Rapid Communications

Measurements of branching fraction and direct *CP* asymmetry in $B^\pm \to K_S^0 K_S^0 K^\pm$ and a search for $B^\pm \to K_S^0 K_S^0 \pi^\pm$

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We study charmless hadronic decays of charged B mesons to the final states $K_S^0 K_S^0 K^\pm$ and $K_S^0 K_S^0 \pi^\pm$ using a 711 fb^{-1} data sample that contains 772×10^6 $B\bar{B}$ pairs and was collected at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB asymmetric-energy e^+e^- collider. For $B^\pm \to K_S^0 K_S^0 K^\pm$, the measured branching fraction and direct CP asymmetry are $[10.42 \pm 0.43(\mathrm{stat}) \pm 0.22(\mathrm{syst})] \times 10^{-6}$ and $[+1.6 \pm 3.9(\mathrm{stat}) \pm 0.9(\mathrm{syst})]\%$, respectively. In the absence of a statistically significant signal for $B^\pm \to K_S^0 K_S^0 \pi^\pm$, we obtain a 90% confidence-level upper limit on its branching fraction as 8.7×10^{-7}.

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Charged B-meson decays to the three-body charmless hadronic final states $K_S^0 K_S^0 K^\pm$ and $K_S^0 K_S^0 \pi^\pm$ mainly proceed via $b \to s$ and $b \to d$ loop transitions, respectively. Figure 1 shows Feynman diagrams of the dominant amplitudes that contribute to these decays. These flavor-changing neutral current transitions, being suppressed in the standard model (SM), are interesting, as they could be sensitive to possible non-SM contributions [1].

Further motivation, especially to study the contributions of various quasi-two-body resonances to inclusive CP asymmetry, comes from the recent results on $B^\pm \to K^+K^-K^\pm$, $K^+K^-\pi^\pm$ and other such three-body decays [2–4]. LHCb has found large asymmetries localized in phase space in $B^\pm \to K^+K^-\pi^\pm$ decays [3]. Recently, Belle has also reported strong evidence for large CP asymmetry at the low K^+K^- invariant mass region of $B^\pm \to K^+K^-\pi^\pm$ [4]. The fact that the $K\bar{K}$ system of $B^\pm \to K_S^0 K_S^0 h^\pm$ ($h = K, \pi$), in contrast to that of $B^\pm \to K^+K^-h^\pm$, cannot form a vector resonance (Bose symmetry) may shed light on the source of large CP violation in the latter decays.

The three-body decay $B^+ \to K_S^0 K_S^0 K^+$ [5] was observed by Belle [6] and subsequently studied by *BABAR* [7]. Belle measured the decay branching fraction as $(13.4 \pm 1.9 \pm 1.5) \times 10^{-6}$ based on a data sample of 70 fb^{-1} [6], and *BABAR* reported a branching fraction of $(10.6 \pm 0.5 \pm 0.3) \times 10^{-6}$ and a CP asymmetry of $(+4^{+4}_{-5} \pm 2)\%$ using 426 fb^{-1} of data [7]. The quoted uncertainties are statistical and systematic, respectively.

The decay $B^+ \to K_S^0 K_S^0 \pi^+$ is suppressed by the squared ratio of CKM matrix [8] elements $|V_{td}/V_{ts}|^2 (= 0.046)$ with respect to $B^+ \to K_S^0 K_S^0 K^+$, and has not yet been observed. The most restrictive limit at 90% confidence level on its branching fraction, $\mathcal{B}(B^+ \to K_S^0 K_S^0 \pi^+) < 5.1 \times 10^{-7}$, comes from *BABAR* [9].

We present an improved measurement of the branching fraction and direct CP asymmetry of the decay $B^+ \to K_S^0 K_S^0 K^+$ as well as a search for $B^+ \to K_S^0 K_S^0 \pi^+$ using a data sample of 711 fb^{-1}, which contains 772×10^6 $B\bar{B}$ pairs and was recorded near the $\Upsilon(4S)$ resonance with the Belle detector [10] at the KEKB e^+e^- collider [11]. The direct CP asymmetry is defined as

$$\mathcal{A}_{CP} = \frac{N(B^- \to K_S^0 K_S^0 h^-) - N(B^+ \to K_S^0 K_S^0 h^+)}{N(B^- \to K_S^0 K_S^0 h^-) + N(B^+ \to K_S^0 K_S^0 h^+)}, \quad (1)$$

where N is the obtained signal yield for the corresponding mode. The detector components relevant for our study are a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov

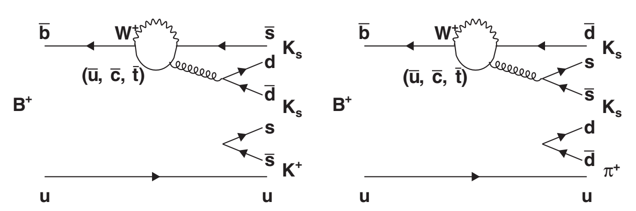


FIG. 1. Feynman diagrams of the dominant amplitudes that contribute to the decays $B^\pm \to K_S^0 K_S^0 K^\pm$ (left) and $B^\pm \to K_S^0 K_S^0 \pi^\pm$ (right).

counters (ACC), and a barrel-like arrangement of time-of-flight scintillation counters (TOF); all located inside a 1.5 T solenoidal magnetic field.

To reconstruct $B^+ \to K_S^0 K_S^0 h^+$ candidates, we begin by identifying charged kaons and pions. A kaon or pion candidate track must have a minimum transverse momentum of 100 MeV/c in the lab frame, and a distance of closest approach with respect to the interaction point (IP) of less than 0.2 cm in the transverse $r-\phi$ plane and less than 5.0 cm along the z axis. Here, the z axis is defined opposite the e^+ beam. Charged tracks are identified as kaons or pions based on a likelihood ratio $\mathcal{R}_{K/\pi} = \mathcal{L}_K/(\mathcal{L}_K + \mathcal{L}_\pi)$, where $\mathcal{L}_K$ and $\mathcal{L}_\pi$ are the individual likelihoods for kaons and pions, respectively, calculated with information from the CDC, ACC, and TOF. Tracks with $\mathcal{R}_{K/\pi} > 0.6$ are identified as kaons, while those with $\mathcal{R}_{K/\pi} < 0.4$ are identified as pions. The efficiency for kaon (pion) identification is 86% (91%), with a pion (kaon) misidentification rate of 9% (14%).

The K_S^0 candidates are reconstructed from pairs of oppositely charged tracks, both assumed to be pions, and are further subject to a selection [12] based on a neural network [13]. The network uses the following input variables: the K_S^0 momentum in the lab frame, the distance along the z axis between the two track helices at their closest approach, the K_S^0 flight length in the $r-\phi$ plane, the angle between the K_S^0 momentum and the vector joining the IP to the K_S^0 decay vertex, the angle between the pion momentum and the lab frame direction in the K_S^0 rest frame, the distances of closest approach in the $r-\phi$ plane between the IP and the two pion helices, the number of hits in the CDC for each pion track, and the presence or absence of hits in the SVD for each pion track. We require that the reconstructed invariant mass be between 491 and 505 MeV/c^2, corresponding to $\pm 3\sigma$ around the nominal K_S^0 mass [14], with σ denoting the experimental resolution.

We identify B-meson candidates using two kinematic variables: the beam-energy constrained mass, $M_{\rm bc} = \sqrt{E_{\rm beam}^2/c^4 - |\sum_i \vec{p}_i/c|^2}$, and the energy difference, $\Delta E = \sum_i E_i - E_{\rm beam}$, where $E_{\rm beam}$ is the beam energy, and $\vec{p}_i$ and E_i are the momentum and energy of the ith daughter of the reconstructed B candidate, all calculated in the center-of-mass frame. For each B candidate, we perform a fit constraining its daughters to come from a common vertex, whose position is consistent with the IP profile. Events with $5.271~{\rm GeV}/c^2 < M_{\rm bc} < 5.287~{\rm GeV}/c^2$ and $-0.10~{\rm GeV} < \Delta E < 0.15$ GeV are retained for further analysis. The $M_{\rm bc}$ requirement corresponds approximately to a $\pm 3\sigma$ window around the nominal B^+ mass [14]. We apply a looser $(-6\sigma, +9\sigma)$ requirement on ΔE, as it is later used to extract the signal yield.

The average number of B candidates per event is 1.1 (1.5) for $B^+ \to K_S^0 K_S^0 K^+$ ($K_S^0 K_S^0 \pi^+$). In the case of multiple candidates, we choose the one with the minimum χ^2 value for the aforementioned vertex fit. This criterion selects the correct B-meson candidate in 75% and 63% of Monte Carlo (MC) events having more than one candidate in $B^+ \to K_S^0 K_S^0 K^+$ and $B^+ \to K_S^0 K_S^0 \pi^+$, respectively.

The dominant background arises from the $e^+e^- \to q\bar{q}$ ($q = u,\ d,\ s,\ c$) continuum process. We use observables based on event topology to suppress it. The event shape in the c.m. frame is expected to be spherical for $B\bar{B}$ events, whereas continuum events are jetlike. We employ a neural network based on NeuroBayes [13] to separate signal from background using the following six input variables: a Fisher discriminant formed from 16 modified Fox-Wolfram moments [15], the cosine of the angle between the B momentum and the z axis, the cosine of the angle between the B thrust and the z axis, the cosine of the angle between the thrust axis of the B candidate and that of the rest of the event, the ratio of the second- to the zeroth-order Fox-Wolfram moments, and the vertex separation along the z axis between the B candidate and the remaining tracks. The first five quantities are calculated in the c.m. frame. The neural network training is performed with simulated signal and $q\bar{q}$ samples each containing 30 000 events after all selection requirements. Using MC events that are independent of the ones used for training, we verify that the network is not overtrained. Signal and background samples are generated with the EvtGen program [16]; for signal we assume a uniform decay in phase space. A GEANT-based [17] simulation is used to model the detector response.

We require the neural network output ($C_{\rm NB}$) to be greater than -0.2 to substantially reduce the continuum background. For both decays, the relative signal efficiency due to this requirement is approximately 91%, and the achieved continuum suppression is close to 84%. The remainder of the $C_{\rm NB}$ distribution strongly peaks near 1.0 for signal, making it challenging to model it analytically. However, its transformed variable

$$C'_{\rm NB} = \ln\left[\frac{C_{\rm NB} - C_{\rm NB,min}}{C_{\rm NB,max} - C_{\rm NB}}\right], \tag{2}$$

where $C_{\rm NB,min} = -0.2$ and $C_{\rm NB,max} \simeq 1.0$, can be parametrized by one or more Gaussian functions. We use $C'_{\rm NB}$ as a fit variable along with ΔE.

The background due to charmed B decays, mediated via the dominant $b \to c$ transition, is studied with an MC sample. The resulting ΔE and $M_{\rm bc}$ distributions are found to peak in the signal region for both $B^+ \to K_S^0 K_S^0 K^+$ and $B^+ \to K_S^0 K_S^0 \pi^+$ decays. For $B^+ \to K_S^0 K_S^0 K^+$, the peaking background predominantly stems from $B^+ \to D^0 K^+$ with $D^0 \to K_S^0 K_S^0$ and from $B^+ \to \chi_{c0}(1{\rm P}) K^+$ with $\chi_{c0}(1{\rm P}) \to K_S^0 K_S^0$. To suppress these backgrounds, we exclude candidates for which $M_{K_S^0 K_S^0}$ lies in the range $[1.85, 1.88]$ GeV/c^2 or $[3.38, 3.45]$ GeV/c^2, corresponding to a $\pm 3\sigma$ window around the nominal D^0 or $\chi_{c0}(1{\rm P})$ mass [14], respectively.

In the case of $B^+ \to K_S^0 K_S^0 \pi^+$, the peaking background largely arises from $B^+ \to D^0 \pi^+$ with $D^0 \to K_S^0 K_S^0$. To suppress it, we exclude candidates for which $M_{K_S^0 K_S^0}$ lies in the aforementioned D^0 mass window. The relative loss of signal efficiency due to these charm vetoes is 3% (1%) for $B^+ \to K_S^0 K_S^0 K^+$ ($K_S^0 K_S^0 \pi^+$).

A few background modes contribute in the $M_{\rm bc}$ signal region, but having their ΔE peak shifted from zero to the positive side for $B^+ \to K_S^0 K_S^0 K^+$ or to the negative side for $B^+ \to K_S^0 K_S^0 \pi^+$. To identify these so-called "feed-across" backgrounds, mostly arising due to $K - \pi$ misidentification, we use a $B\bar{B}$ MC sample in which one of the B mesons decays via $b \to u, d, s$ transitions, along with the charmed $B\bar{B}$ sample. For $B^+ \to K_S^0 K_S^0 \pi^+$, the feed-across background includes contributions from $B^+ \to K_S^0 K_S^0 K^+$ as well as $B^+ \to D^0 K^+$ and $B^+ \to \chi_{c0}(1P) K^+$ that survive the D^0 and $\chi_{c0}(1P)$ vetoes. For $B^+ \to K_S^0 K_S^0 K^+$, it comes entirely from $B^+ \to K_S^0 K_S^0 \pi^+$. All other events coming from neither the signal, the continuum, nor the feed-across components form the so-called "combinatorial" $B\bar{B}$ background.

After all selection requirements, the efficiencies for correctly reconstructed signal events are 24% for $B^+ \to K_S^0 K_S^0 K^+$ and 26% for $B^+ \to K_S^0 K_S^0 \pi^+$. The fractions of misreconstructed signal events for which one of the daughter particles comes from the other B-meson decay are 0.5% for $B^+ \to K_S^0 K_S^0 K^+$ and 1.1% for $B^+ \to K_S^0 K_S^0 \pi^+$. We consider these events as part of the signal.

The signal yield and $\mathcal{A}_{CP}$ are obtained with an unbinned extended maximum likelihood fit to the two-dimensional distribution of ΔE and $C'_{\rm NB}$. The extended likelihood function is

$$\mathcal{L} = \frac{e^{-\sum_j n_j}}{N!} \prod_i \left[\sum_j n_j \mathcal{P}_j^i \right], \qquad (3)$$

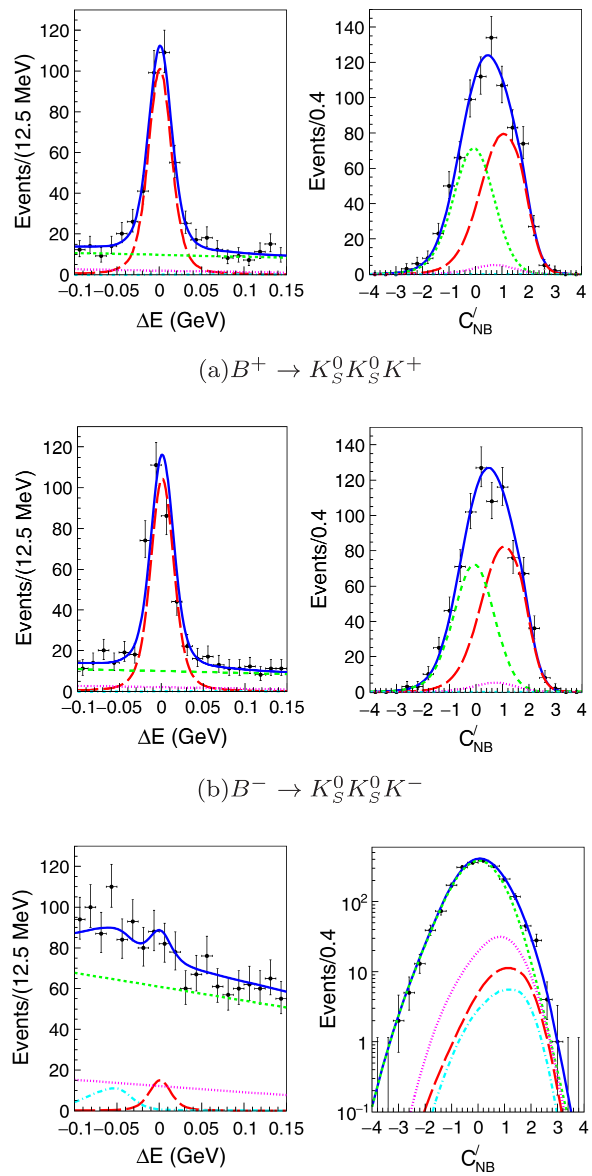
where

$$\mathcal{P}_j^i \equiv \frac{1}{2}(1 - q^i \mathcal{A}_{CP,j}) \times \mathcal{P}_j(\Delta E^i) \times \mathcal{P}_j(C'^i_{\rm NB}). \qquad (4)$$

Here, N is the total number of events, i is the event index, and n_j is the yield of the event category j ($j \equiv$ signal, $q\bar{q}$, combinatorial, and feed-across). $\mathcal{P}_j$ and $\mathcal{A}_{CP,j}$ are the probability density function (PDF) and the direct CP asymmetry corresponding to the category j, and q^i is the electric charge of the B candidate in event i. As the correlation between ΔE and $C'_{\rm NB}$ is small (the linear correlation coefficient ranges from 0.5% to 7.0%), the product of two individual PDFs is a good approximation for the total PDF. We apply a tight requirement on $M_{\rm bc}$ instead

TABLE I. List of PDFs used to model the ΔE and $C'_{\rm NB}$ distributions for various event categories for $B^+ \to K_S^0 K_S^0 K^+$. "G," "AG," and "Poly1" denote Gaussian, asymmetric Gaussian, and first-order polynomial, respectively.

Event category	ΔE	$C'_{\rm NB}$
Signal	3 G	G + AG
Continuum $q\bar{q}$	Poly1	2 G
Combinatorial $B\bar{B}$	Poly1	2 G
Feed-across	G + Poly1	G



(a) $B^+ \to K_S^0 K_S^0 K^+$

(b) $B^- \to K_S^0 K_S^0 K^-$

(c) $B^\pm \to K_S^0 K_S^0 \pi^\pm$

FIG. 2. Projections of the two-dimensional simultaneous fit to ΔE for $C'_{\rm NB} > 0.0$ and $C'_{\rm NB}$ for $|\Delta E| < 50$ MeV. Black points with error bars are the data, solid blue curves are the total PDF, long-dashed red curves are the signal, dashed green curves are the continuum background, dotted magenta curves are the combinatorial $B\bar{B}$ background, and dash-dotted cyan curves are the feed-across background.

TABLE II. Efficiency, differential branching fraction, and $\mathcal{A}_{CP}$ in each $M_{K_S^0 K_S^0}$ bin for $B^+ \to K_S^0 K_S^0 K^+$.

$M_{K_S^0 K_S^0}$ (GeV/c^2)	Efficiency (%)	$d\mathcal{B}/dM \times 10^{-6}$ (c^2/ GeV)	$\mathcal{A}_{CP}$ (%)
1.0–1.1	24.0 ± 0.4	$10.40 \pm 1.24 \pm 0.38$	$-3.9 \pm 10.9 \pm 0.9$
1.1–1.3	23.4 ± 0.2	$8.60 \pm 0.85 \pm 0.32$	$-0.1 \pm 9.3 \pm 0.9$
1.3–1.6	22.9 ± 0.1	$10.23 \pm 0.73 \pm 0.38$	$+6.6 \pm 6.9 \pm 0.9$
1.6–2.0	21.8 ± 0.1	$3.93 \pm 0.43 \pm 0.15$	$+16.1 \pm 10.3 \pm 0.9$
2.0–2.3	24.1 ± 0.1	$3.90 \pm 0.47 \pm 0.15$	$-3.3 \pm 11.3 \pm 0.9$
2.3–2.7	25.2 ± 0.1	$2.45 \pm 0.33 \pm 0.09$	$-5.7 \pm 12.2 \pm 1.0$
2.7–5.0	26.3 ± 0.0	$0.35 \pm 0.07 \pm 0.01$	$-31.9 \pm 19.7 \pm 1.2$

of including it as a fit variable, since it exhibits a large correlation with ΔE for the signal and feed-across background. We choose ΔE over $M_{\rm bc}$ in the fit because the former is a better variable to distinguish signal from feed-across background. To account for crossfeed between the two channels, they are fitted simultaneously, with the $B^+ \to K_S^0 K_S^0 K^+$ branching fraction in the correctly reconstructed sample determining the normalization of the crossfeed in the $B^+ \to K_S^0 K_S^0 \pi^+$ fit region, and vice versa.

Table I lists the PDFs used to model the ΔE and $C'_{\rm NB}$ distributions for various event categories for $B^+ \to K_S^0 K_S^0 K^+$. For $B^+ \to K_S^0 K_S^0 \pi^+$, we use the same PDF shapes except for the feed-across background component, where we add an asymmetric Gaussian function to the PDFs in Table I to accurately describe ΔE and $C'_{\rm NB}$ distributions. The free parameters in the fit are the continuum background yields and the branching fractions of $B^+ \to K_S^0 K_S^0 K^+$ and $B^+ \to K_S^0 K_S^0 \pi^+$, and the signal $\mathcal{A}_{CP}$ for $B^+ \to K_S^0 K_S^0 K^+$. In addition, the following PDF shape parameters of the continuum background are floated in the fit for both $B^+ \to K_S^0 K_S^0 K^+$ and $K_S^0 K_S^0 \pi^+$: the slope of the first-order polynomial used for ΔE and the mean and width of the dominant Gaussian component used to model $C'_{\rm NB}$. The combinatorial $B\bar{B}$ yields are fixed to the MC values due to their correlation with the continuum yields. This is because $C'_{\rm NB}$ is the only variable that offers some discrimination between the two background categories. To improve the overall fit stability, $\mathcal{A}_{CP}$ for all components but for the $B^+ \to K_S^0 K_S^0 K^+$ signal are fixed to zero. The other PDF shape parameters for signal and background components are fixed to the corresponding MC expectations for both decays. We correct the signal ΔE and $C'_{\rm NB}$ PDF shapes for possible data-MC differences, according to the values obtained with a control sample of $B^+ \to \bar{D}^0 \pi^+$ with $\bar{D}^0 \to K_S^0 \pi^+ \pi^-$. The same correction factors are also applied for the feed-across background component of $B^+ \to K_S^0 K_S^0 \pi^+$.

We determine the branching fraction as

$$\mathcal{B}(B^+ \to K_S^0 K_S^0 h^+) = \frac{n_{\rm sig}}{\epsilon \times N_{B\bar{B}} \times [\mathcal{B}(K_S^0 \to \pi^+ \pi^-)]^2}, \quad (5)$$

where $n_{\rm sig}$, ϵ, and $N_{B\bar{B}}$ are the total signal yield, average detection efficiency, and number of $B\bar{B}$ pairs, respectively. Figure 2 shows signal-enhanced ΔE and $C'_{\rm NB}$ projections of the separate fit to B^+ and B^- samples for $B^+ \to K_S^0 K_S^0 K^+$ and of the charge-combined fit for $B^+ \to K_S^0 K_S^0 \pi^+$. For $B^+ \to K_S^0 K_S^0 \pi^+$, we fit a total of 5103 candidate events to obtain a branching fraction of

$$\mathcal{B}(B^+ \to K_S^0 K_S^0 \pi^+) = (6.5 \pm 2.6 \pm 0.4) \times 10^{-7}, \quad (6)$$

where the first uncertainty is statistical and the second is systematic (described below). Its signal significance is estimated as $\sqrt{-2\ln(\mathcal{L}_0/\mathcal{L}_{\rm max})}$, where $\mathcal{L}_0$ and $\mathcal{L}_{\rm max}$ are the likelihood values for the fit with the branching fraction fixed to zero and for the best-fit case, respectively. Including systematic uncertainties by convolving the likelihood with a Gaussian function of width equal to the systematic uncertainty, we determine the significance to be 2.5 standard deviations. In view of the significance being less than 3 standard deviations, we set an upper limit on the branching fraction of $B^+ \to K_S^0 K_S^0 \pi^+$. We integrate the convolved likelihood over the branching fraction to obtain the upper limit of 8.7×10^{-7} at 90% confidence level. This limit is similar to that of *BABAR* [9].

For $B^+ \to K_S^0 K_S^0 K^+$, we perform the fit for 2709 candidate events in seven unequal bins of $M_{K_S^0 K_S^0}$ to decipher contributions from possible quasi-two-body

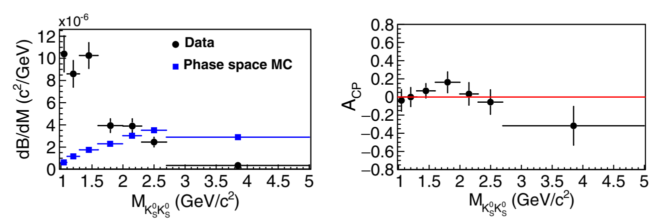


FIG. 3. Differential branching fraction (left) and $\mathcal{A}_{CP}$ (right) as functions of $M_{K_S^0 K_S^0}$ for $B^+ \to K_S^0 K_S^0 K^+$. Black points with error bars are the results from the two-dimensional fits to data and include systematic uncertainties. Blue squares in the left plot show the expectation from a phase-space MC sample, and the red line in the right plot indicates a zero *CP* asymmetry.

TABLE III. Systematic uncertainties in the branching fraction of $B^+ \to K_S^0 K_S^0 \pi^+$.

Source	Relative uncertainty in $\mathcal{B}$ (%)
Tracking	0.35
Particle identification	0.80
Number of $B\bar{B}$ pairs	1.37
Continuum suppression	0.34
Requirement on $M_{\rm bc}$	0.03
K_S^0 reconstruction	3.22
Fit bias	1.86
Signal PDF	1.30
Combinatorial $B\bar{B}$ PDF	+1.31, −1.98
Feed-across PDF	+3.57, −4.10
Fixed background yield	+2.63, −2.27
Fixed background $\mathcal{A}_{CP}$	0.50
Total	+6.30, −6.67

resonances. The efficiency, differential branching fraction, and $\mathcal{A}_{CP}$ thus obtained are listed in Table II. Figure 3 shows the differential branching fraction and $\mathcal{A}_{CP}$ plotted as a function of $M_{K_S^0 K_S^0}$. We observe an excess of events around 1.5 GeV$/c^2$ beyond the expectation of a phase-space MC sample. No significant evidence for CP asymmetry is found in any of the bins. Upon inspection, no peaking structure beyond kinematic reflection is seen in the $M_{K_S^0 K^+}$ distribution. We calculate the branching fraction by integrating the differential branching fraction over the entire $M_{K_S^0 K_S^0}$ range:

$$\mathcal{B}(B^+ \to K_S^0 K_S^0 K^+) = (10.42 \pm 0.43 \pm 0.22) \times 10^{-6}, \quad (7)$$

where the first uncertainty is statistical and the second is systematic. The $\mathcal{A}_{CP}$ over the full $M_{K_S^0 K_S^0}$ range is

$$\mathcal{A}_{CP}(B^+ \to K_S^0 K_S^0 K^+) = (+1.6 \pm 3.9 \pm 0.9)\%. \quad (8)$$

This is obtained by weighting the $\mathcal{A}_{CP}$ value in each bin with the obtained branching fraction in that bin. As the statistical uncertainties are bin independent, their total contribution is a quadratic sum. For the systematic uncertainties, the contributions from the bin-correlated sources are linearly added, and those from the bin-uncorrelated sources are added in quadrature. The results agree with *BABAR* [7], which reported an $\mathcal{A}_{CP}$ consistent with zero

TABLE IV. Systematic uncertainties in the differential branching fraction and $\mathcal{A}_{CP}$ in $M_{K_S^0 K_S^0}$ bins for $B^+ \to K_S^0 K_S^0 K^+$. "†" indicates that the uncertainty is independent of $M_{K_S^0 K_S^0}$, with the listed value being applicable for all the bins. An ellipsis indicates a value below 0.05% in $d\mathcal{B}/dM$ and below 0.001% in $\mathcal{A}_{CP}$.

$M_{K_S^0 K_S^0}$ (GeV$/c^2$)	1.0–1.1	1.1–1.3	1.3–1.6	1.6–2.0	2.0–2.3	2.3–2.7	2.7–5.0
Source				Relative uncertainty in $d\mathcal{B}/dM$ (%)			
Tracking†				0.35			
Particle identification†				0.80			
Number of $B\bar{B}$ pairs†				1.37			
Continuum suppression†				0.34			
Requirement on $M_{\rm bc}$†				0.03			
K_S^0 reconstruction†				3.22			
Fit bias†				0.53			
Signal PDF	$^{+0.33}_{-0.27}$	$^{+0.63}_{-0.48}$	$^{+0.46}_{-0.44}$	$^{+0.22}_{-0.63}$	$^{+0.52}_{-0.38}$	0.67	1.10
Combinatorial $B\bar{B}$ PDF	0.09	$^{+0.08}_{-0.13}$	0.12	$^{+0.17}_{-0.21}$	$^{+0.26}_{-0.34}$	0.40	0.40
Feed-across PDF	…	…	…	…	…	…	…
Fixed background yield	…	0.10	0.10	0.23	…	0.11	0.60
Fixed background $\mathcal{A}_{CP}$	…	…	…	0.20	0.10	…	0.13
Total	±3.68	±3.72	±3.69	±3.73	±3.72	±3.75	±3.89

$M_{K_S^0 K_S^0}$(GeV$/c^2$)	1.0–1.1	1.1–1.3	1.3–1.6	1.6–2.0	2.0–2.3	2.3–2.7	2.7–5.0
Source				Absolute uncertainty in $\mathcal{A}_{CP}$			
Signal PDF	0.001	0.002	0.001	0.002	0.001	0.001	0.004
Combinatorial $B\bar{B}$ PDF	0.001	0.001	0.001	…	0.001	0.002	0.001
Feed-across PDF	…	…	…	…	…	…	…
Fixed background yield	…	…	0.001	0.001	0.001	0.001	0.004
Fixed background $\mathcal{A}_{CP}$	…	…	0.001	0.001	0.001	0.002	0.006
Detector bias†				0.009			
Total	±0.009	±0.009	±0.009	±0.009	±0.009	±0.010	±0.012

as well as the presence of quasi-two-body resonances $f_0(980)$, $f_0(1500)$, and $f_2'(1525)$ in the low $M_{K_S^0K_S^0}$ region.

Major sources of systematic uncertainty in the branching fractions are similar for both $B^+ \to K_S^0K_S^0K^+$ and $K_S^0K_S^0\pi^+$ decays. These are listed along with their contributions in Tables III and IV. We use partially reconstructed $D^{*+} \to D^0\pi^+$ with $D^0 \to K_S^0\pi^+\pi^-$ decays to assign the systematic uncertainty due to charged-track reconstruction (0.35% per track). The $D^{*+} \to D^0\pi^+$ with $D^0 \to K^-\pi^+$ sample is used to determine the systematic uncertainty due to particle identification. The uncertainty due to the number of $B\bar{B}$ pairs is 1.37%. The uncertainties due to continuum suppression and $M_{\rm bc}$ requirements are estimated with the control sample of $B^+ \to \bar{D}^0\pi^+$ with $\bar{D}^0 \to K_S^0\pi^-\pi^+$. The uncertainty arising due to K_S^0 reconstruction is estimated from $D^0 \to K_S^0K_S^0$ decays [18]. A potential fit bias is checked by performing an ensemble test comprising 1000 pseudoexperiments in which signal events are drawn from the corresponding MC sample and background events are generated according to their PDF shapes. The uncertainties due to signal PDF shape are estimated by varying the correction factors by $\pm 1\sigma$ of their statistical uncertainty. Similarly, the uncertainties due to background PDF shape are calculated by varying all fixed parameters by $\pm 1\sigma$. We evaluate the uncertainty due to fixed background yields by varying them up and down by 20% of their MC values. The uncertainty due to fixed background $\mathcal{A}_{CP}$ is estimated by varying the $\mathcal{A}_{CP}$ values up and down by one unit of their statistical uncertainties. As for a possible systematics due to efficiency variation across the Dalitz plot in the $B^+ \to K_S^0K_S^0\pi^+$ channel, we find its impact to be negligible.

Systematic uncertainties in $\mathcal{A}_{CP}$ are listed in Table IV. The systematic uncertainties due to the PDF modeling, fixed background yields, and $\mathcal{A}_{CP}$ are estimated with the same procedure as for the branching fraction. Uncertainties due to the intrinsic detector bias on charged particle detection are evaluated with the samples of $D^+ \to \phi\pi^+$ and $D_s^+ \to \phi\pi^+$ in conjunction with $D^0 \to K^-\pi^+$ [19]. The total systematic uncertainty is calculated by summing all individual contributions in quadrature.

In summary, we have reported measurements of the charmless three-body decays $B^+ \to K_S^0K_S^0K^+$ and $B^+ \to K_S^0K_S^0\pi^+$ using the full $\Upsilon(4S)$ data sample collected with the Belle detector. We perform a two-dimensional simultaneous fit to extract the signal yields of both decays. For $B^+ \to K_S^0K_S^0\pi^+$, a 90% confidence-level upper limit is set on the branching fraction at 8.7×10^{-7}. We measure the branching fraction and $\mathcal{A}_{CP}$ of $B^+ \to K_S^0K_S^0K^+$ to be $\mathcal{B}(B^+ \to K_S^0K_S^0K^+) = (10.42 \pm 0.43 \pm 0.22) \times 10^{-6}$ and $\mathcal{A}_{CP}(B^+ \to K_S^0K_S^0K^+) = (+1.6 \pm 3.9 \pm 0.9)\%$. These results supersede Belle's earlier measurements [6] and are consistent with those of *BABAR* [7,9].

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