underline{\lambda}(H)\|e_{0f}\|^2 + \|e_{0f}\|\|\dot{f}_0\| \leq -\underline{\lambda}(H)\|e_{0f}\|^2 + q_0\|e_{0f}\|.$$

The above inequality can be rewritten as

$$\dot{V}_2 \leq -2\underline{\lambda}(H)V_2 + \sqrt{2}q_0\sqrt{V_2}.$$

It then follows that

$$\sqrt{V_2(t)} \leq \sqrt{V_2(0)}e^{-\underline{\lambda}(H)t} + \frac{\sqrt{2}q_0}{2\underline{\lambda}(H)}(1 - e^{-\underline{\lambda}(H)t}) \leq \sqrt{V_2(0)} + \frac{\sqrt{2}q_0}{2\underline{\lambda}(H)}. \quad (12)$$

Then, (11a) can result from (12) because $\sqrt{V_2} = \frac{\sqrt{2}}{2}\|e_{0f}\|$. Meanwhile, for the first inequality in (12), taking the limits of both sides as $t \rightarrow \infty$ would yield (11b).

For the estimation of f_i , define the error as $e_{f,i} = \hat{f}_i - f_i$ and further the vector $e_f = [e_{f,1} \ e_{f,2} \ \dots \ e_{f,N}]^\top$. By (7), the dynamics of e_f is governed by

$$\dot{e}_f = -le_f - \dot{f}, \quad (13)$$

where $\dot{f} = [\dot{f}_1 \ \dot{f}_2 \ \dots \ \dot{f}_N]^\top$. The next lemma shows that the error e_f is bounded under Assumption 2. Its proof is similar to that of Lemma 5 and thus omitted here.

Lemma 6. *If Assumption 2 holds, then*

$$\begin{aligned} \|e_f(t)\| &\leq \|e_f(0)\| + \frac{q_1}{l}, \quad t > 0 \\ \lim_{t \rightarrow \infty} \|e_f(t)\| &\leq \frac{q_1}{l}. \end{aligned}$$

With the above results, we are now in a good position to characterize the properties of the tracking error. Define follower i 's tracking error as $e_i = x_i - x_0$, and put together e_i for $i = 1, 2, \dots, N$ to form the vector $e = [e_1 \ e_2 \ \dots \ e_N]^\top$. Using (3) and (4), it can be derived that the dynamics of e can be described as

$$\dot{e} = -kHe + e_{0f} + e_u - e_f. \quad (14)$$

The following theorem provides a key technical result.

Theorem 1. *Suppose that Assumptions 1 and 2 hold. Then,*

$$\|e(t)\| \leq \|e(0)\| + \frac{\|e_{0f}(0)\| + \|e_u(0)\| + \|e_f(0)\| + \frac{q_0}{\underline{\lambda}(H)} + \frac{q_1}{l}}{k\underline{\lambda}(H)}, \quad (15a)$$

$$\lim_{t \rightarrow \infty} \|e\| \leq \frac{\frac{q_0}{\underline{\lambda}(H)} + \frac{q_1}{l}}{k\underline{\lambda}(H)}. \quad (15b)$$

Proof: Take the Lyapunov function candidate $V_3(e) = \frac{1}{2}e^\top e$ for (14). Consider its derivative:

$$\begin{aligned} \dot{V}_3 &= -ke^\top He + e^\top e_{0f} + e^\top e_u - e^\top e_f \\ &\leq -k\underline{\lambda}(H)\|e\|^2 + \|e\| \cdot \|e_{0f}\| + \|e\| \cdot \|e_u\| + \|e\| \cdot \|e_f\|, \end{aligned}$$

where $\underline{\lambda}(H) > 0$. Equivalently, we have

$$\dot{V}_3 \leq -2k\underline{\lambda}(H)V_3 + \sqrt{2}(\|e_{0f}\| + \|e_u\| + \|e_f\|)\sqrt{V_3}.$$

Then,

$$\begin{aligned}\sqrt{V_3(t)} &\leq \sqrt{V_3(0)}e^{-k\lambda(H)t} + \frac{\sqrt{2}(\|e_{0f}(t)\| + \|e_u(t)\| + \|e_f(t)\|)}{2k\lambda(H)}(1 - e^{-k\lambda(H)t}) \\ &\leq \sqrt{V_3(0)} + \frac{\sqrt{2}(\|e_{0f}(t)\| + \|e_u(t)\| + \|e_f(t)\|)}{2k\lambda(H)},\end{aligned}$$

which, based on Lemmas 4-6, indicates (15a)-(15b). •

Theorem 1 shows that the proposed observer-based controller can make each follower track the leader with bounded position errors despite the disturbances. Therefore, we can say that the influence of the disturbances is effectively suppressed and that tracking is achieved in a practically meaningful manner.

Remark 2. For the proposed controller, the tracking performance will be further improved if the disturbances satisfy some stricter conditions. In particular, it is noteworthy that perfect or zero-error tracking can be attained if the disturbances see their rates of change gradually settle down to zero, i.e., $\dot{f}_i(t) \rightarrow 0$ as $t \rightarrow \infty$ for $i = 0, 1, \dots, N$. The proof can be developed following similar lines as above and is omitted here. •

4. Second-Order Leader-Follower Tracking

This section considers leader-follower tracking control for a second-order MAS. An agent's dynamics now involves position, velocity, acceleration and disturbance:

$$\begin{cases} \dot{x}_i = v_i, & x_i \in \mathbb{R}, \\ \dot{v}_i = u_i + f_i, & v_i \in \mathbb{R}, \end{cases} \quad (16)$$

for $i = 0, 1, \dots, N$. Here, x_i is the position, v_i the velocity, u_i the acceleration input, and f_i the disturbance. Still, agent 0 is the leader, and the others are followers numbered from 1 to N . We continue to apply Assumptions 1-2 here and set the objective of making the followers achieve bounded-error tracking of the leader in the presence of the disturbances.

For a general problem formulation, we further assume that no velocity sensor is deployed on the leader and followers. Hence, there are no velocity measurements throughout the tracking process. The absence of the velocity information, together with the unknown disturbances, makes the tracking control problem more complex than in the first-order case, thus requiring a substantial sophistication of the observer-based control approach in Section 3. Here, we will custom build an observer-based tracking controller and develop new velocity and disturbance observers.

Consider follower i . We propose the following distributed controller:

$$u_i = -k \left[\sum_{j \in \mathcal{N}_i} a_{ij}(x_i - x_j) + b_i(x_i - x_0) \right] - (\hat{v}_i - \hat{v}_{0,i}) + \hat{u}_{0,i} + \hat{f}_{0,i} - \hat{f}_i, \quad (17)$$

where $k > 0$ is the control gain. In addition, $\hat{u}_{0,i}$, $\hat{v}_{0,i}$, $\hat{f}_{0,i}$, $\hat{v}_i$ and $\hat{f}_i$ are, respectively, follower i 's estimates of u_0 , v_0 , f_0 , v_i and f_i . The terms $-\sum_{j \in \mathcal{N}_i} a_{ij}(x_i - x_j) - b_i(x_i - x_0)$ and $-(\hat{v}_i - \hat{v}_{0,i})$ are used to enable the follower to track the leader in both position and velocity; the term $\hat{u}_{0,i} + \hat{f}_{0,i}$ is used to make the follower steer itself with a maneuvering input close to the combined input and disturbance driving the leader; the term $-\hat{f}_i$ is meant to offset the local disturbance.

With the above controller structure, we need to construct observers that can obtain the needed estimates. First, it is noted that the input observer of u_0 in (5) can be applied here without any change, and its convergence property as shown in Lemma 4 also holds in this case. Then, we develop the following observer such that follower i can estimate the leader's unknown velocity:

$$\dot{z}_{v0,i} = -b_i z_{v0,i} - b_i^2 x_0 - \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{v}_{0,i} - \hat{v}_{0,j}) + \hat{f}_{0,i} + \hat{u}_{0,i}, \quad (18a)$$

$$\hat{v}_{0,i} = z_{v0,i} + b_i x_0, \quad (18b)$$

where $z_{v0,i}$ is the internal state of this observer. Follower i 's observer for the leader's disturbance is then proposed as

$$\dot{z}_{f0,i} = -z_{f0,i} - \hat{v}_{0,i} - \hat{u}_{0,i}, \quad (19a)$$

$$\hat{f}_{0,i} = z_{f0,i} + \hat{v}_{0,i}. \quad (19b)$$

The next observer enables follower i to estimate its own velocity:

$$\dot{z}_{v,i} = -l z_{v,i} - l^2 x_i + \hat{f}_i + u_i, \quad (20a)$$

$$\hat{v}_i = z_{v,i} + l x_i. \quad (20b)$$

Here, $l > 0$ is the gain for this observer, and $z_{v,i}$ the internal state. The final observer is aimed to allow follower i to infer its local disturbance. It is designed as

$$\dot{z}_{f,i} = -z_{f,i} - \hat{v}_i - u_i, \quad (21a)$$

$$\hat{f}_i = z_{f,i} + \hat{v}_i, \quad (21b)$$

where $z_{f,i}$ is the internal state.

From above, a complete observer-based distributed tracking controller can be built by putting together the control law (17) and the observers in (5) and (18)-(21). The following theorem is the main result about the closed-loop stability of the proposed controller.

Theorem 2. *Suppose that Assumptions 1-2 hold and apply the proposed distributed tracking controller given in (5) and (17)-(21) to the MAS in (16). Then, there exist $\delta > 0$ and $\epsilon > 0$ such that*

$$\left\| \begin{bmatrix} x_i(t) - x_0(t) \\ v_i(t) - v_0(t) \end{bmatrix} \right\| \leq \delta, \quad t > 0, \quad (22a)$$

$$\lim_{t \rightarrow \infty} \left\| \begin{bmatrix} x_i(t) - x_0(t) \\ v_i(t) - v_0(t) \end{bmatrix} \right\| \leq \epsilon, \quad (22b)$$

for $i = 1, 2, \dots, N$.

Proof: The proof is organized into three parts. Part a) proves that the coupled observers in (18)-(19) yield bounded-error estimation of v_0 and f_0 ; Part b) shows that the observers in (20)-(21) lead to bounded errors when estimating v_i and f_i ; finally, based on Parts a) and b), Part c) demonstrates the upper boundedness of the position and velocity tracking errors when the proposed controller is applied.

Part a): Define the estimation errors of the observers in (18)-(19) as $e_{0v,i} = \hat{v}_{0,i} - v_0$ and $e_{0f,i} = \hat{f}_{0,i} - f_0$. According to (16) and (18)-(19), their dynamics can be written as

$$\dot{e}_{0v,i} = -b_i e_{0v,i} - \sum_{j \in \mathcal{N}_i} a_{ij} (\hat{v}_{0,i} - \hat{v}_{0,j}) + \hat{u}_{0,i} - u_0 + \hat{f}_{0,i} - f_0,$$

$$\dot{e}_{0f,i} = -b_i e_{0v,i} - \sum_{j \in \mathcal{N}_i} a_{ij} (\hat{v}_{0,i} - \hat{v}_{0,j}) - \dot{f}_0.$$

Defining $e_{0vf} = [e_{0v,1} \quad \dots \quad e_{0v,N} \quad e_{0f,1} \quad \dots \quad e_{0f,N}]^\top$, we have

$$\dot{e}_{0vf} = Q_1 e_{0vf} + \ell_1, \quad (23)$$

where

$$Q_1 = \begin{bmatrix} -H & I \\ -H & 0 \end{bmatrix}, \quad \ell_1 = \begin{bmatrix} e_{0u} \\ -\dot{f}_0 \mathbf{1} \end{bmatrix}.$$

The characteristic polynomial of Q_1 is given by

$$\det(sI - Q_1) = \det \left(\begin{bmatrix} sI + H & -I \\ H & sI \end{bmatrix} \right) = \det(s^2 I + Hs + H) = \prod_{i=1}^N [s^2 + \lambda_i(H)s + \lambda_i(H)].$$

As is seen from above, the poles of Q_1 is stable since H is positive definite. Then, there must exist a positive definite matrix P_1 such that

$$P_1 Q_1 + Q_1^\top P_1 = -I.$$

For (23), take a Lyapunov function $V_4(e_{0vf}) = \frac{1}{2} e_{0vf}^\top P_1 e_{0vf}$. Consider its derivative:

$$\begin{aligned} \dot{V}_4 &= -\frac{1}{2} e_{0vf}^\top (P_1 Q_1 + Q_1^\top P_1) e_{0vf} + e_{0vf}^\top P_1 \ell_1 \\ &\leq -\frac{1}{2} \|e_{0vf}\|^2 + \|e_{0vf}\| \|P_1\| \|\ell_1\| \\ &\leq -\frac{1}{2} \|e_{0vf}\|^2 + \sqrt{e_u^2(0) + q_0^2} \|P_1\| \|e_{0vf}\|, \end{aligned}$$

where the fact suggested by Lemma 4 that e_u exponentially decreases to zero is used. The above inequality can be written equivalently as

$$\dot{V}_4 \leq -\alpha_1 V_4 + \beta_1 \sqrt{V_4},$$

with $\alpha_1 = 1/\bar{\lambda}(P_1)$ and $\beta_1 = \sqrt{2(e_u^2(0) + q_0^2)} \|P_1\| / \underline{\lambda}(P_1)$. Hence,

$$\sqrt{V_4(t)} \leq \sqrt{V_4(0)} e^{-\frac{\alpha_1 t}{2}} + \frac{\beta_1}{\alpha_1} (1 - e^{-\frac{\alpha_1 t}{2}}) \leq \sqrt{V_4(0)} + \frac{\beta_1}{\alpha_1}. \quad (24)$$

It then follows from (24) that

$$\begin{aligned} \|e_{0vf}(t)\| &\leq \sqrt{\frac{\bar{\lambda}(P_1)}{\underline{\lambda}(P_1)}} \|e_{0vf}(0)\| + \frac{\beta_1}{\alpha_1} = \sqrt{\frac{\bar{\lambda}(P_1)}{\underline{\lambda}(P_1)}} \|e_{0vf}(0)\| + \frac{\bar{\lambda}(P_1) \sqrt{2(e_u^2(0) + q_0^2)} \|P_1\|}{\underline{\lambda}(P_1)}, \\ \lim_{t \rightarrow \infty} \|e_{0vf}(t)\| &\leq \frac{\sqrt{2} \bar{\lambda}(P_1) q_0 \|P_1\|}{\underline{\lambda}(P_1)}. \end{aligned}$$

Part b): Consider the observers for v_i and f_i . Define their respective estimation errors as $e_{v,i} = \hat{v}_i - v_i$ and $e_{f,i} = \hat{f}_i - f_i$. Their dynamics can be described as

$$\begin{aligned} \dot{e}_{v,i} &= -l e_{v,i} + e_{f,i}, \\ \dot{e}_{f,i} &= -l e_{v,i} - \dot{f}_i. \end{aligned}$$

Defining $e_{vf} = [e_{v,1} \ \dots \ e_{v,N} \ e_{f,1} \ \dots \ e_{f,N}]^\top$, we have

$$\dot{e}_{vf} = Q_2 e_{vf} + \ell_2,$$

where

$$Q_2 = \begin{bmatrix} -lI & I \\ -lI & 0 \end{bmatrix}, \quad \ell_2 = [0 \quad -\dot{f}_1 \quad -\dot{f}_2 \quad \dots \quad -\dot{f}_N]^\top.$$

Following similar lines to Part a), we can obtain that e_{vf} is upper bounded:

$$\begin{aligned} \|e_{vf}(t)\| &\leq \sqrt{\frac{\bar{\lambda}(P_2)}{\underline{\lambda}(P_2)}} \|e_{vf}(0)\| + \frac{\beta_2}{\alpha_2} = \sqrt{\frac{\bar{\lambda}(P_2)}{\underline{\lambda}(P_2)}} \|e_{vf}(0)\| + \frac{\sqrt{2} \bar{\lambda}(P_2) q_1 \|P_2\|}{\underline{\lambda}(P_2)}, \\ \lim_{t \rightarrow \infty} \|e_{vf}(t)\| &\leq \frac{\sqrt{2} \bar{\lambda}(P_2) q_1 \|P_2\|}{\underline{\lambda}(P_2)}, \end{aligned}$$

where P_2 is a positive definite matrix satisfying $P_2 Q_2 + Q_2^\top P_2 = -I$.

Part c): Based on Parts a) and b), now let us move on to analyze the tracking performance under the controller in (17). Note that the position and velocity tracking errors are governed by

$$\begin{aligned}\dot{x}_i - \dot{x}_0 &= v_i - v_0, \\ \dot{v}_i - \dot{v}_0 &= -k_1 \left[\sum_{j \in \mathcal{N}_i} a_{ij}(x_i - x_j) + b_i(x_i - x_0) \right] - (v_i - v_0) - (\hat{v}_i - v_i) + (\hat{v}_{0,i} - v_0) \\ &\quad + \hat{f}_{0,i} - f_0 + f_i - \hat{f}_i + \hat{u}_{0,i} - u_0,\end{aligned}$$

for $i = 1, 2, \dots, N$. Define

$$e = \begin{bmatrix} x_1 - x_0 & \cdots & x_N - x_0 & v_1 - v_0 & \cdots & v_N - v_0 \end{bmatrix}^\top.$$

Then,

$$\dot{e} = Q_3 e + \ell_3, \quad (25)$$

where

$$Q_3 = \begin{bmatrix} 0 & I \\ -k_1 H & -I \end{bmatrix}, \quad \ell_3 = \begin{bmatrix} 0 \\ -e_v + e_{0v} + e_{0f} - e_f + e_u \end{bmatrix}.$$

The characteristic polynomial of Q_3 is

$$\det(sI - Q_3) = \det \begin{bmatrix} sI & -I \\ kH & sI + I \end{bmatrix} = \det(s^2 I + sI + kH) = \prod_{i=1}^n [s^2 + s + k\lambda_i(H)].$$

It is seen from above that Q_3 is stable because H is positive definite and $k > 0$. If Q_3 is stable, there exists a positive definite matrix P_3 such that

$$P_3 Q_3 + Q_3^\top P_3 = -I.$$

Define $V_5(e) = \frac{1}{2} e^\top P_3 e$ for (25). Then,

$$\begin{aligned}\dot{V}_5 &= -\frac{1}{2} e^\top (P_3 Q_3 + Q_3^\top P_3) e + e^\top P_3 \ell_3 \\ &\leq -\frac{1}{2} \|e\|^2 + \|e\| \cdot \|P_3\| \cdot \|\ell_3\| \\ &\leq -\frac{1}{2} \|e\|^2 + (\sqrt{2}\|e_{vf}\| + \sqrt{2}\|e_{0vf}\| + \|e_u\|) \cdot \|P_3\| \cdot \|e\|.\end{aligned}$$

It can be rewritten as

$$\dot{V}_5 \leq -\alpha_3 V_5 + \beta_3 \sqrt{V_5},$$

where $\alpha_3 = 1/\bar{\lambda}(P_3)$ and $\beta_3 = (2\|e_{vf}\| + 2\|e_{0vf}\| + \sqrt{2}\|e_u\|)\|P_3\|/\underline{\lambda}(P_3)$. Therefore, we have

$$\sqrt{V_5} \leq \sqrt{V_5(0)} e^{-\frac{\alpha_3 t}{2}} + \frac{\beta_3}{\alpha_3} (1 - e^{-\frac{\alpha_3 t}{2}}) \leq \sqrt{V_5(0)} + \frac{\beta_3}{\alpha_3}.$$

It then follows that e satisfies

$$\begin{aligned}\|e(t)\| &\leq \sqrt{\frac{\bar{\lambda}(P_3)}{\underline{\lambda}(P_3)}} \|e(0)\| + \frac{\beta_3}{\alpha_3} \leq \sqrt{\frac{\bar{\lambda}(P_3)}{\underline{\lambda}(P_3)}} \|e(0)\| + \frac{\bar{\lambda}(P_3)\|P_3\|}{\underline{\lambda}(P_3)} \left(2\sqrt{\frac{\bar{\lambda}(P_2)}{\underline{\lambda}(P_2)}} \|e_{vf}(0)\| \right. \\ &\quad \left. + \frac{2\sqrt{2}\bar{\lambda}(P_2)q_1\|P_2\|}{\underline{\lambda}(P_2)} + 2\sqrt{\frac{\bar{\lambda}(P_1)}{\underline{\lambda}(P_1)}} \|e_{0vf}(0)\| + \frac{2\bar{\lambda}(P_1)\sqrt{2(e_u^2(0) + q_0^2)\|P_1\|}}{\underline{\lambda}(P_1)} + \sqrt{2}\|e_u(0)\| \right), \quad (26)\end{aligned}$$

$$\lim_{t \rightarrow \infty} \|e(t)\| \leq \frac{\bar{\lambda}(P_3)\|P_3\|}{\underline{\lambda}(P_3)} \left(\frac{2\sqrt{2}\bar{\lambda}(P_2)q_1\|P_2\|}{\underline{\lambda}(P_2)} + \frac{2\sqrt{2}\bar{\lambda}(P_1)q_0\|P_1\|}{\underline{\lambda}(P_1)} \right). \quad (27)$$

By (26)-(27), there exist δ and ϵ such that (22a)-(22b) hold. This completes the proof. •

Theorem 2 reveals that, for a second-order MAS, the proposed observer-based controller can enable a follower to track the leader with bounded position and velocity errors when the disturbances are bounded in rates of change. Such an effectiveness is mainly attributed to the proposed observers, through which a follower can estimate the disturbance and velocity variables for tracking control. Further, similar to Remark 2, the tracking error $e_i(t) \rightarrow 0$ as $t \rightarrow \infty$ if $\dot{f}_i(t) \rightarrow 0$.

5. NUMERICAL STUDY

This section presents an illustrative simulation example to validate the proposed results. We consider a second-order MAS consisting of one leader and five followers, which share a communication topology shown in Figure 1(a). Vertex 0 is the leader, and vertices numbered from 1 to 5 are followers. The leader will only send information updates to follower 1, which is its only neighbor. The followers maintain bidirectional communication with their neighbors. For the topology graph, the edge-based weights are set to be unit for simplicity. The corresponding Laplacian matrix L is then given as follows:

$$L = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}.$$

Based on the communication topology, the leader adjacency matrix is $B = \text{diag}(1, 0, 0, 0, 0)$. We choose $l = 1$ and $k = 0.5$, respectively. The initial conditions include

$$x(0) = [0 \quad 3 \quad 0 \quad -2 \quad 1 \quad -1]^\top, \quad v(0) = [0 \quad 1 \quad -2 \quad 3 \quad 0 \quad -1]^\top.$$

Further,

$$u_0(t) = -2 \cos(0.1\pi t), \quad f_0(t) = -\cos(0.1\pi t), \quad f(t) = [0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5]^\top t.$$

Note that the disturbances enforced on the followers are bounded in rates of change but linearly diverge through time. This extreme setting is used to illustrate the effectiveness of disturbance rejection here. Apply the observer-based control approach in Section 4 to the MAS. The observer-based control approach in Section 4 is applied, with the simulation results outlined in Figure 1. Figure 1(b) and 1(c) demonstrate that the followers maintain bounded position and velocity tracking errors, which is in agreement with the results in Theorem 2. It is shown in Figure 1(d) that the observer for u_0 can gradually achieve accurate estimation through time. This is because the leader can send u_0 to its neighbor follower i , and with the implicit information propagation, the other followers can eventually estimate u_0 precisely. By comparison, the estimation of v_0 , f_0 , v_i and f_i is less accurate, as is seen in Figures 1(e)-1(h), because there is no measurement of them available. However, the differences or estimation errors are still bounded, matching the expectation as suggested by the theoretical analysis.

6. Conclusion

MASs have attracted significant research interest in the past decade due to their increasing applications. In this paper, we have studied leader-follower tracking for the first- and second-order MASs with unknown disturbances. Departing from the literature, we have considered a much less restrictive setting about disturbances. Specifically, disturbances can be applied to all the leader and followers and assumed to be bounded just in rates of change. This considerably relaxes the usual setting that only followers are affected by magnitude-bounded disturbances. To solve this problem, we have developed observer-based tracking control approaches, which particularly included the design of novel distributed disturbance observers for followers to estimate the leader's unknown disturbance. We have proved that the proposed approaches can enable bounded-error tracking in the considered disturbance setting. Simulation results further demonstrated the effectiveness of the proposed approaches.

The proposed framework and methodology provide an ample scope for future work. They can be extended to more sophisticated MASs, such as those with nonlinear dynamics, multiple leaders, or a directed communication topology.

In addition, the communication protocols or malicious cyber attacks are of growing importance for the design of future MASs. It will be an interesting question to leverage the proposed observer-based framework to address these challenges.

References

- [1] G. Wen, Z. Peng, A. Rahmani, Y. Yu, Distributed leader-following consensus for second-order multi-agent systems with nonlinear inherent dynamics, *International Journal of Systems Science* 45 (9) (2014) 1892–1901.
- [2] G. Shi, Y. Hong, Global target aggregation and state agreement of nonlinear multi-agent systems with switching topologies, *Automatica* 45 (5) (2009) 1165–1175.
- [3] Y. Hong, G. Chen, L. Bushnell, Distributed observers design for leader-following control of multi-agent networks, *Automatica* 44 (3) (2008) 846–850.
- [4] Z. Li, Z. Duan, G. Chen, L. Huang, Consensus of multiagent systems and synchronization of complex networks: A unified viewpoint, *IEEE Transactions on Circuits and Systems I: Regular Papers* 57 (1) (2010) 213–224.
- [5] W. Zhu, D. Cheng, Leader-following consensus of second-order agents with multiple time-varying delays, *Automatica* 46 (12) (2010) 1994–1999.
- [6] F. A. Yaghmaie, F. L. Lewis, R. Su, Leader-follower output consensus of linear heterogeneous multi-agent systems via output feedback, in: *Decision and Control (CDC), 2015 IEEE 54th Annual Conference on*, IEEE, 2015, pp. 4127–4132.
- [7] C. Ma, T. Li, J. Zhang, Consensus control for leader-following multi-agent systems with measurement noises, *Journal of Systems Science and Complexity* 23 (1) (2010) 35–49.
- [8] M. Nourian, P. E. Caines, R. P. Malhamé, M. Huang, Mean field lqg control in leader-follower stochastic multi-agent systems: Likelihood ratio based adaptation, *IEEE Transactions on Automatic Control* 57 (11) (2012) 2801–2816.
- [9] M. Ji, A. Muhammad, M. Egerstedt, Leader-based multi-agent coordination: Controllability and optimal control, in: *American Control Conference*, 2006, IEEE, 2006, pp. 1358–1363.
- [10] A. K. Bondhus, K. Y. Pettersen, J. T. Gravdahl, Leader/follower synchronization of satellite attitude without angular velocity measurements, in: *Decision and Control, 2005 and 2005 European Control Conference. CDC-ECC'05. 44th IEEE Conference on*, IEEE, 2005, pp. 7270–7277.
- [11] M. Defoort, A. Polyakov, G. Demesure, M. Djemai, K. Veluvolu, Leader-follower fixed-time consensus for multi-agent systems with unknown non-linear inherent dynamics, *IET Control Theory & Applications* 9 (14) (2015) 2165–2170.
- [12] J. Wu, Y. Shi, Consensus in multi-agent systems with random delays governed by a markov chain, *Systems & Control Letters* 60 (10) (2011) 863–870.
- [13] D. V. Dimarogonas, P. Tsiotras, K. J. Kyriakopoulos, Leader–follower cooperative attitude control of multiple rigid bodies, *Systems & Control Letters* 58 (6) (2009) 429–435.
- [14] S. Mou, M. Cao, A. S. Morse, Target-point formation control, *Automatica* 61 (2015) 113 – 118.
- [15] C. Yan, H. Fang, H. Chao, Energy-aware leader-follower tracking control for electric-powered multi-agent systems, *Control Engineering Practice* 79 (2018) 209–218.
- [16] C. Yan, H. Fang, Observer-based distributed leader-follower tracking control: A new perspective and results, *International Journal of Control* In press.
- [17] C. Yan, H. Fang, Leader-follower tracking control for multi-agent systems based on input observer design, in: *Proceedings of American Control Conference*, 2018, pp. 478–483.
- [18] C. Yan, H. Fang, H. Chao, Battery-aware time/range-extended leader-follower tracking for a multi-agent system, in: *Proceedings of Annual American Control Conference*, 2018, pp. 3887–3893.
- [19] S. Li, H. Du, X. Lin, Finite-time consensus algorithm for multi-agent systems with double-integrator dynamics, *Automatica* 47 (8) (2011) 1706–1712.
- [20] Y. Zhang, Y. Yang, Y. Zhao, G. Wen, Distributed finite-time tracking control for nonlinear multi-agent systems subject to external disturbances, *International Journal of Control* 86 (1) (2013) 29–40.
- [21] W. Cao, J. Zhang, W. Ren, Leader–follower consensus of linear multi-agent systems with unknown external disturbances, *Systems & Control Letters* 82 (2015) 64–70.
- [22] J. Sun, Z. Geng, Y. Lv, Z. Li, Z. Ding, Distributed adaptive consensus disturbance rejection for multi-agent systems on directed graphs, *IEEE Transactions on Control of Network Systems* In press.
- [23] Y. Zhao, Z. Duan, G. Wen, Y. Zhang, Distributed finite-time tracking control for multi-agent systems: An observer-based approach, *Systems & Control Letters* 62 (1) (2013) 22–28.
- [24] Y. Zhao, Z. Duan, G. Wen, G. Chen, Distributed finite-time tracking for a multi-agent system under a leader with bounded unknown acceleration, *Systems & Control Letters* 81 (2015) 8–13.
- [25] H. Du, S. Li, P. Shi, Robust consensus algorithm for second-order multi-agent systems with external disturbances, *International Journal of Control* 85 (12) (2012) 1913–1928.
- [26] W. Ren, Y. Cao, *Distributed Coordination of Multi-Agent Networks: Emergent Problems, Models, and Issues*, Springer Science & Business Media, 2010.
- [27] J. Hu, Y. Hong, Leader-following coordination of multi-agent systems with coupling time delays, *Physica A: Statistical Mechanics and its Applications* 374 (2) (2007) 853–863.
- [28] F. Clarke, *Optimization and Nonsmooth Analysis*, John Wiley & Sons New York, NY, USA, 1983.
- [29] J. Yang, S. Li, X. Yu, Sliding-mode control for systems with mismatched uncertainties via a disturbance observer, *IEEE Transactions on Industrial Electronics* 60 (1) (2013) 160–169.
- [30] H. K. Khalil, *Nonlinear Systems*, Prentice-Hall, New Jersey, 1996.

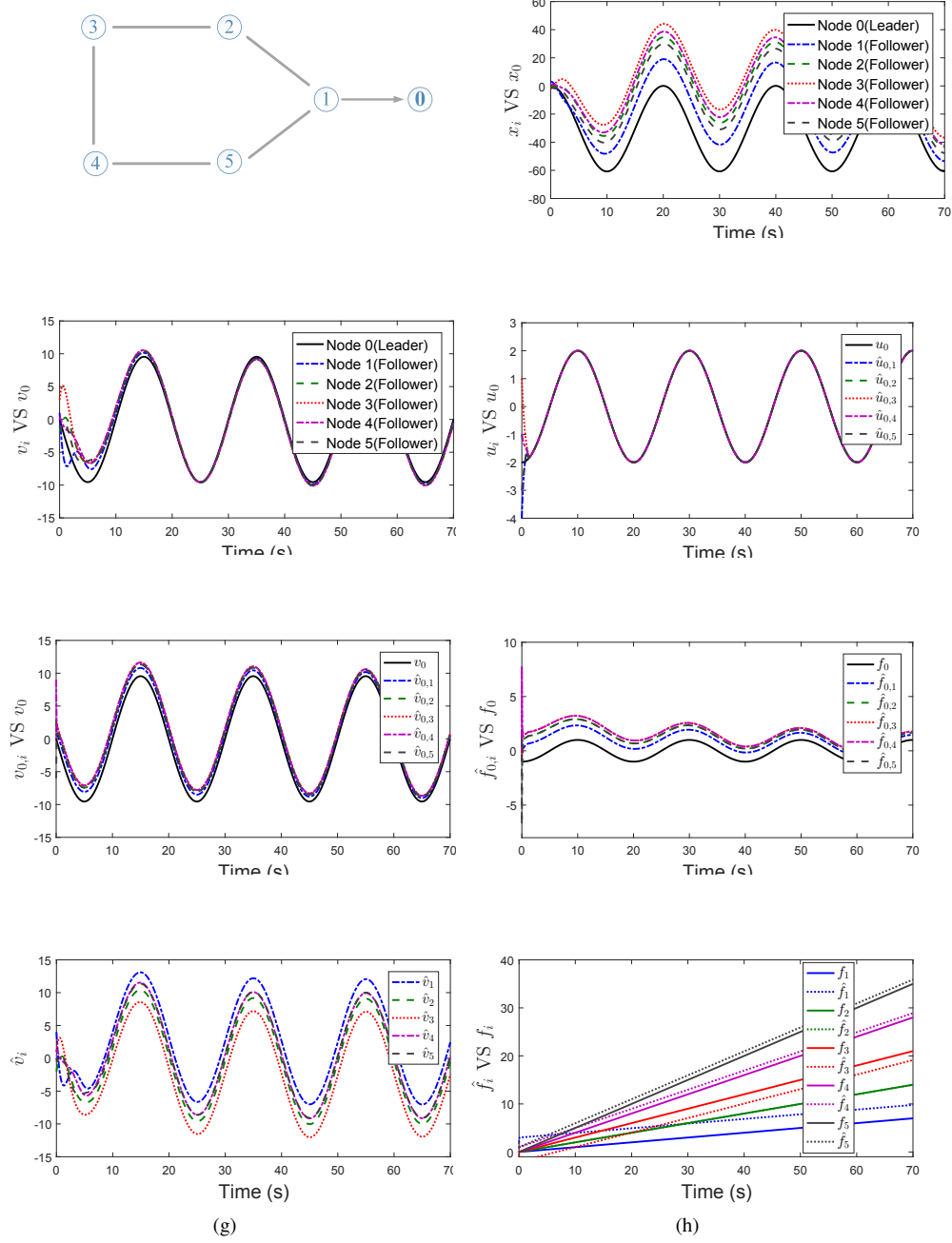


Figure 1: Second-order MAS tracking control: (a) communication topology; (b) leader's and followers' position trajectory profiles; (c) leader's and followers' velocity profiles; (d) leader's acceleration profile and the estimation by each follower; (e) leader's velocity profile and the estimation by each follower; (f) leader's disturbance profile and the estimation by each follower; (g) followers' estimation of their own velocities; (h) followers' disturbance profiles and the estimation on their own.