

Stability Analysis of an Unsaturated Silty Slope under Nonisothermal Conditions

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ABSTRACT

Natural and man-made unsaturated soil slopes can be subjected to different temperatures due to soil-atmospheric interaction, thermally-active earthen systems, and human-induced activities. Such nonisothermal conditions can affect the stress state variables, leading to changes in the soil strength and stability of slope. The main objective of this study is to investigate the stability of unsaturated soil slopes under elevated temperatures. For this purpose, Bishop's effective stress expression for unsaturated soils is extended to account for the effect of temperature on matric suction and effective degree of saturation. The effective stress expression includes a nonisothermal soil water retention curve model to consider the thermal effects on the surface tension, soil-water contact angle, and adsorption by the enthalpy of immersion. The proposed effective stress model is then incorporated into an infinite slope stability analysis of a hypothetical silty soil under no-flow (hydrostatic) condition. The formulations are used to monitor matric suction, suction stress, and factor of safety of slope for different temperatures. The results suggest that the effects of temperature on slope stability can be significant. It was found that by increasing temperature from 25°C to 40°C and 60°C, the matric suction decreases by approximately 24% and 39%, suction stress increases by approximately 30% and 45%, and the FOS decreases approximately by 5% and 11%, respectively.

INTRODUCTION

Examining the stability of unsaturated slopes is an important and challenging aspect of geotechnical engineering. Among others, the effect of temperature on the stability of unsaturated slopes is an important issue that certainly warrants further investigation. Natural and man-made unsaturated slopes can be subjected to elevated temperatures due to soil-atmospheric interaction, thermally-active earthen systems, and human-induced activities. Further, recorded observations and projected patterns all suggest increases in temperature, warm spells, short-term heatwaves, and concurrent drought and heat waves in several regions (e.g., Mazdiasni and AghaKouchak, 2015; Shukla et al., 2015). These observations and projections further highlight the need to properly study the effects of temperature on the stability of natural and man-made unsaturated slopes in a changing climate (e.g., Damiano and Mercogliano, 2013; Vardon 2015; Robinson and Vahedifard 2016; Vahedifard et al., 2015; 2016; 2017; 2018a).

Temperature changes can affect the stability of a slope in several ways. For example, changes in temperature can increase the near surface permeability of the soils, and this would facilitate seepage flow parallel to the slope and reduce the factor of safety (FOS) of the slope (Greenway 1987; Pradel and Raad 1993; Bo et al., 2008). Increased temperature can also cause desiccation cracking and shrinkage in clays. While in some cases the lower moisture content will increase soil strength and benefit soil suction, it can conversely result in losses of soil cohesion. An example of this issue is fissuring of a clay deposit, which could accelerate infiltration into the slope with the associated decrease in FOS (Li and Zhang 2010; Wang et al. 2011). Another important effect of nonisothermal conditions can be through temperature-induced changes in stress state variables and the soil water retention curve (SWRC), which can lead to changes in the soil strength and stability of slope. Uchaipichat (2017) indicated that the FOS of silty slopes under undrained conditions decreases with increasing temperature (25 to 60 °C) for different values of matric suction (0 to 100 kPa). The experimental results have shown that an elevated temperature causes a downward shift in the SWRC shape, leading to a decrease in matric suction under a constant water content (e.g., Wan et al., 2015; Roshani and Sedano, 2016). This decrease in matric suction can lead to reduction in effective stresses and consequently, FOS.

The main objective of this study is to investigate the stability of unsaturated soil slopes under elevated temperatures. For this purpose, recently developed models of nonisothermal SWRC (Vahedifard et al., 2018b) and nonisothermal effective stress of unsaturated soils are incorporated into an effective stress-based infinite slope stability analysis. Bishop's effective stress expression for unsaturated soils is extended to account for the effect of temperature on matric suction and effective degree of saturation. The effective stress expression includes a nonisothermal SWRC model proposed by Vahedifard et al. (2018b), which considers the thermal effect on the surface tension, soil-water contact angle, and adsorption by the enthalpy of immersion. The proposed formulations are then used to monitor matric suction, suction stress, and FOS of a hypothetical silty slope under no-flow (hydrostatic) conditions at different temperatures.

FORMULATIONS OF NONISOTHERMAL INFINITE SLOPE STABILITY

Effective stress for unsaturated soils

In the current study, we adopt the generalized effective stress (σ') expression for unsaturated soils given by Bishop (1959) as:

$$\sigma' = \sigma - u_a + \chi\psi \quad (1)$$

where σ is total stress, u_a is pore air pressure, χ is Bishop's effective stress parameter, and ψ is matric suction.

The term $-\chi\psi$ in Bishop's effective stress expression is referred to as suction stress (e.g., Karube et al., 1996; Lu and Likos, 2006). Using this definition, Bishop's effective stress expression that unifies both saturated and unsaturated conditions can be rewritten as:

$$\sigma' = \sigma - u_a - \sigma^s \quad (2)$$

where σ^s is the suction stress which is defined as (Lu and Likos, 2006):

$$\sigma^s = -\psi \times S_e \quad (3)$$

where S_e is the effective degree of saturation and can be obtained either by using normalized degree of saturation or the SWRC model by van Genuchten (1980). As defined, S_e is given by:

$$S_e = \frac{S - S_r}{1 - S_r} = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \left(\frac{1}{1 + (\alpha\psi)^n} \right)^m \quad (4)$$

where S is the degree of saturation, S_r is the residual saturation, θ , θ_s , and θ_r are the total, saturated and residual water contents, respectively, α is a fitting parameter inversely related to the air-entry suction (1/kPa), n is the pore-size distribution fitting parameter, and $m = (1 - 1/n)$ is a fitting parameter representing the overall geometry of the SWRC.

Nonisothermal SWRC, suction stress and effective stress

In this study, we use the nonisothermal SWRC formulations by Vahedifard et al. (2018b), which considers the effect of temperature on capillarity as a function of surface tension, contact angle and enthalpy of immersion per unit area. The SWRC formulations are then employed to develop expressions for effective degree of saturation, suction stress and effective stress under nonisothermal conditions.

Using the van Genuchten (1980) SWRC model, the nonisothermal effective degree of saturation, suction stress and the effective stress can be written as (Vahedifard et al., 2018b):

$$S_e = \left\{ 1 + \left[\alpha \psi \left(\frac{\beta_{T_r} + T_r}{\beta + T} \right) \right]^n \right\}^{-(1-1/n)} \quad (5)$$

$$\sigma^s = - \left\{ 1 + \left[\alpha \psi \left(\frac{\beta_{T_r} + T_r}{\beta + T} \right) \right]^n \right\}^{-(1-1/n)} \psi \left(\frac{\beta_{T_r} + T_r}{\beta + T} \right) \quad (6)$$

$$\sigma' = (\sigma - u_a) + \left\{ 1 + \left[\alpha \psi \left(\frac{\beta_{T_r} + T_r}{\beta + T} \right) \right]^n \right\}^{-(1-1/n)} \psi \left(\frac{\beta_{T_r} + T_r}{\beta + T} \right) \quad (7)$$

where β and β_{T_r} are the linear regression parameters, which depend on surface tension, enthalpy of immersion and contact angle. More details for determining the nonisothermal SWRCs can be found in Vahedifard et al. (2018b).

Infinite slope stability under nonisothermal conditions

In several cases, such as well-drained colluvial, residual, and unconsolidated loose surficial soils, infinite slope instability may occur within the unsaturated zone. The effective stress principle is commonly adopted to analyze infinite slope failure problems in unsaturated soils (e.g., Lu and Godt, 2008). Recent studies of slope-stability analyses under isothermal condition have been based on the effective stress principle in which negative pore pressure is quantified by unsaturated seepage theories (e.g., Lu and Godt, 2008; Godt et al., 2012). In this study, we extend the infinite slope stability formulation under unsaturated steady flow by Lu and Godt (2008) to nonisothermal conditions. This paper presents formulations and results only for no-flow (hydrostatic) condition.

The FOS of infinite soil slopes is determined commonly as ratio of shear strength to shear stress of soil. For the FOS under unsaturated conditions, the generalized suction stress can be inserted into the well-known equation for infinite slope stability (Duncan and Wright, 2005; Lu and Godt, 2008):

$$FOS = \frac{\tan \phi'}{\tan \beta'} + \frac{2c'}{\gamma H_{ss} \sin 2\beta'} - \frac{\sigma^s (\tan \beta' + \cot \beta') \tan \phi'}{\gamma H_{ss}} \quad (8)$$

where, ϕ' is the friction angle, c' is the cohesion, β' is the angle of slope with horizontal, γ is the unit weight of soil, $H_{ss} = (H_{wt} - z)$ is the height of sliding surface, with H_{wt} being the total height between the water table and the ground surface, and z is the depth above water table. The geometry and definition of variables are shown in Figure 1.

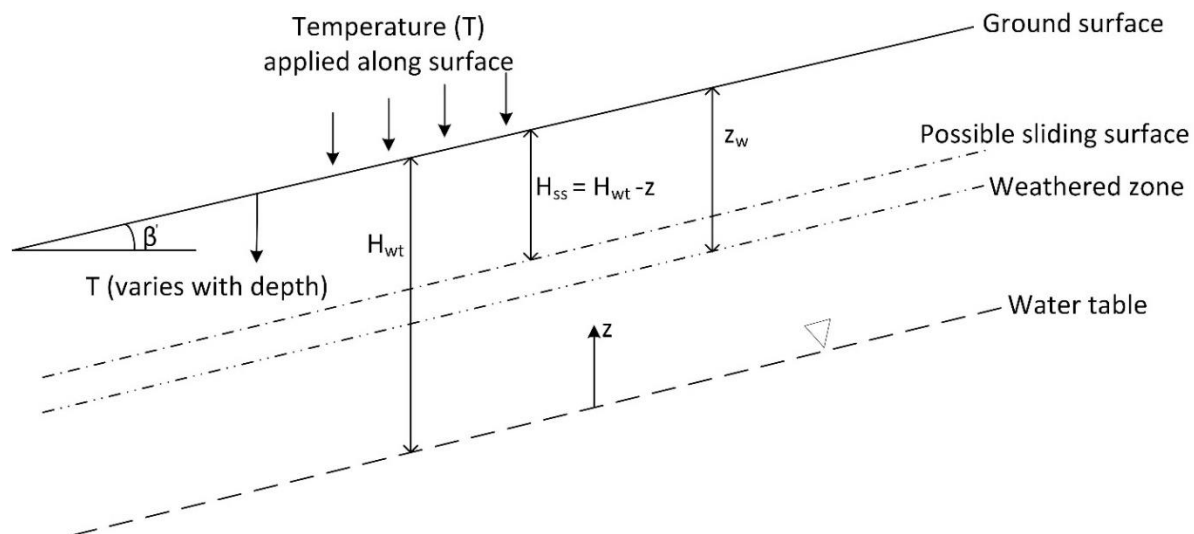


Figure 1. Schematic diagram of infinite slope stability for unsaturated soils under thermal load (modified after Lu and Godt, 2008).

The depth-dependent friction angle can be given by:

$$\phi' = \phi'_o + \frac{\Delta\phi}{1 + \frac{z_w}{H_{ss}}} \quad (9)$$

where, ϕ'_o is the friction angle at the ground surface and $\Delta\phi$ is the change of friction angle within the weathering zone z_w .

The soil temperature would vary with depth below the ground surface due to changes in thermal properties of soil such as thermal conductivity. The variation of temperature with respect to depth can be rewritten from Fourier's law as:

$$\frac{dT}{dz} = -\frac{q_h}{\lambda} \quad (10)$$

where, q_h is the heat flux (Wm^{-2}), which depends on type of material and net radiation and λ is the thermal conductivity (W/mK), which can be calculated based on the SWRC and is expressed in terms of degree of saturation as (Lu and Dong 2015):

$$\frac{\lambda - \lambda_{dry}}{\lambda_{sat} - \lambda_{dry}} = 1 - \left[1 + \left(\frac{S}{S_f} \right)^m \right]^{1/m-1} \quad (11)$$

where, λ_{sat} , λ_{dry} are thermal conductivities at saturated (maximum) and dry (minimum) states,

respectively, S_j is the degree of saturation at which funicular regime is onset, and m is defined as the pore fluid network connectivity parameter that could be related to the pore-size parameter n in van Genuchten's SWRC model. In the current study, the parameters $\lambda_{dry} = 0.198 \text{ W/mK}$, $\lambda_{sat} = 1.216 \text{ W/mK}$, $S_j = 0.246$ to calculate thermal conductivity for silty soil are obtained from those reported in Lu and Dong (2015). Figure 2 shows the temperature gradient for three surface temperatures of 25, 40 and 60 °C, respectively. Assuming a constant heat flux of 3 Wm^{-2} , the temperature decreases with depth. This is due to the increase in thermal conductivity as the soil becomes saturated towards the water table.

With the conceptualization of a weathered soil mantle, a generalized FOS equation under steady vertical seepage for both saturated and unsaturated conditions can be written as (Lu and Godt, 2008):

$$FOS(z) = \frac{\tan \phi'(z)}{\tan \beta'} + \frac{2c'}{\gamma(H_{wt} - z) \sin 2\beta'} - \frac{\sigma^s(T(z))(\tan \beta' + \cot \beta') \tan \phi'(z)}{\gamma(H_{wt} - z)} \quad (12)$$

The above equation describes the friction angle as a function of depth, suction stress as a function of temperature and temperature as a function of depth. After determining the values of nonisothermal suction stress for hydrostatic condition (Eq. 6), hydraulic properties, shear strength parameters, weathering features, and temperature dependency on depth, Eq. 12 can be readily used to investigate the stability of unsaturated infinite soil slopes under nonisothermal conditions.

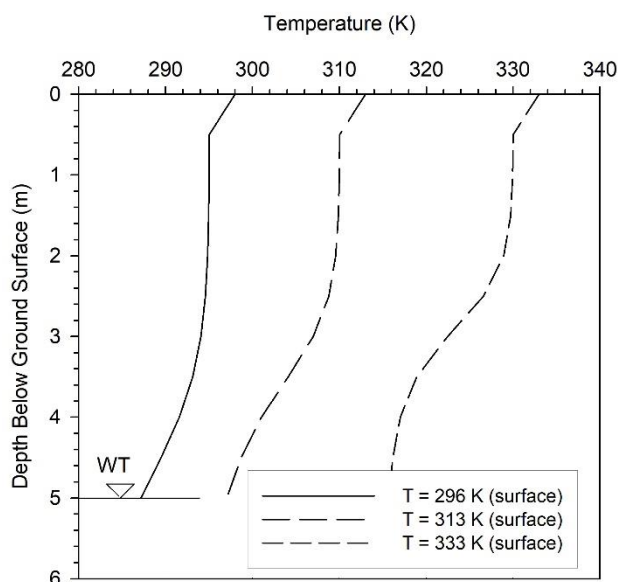


Figure 2. Variation of temperatures with depth in silty soil for surface temperatures of 25 °C (296 K), 40 °C (313 K) and 60 °C (333 K).

RESULTS AND DISCUSSION

The nonisothermal SWRC, suction stress and FOS under hydrostatic conditions can be calculated using Eqs. 5, 6 and 12, respectively, using the input parameters $n = 3.0$, $\alpha = 0.05 \text{ kPa}^{-1}$, $\phi_o = 33^\circ$, $\Delta\phi = 15^\circ$, $c' = 2.0 \text{ kPa}$, $z_w = 1.5 \text{ m}$, $\beta' = 45^\circ$ for silty soil (Lu and Godt 2008). Figure 3(a) shows the matric suction and effective degree of saturation at temperatures of

25, 40, and 60 °C, respectively. It is evident from the results that an elevated temperature can cause downward shift in the SWRCs, which implies that at a specified effective degree of saturation, the matric suction decreases with an increase in temperature. For example, at the effective degree of saturation of 0.35, matric suction decreases by approximately 24% and 39% by increasing temperature from 25 °C to 40 °C, and 60 °C, respectively. This could be due to changes in surface tension of the pore water, soil-water contact angle and enthalpy of immersion with temperature (e.g., Grant and Salehzadeh, 1996; Roshani and Sedano, 2016; Vahedifard et al., 2018b).

The changes in suction stress with matric suction at temperatures of 25, 40, and 60 °C are shown in Figure 3(b). For silty soil, the variation in nonisothermal suction stresses for different matric suction values is non-monotonic in nature. The trend of suction stress is affected by temperature-induced changes in matric suction and effective degree of saturation; this, in turn, depends on capillarity and adsorption regimes. Two distinct behaviors can be noted. Firstly, at relatively low matric suction range (0 to 15 kPa), both suction stress and matric suction decreases with an increase in temperature. Secondly, at relatively high matric suction range (20 to 50 kPa), suction stress increases with temperature. These two observed phenomena could decrease stability of slope due to reduction of resisting forces.

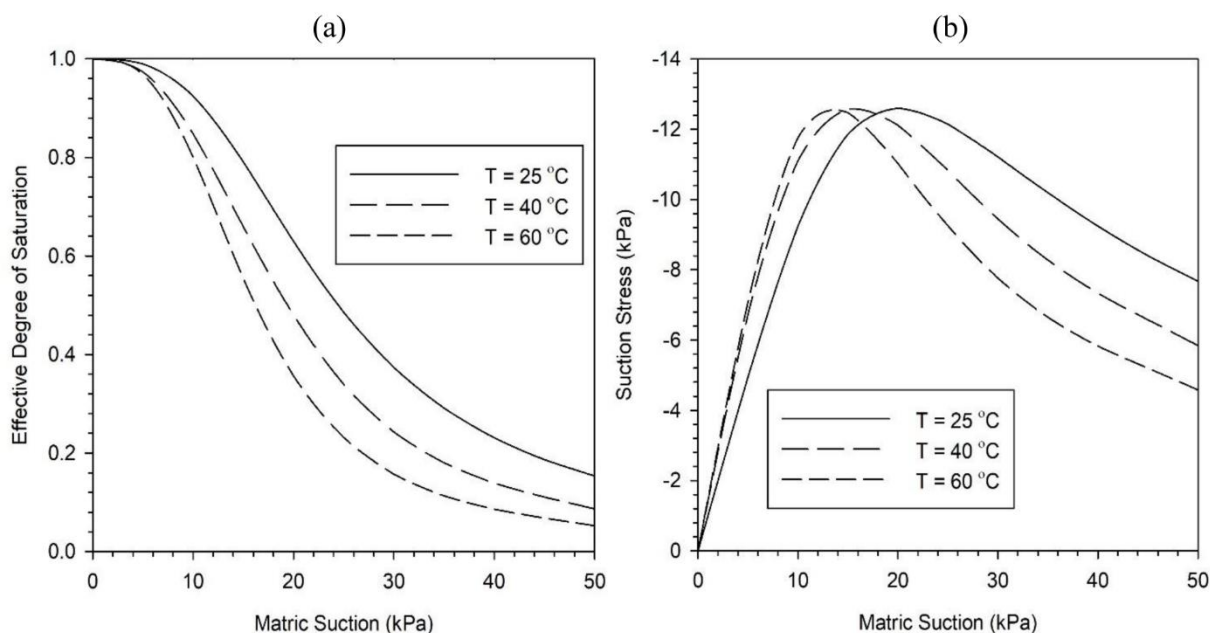


Figure 3. (a) Effective degree of saturation and matric suction and (b) Suction and matric suction, for silty soil at temperatures 25, 40 and 60 °C.

Figure 4(a) demonstrates the FOS and suction stress at temperatures of 25, 40, and 60 °C, respectively. Initially, the FOS remains almost constant for different suction stresses and temperatures. However, for a given FOS after peak suction stress, the suction stress increases with temperature. For instance, at FOS of 1.5, suction stress increases by approximately 30% and 45% as temperature increase from 25 °C to 40 °C, and 60 °C, respectively. This suggests that high suction stress is required to maintain the same FOS of the slope for higher temperatures. The changes in suction stress of silty soil mainly depends on physicochemical forces like van der Waals forces near the contacts of soil particles. These physicochemical forces at higher temperatures can be explained by Plum and Esrig (1969), which indicates that the heating of soil

can increase repulsive forces and decrease the diffuse double layer thickness, thereby, decreasing the matric suction at inter-particle contacts. Furthermore, Kenney (1966) suggested that the expansion of the diffuse double layer due to heating decreases the particle contacts, which allows shear failures to occur at these contacts.

Figure 4(b) shows the variation in FOS with distance above water table at temperatures of 25, 40, and 60 °C, respectively. As temperature increases, the FOS increases slightly near the water table, and then drops as the distance from the water table increases. For example, at depth above the water table of 3m, FOS decreases approximately by 5% and 11% by increasing temperature from 25 °C to 40 °C, and 60 °C, respectively. It is worth noting that significant reduction in FOS with temperature is more pronounced in the weathered zone, which is more critical in this case (i.e., 3.0 to 5.0 m above water table). Most of the slope failures are likely to occur in the critical zone along the weak slip surface. The temperature rise in the weak zone can reduce the critical FOS of the slope and may cause failure to occur sooner than failure of slopes at ambient temperature.

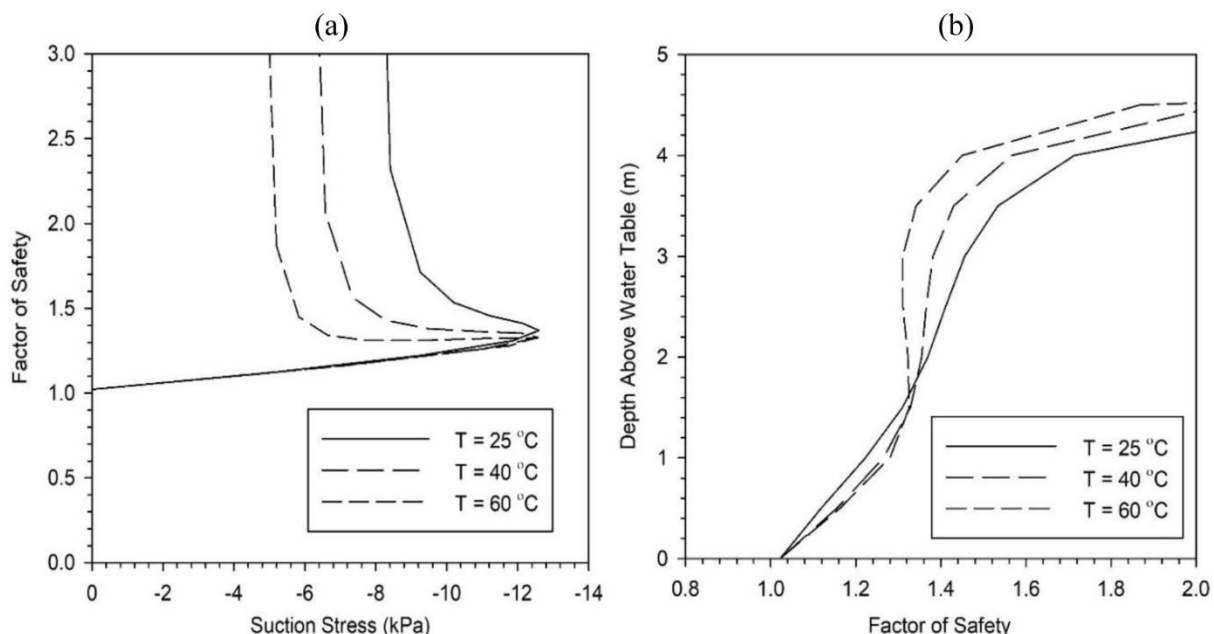


Figure 4. (a) Variation of factor of safety with suction stress and (b) Variation of factor of safety with depth above water table, for silty soil at temperatures of 25, 40 and 60 °C.

CONCLUSIONS

Employing the notion of effective stress principles for unsaturated soils, a closed-form expression to assess the stability of infinite slopes for nonisothermal and no-flow conditions is presented in this paper. This framework mainly relies on the concepts of temperature dependency of SWRC, suction stress and matric suction. The nonisothermal SWRC model is used to consider the thermal effects on the surface tension of water, enthalpy of immersion and the contact angle.

To explicitly evaluate the temperature effects on infinite slope instability, the variation in matric suction, suction stress, and FOS at temperatures of 25, 40, and 60 °C are studied for a hypothetical silty soil slope under hydrostatic conditions. Results indicated a decrease in matric suction and increase in suction stress for elevated temperatures. The results also suggest that temperature can have significant effect on effective stress parameters and thereby stability of

infinite soil slopes. The proposed framework can be extended for different soil types. Depending on availability of hydraulic properties, the proposed approach can also be applied to other existing SWRC models. The present study provides further insight into infiltration and unsteady flow conditions for unsaturated soils.

ACKNOWLEDGEMENTS

This material is based upon work supported in part by the National Science Foundation under Grant No. CMMI-1634748. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

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