

Sinusoidal Parameter Estimation from Signed Measurements Obtained via Time-Varying Thresholds

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Abstract—We consider the problem of sinusoidal parameter estimation using signed observations obtained via one-bit sampling with time-varying thresholds. In a previous paper, a relaxation-based algorithm, referred to as 1bRELAX, has been proposed to iteratively maximize the likelihood function. However, 1bRELAX can only be used in applications involving a small number of sinusoids due to the time-consuming exhaustive search procedure needed in each iteration. In this paper, we present a majorization-minimization (MM) based 1bRELAX algorithm, referred to as 1bMMRELAX, to enhance the computational efficiency of 1bRELAX. Using the MM technique, 1bMMRELAX maximizes the likelihood function iteratively using simple FFT operations to reduce the computational cost of 1bRELAX while maintaining its excellent estimation accuracy. Numerical examples are presented to demonstrate the effectiveness of the proposed method.

I. INTRODUCTION

Low-resolution quantization technology has attracted much attention due to its low cost and low power consumption advantages and for allowing for ultra-high sampling rates [1]. It has many potential applications, including spectral sensing for cognitive radios and radars [2], [3], channel estimation for massive multiple-input multiple-output (MIMO) systems [4]–[12], and target detection using automotive radars for autonomous driving [13]–[15]. As an extreme form of low-resolution quantization, one-bit sampling (which quantizes the signals using a simple comparator with some reference levels) has been studied from many aspects [4]–[12], [16]–[28]. This paper considers the problem of sinusoidal parameter estimation using signed measurements via one-bit sampling with time-varying thresholds, which allows for accurate amplitude estimation [29]–[31].

One-bit sampling with time-varying thresholds has been recently considered to enable accurate amplitude estimation from signed measurements [29]–[31]. In [30], the maximum-likelihood (ML) estimator and the corresponding Cramer-Rao bounds (CRBs) are presented and it was shown that one-bit sampling with time-varying thresholds allows accurate amplitude estimation. Since the direct implementation of the

ML estimator is computationally prohibitive, a relaxation-based method, referred to as 1bRELAX, is proposed in [31] to iteratively maximize the likelihood function. The 1bRELAX algorithm provides a good performance but still suffers from a high computational burden due to the time-consuming exhaustive search needed for each iteration step. Additionally, sparse methods based on the l_1 penalty and logarithm penalty are proposed in [29], but the numerical examples in [31] have demonstrated that 1bRELAX outperforms these sparse methods.

In this paper, we present a computationally more efficient algorithm than 1bRELAX, referred to as majorization-minimization (MM) based 1bRELAX, or 1bMMRELAX. Using the MM approach [32], [33], 1bMMRELAX maximizes the likelihood function iteratively via simple FFT operations instead of the more computationally intensive search procedures of 1bRELAX. Numerical examples are presented to show that 1bMMRELAX can reduce the computational cost while providing a similar estimation accuracy as compared to the 1bRELAX algorithm.

II. BACKGROUND

A. Signal Model

Consider a sinusoidal signal $s_t(\boldsymbol{\theta})$:

$$\begin{aligned} s_t(\boldsymbol{\theta}) &= \sum_{k=1}^K A_k \sin(\omega_k t + \phi_k) \\ &= \sum_{k=1}^K a_k \sin(\omega_k t) + b_k \cos(\omega_k t), \end{aligned} \quad (1)$$

where K is the number of sinusoids, A_k , ω_k , and ϕ_k denote the amplitude, frequency, and phase of the k th sinusoidal component, respectively, and t denotes the time variable. The unknown sinusoidal parameter vector is denoted by $\boldsymbol{\theta} = [a_1, b_1, \omega_1, \dots, a_K, b_K, \omega_K]^T \in \mathbb{R}^{3K}$ with $a_k = A_k \cos \phi_k \in \mathbb{R}$ and $b_k = A_k \sin \phi_k \in \mathbb{R}$. $(\cdot)^T$ is the transpose operator and the symbol $\mathbb{R}$ means the real space.

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Suppose we have N noisy, signed measurements, obtained via one-bit sampling with time-varying thresholds, given by:

$$y_n = \text{sign}(s_n(\boldsymbol{\theta}) + e_n - h_n), \quad (2)$$

where e_n is the additive noise, h_n is the known time-varying threshold which allows for the accurate amplitude estimation, $n, n = 0, 1, \dots, N-1$, is the time index, and $\text{sign}(\cdot)$ is the sign operator defined as:

$$\text{sign}(x) = \begin{cases} 1 & \text{if } x \geq 0, \\ -1 & \text{if } x < 0. \end{cases} \quad (3)$$

Under the assumption that the additive noise e_n is i.i.d. Gaussian with zero-mean and known variance σ^2 , the likelihood function is given by [30], [31]:

$$\begin{aligned} L(\boldsymbol{\theta}) &= \prod_{n=0}^{N-1} \Phi\left(y_n \frac{s_n(\boldsymbol{\theta}) - h_n}{\sigma}\right) \\ &= \prod_{n=0}^{N-1} \Phi\left(y_n \frac{\left(\sum_{k=1}^K a_k \sin(\omega_k n) + b_k \cos(\omega_k n)\right) - h_n}{\sigma}\right), \end{aligned} \quad (4)$$

where $\Phi(x)$ denotes the cumulative distribution function (cdf) of the standard normal distribution.

We are interested in recovering the parameter vector $\boldsymbol{\theta}$ based on the signed measurements $\mathbf{y} = [y_0, y_1, \dots, y_{N-1}]^T$.

B. Maximum Likelihood Algorithm

The ML estimate of the parameter vector $\boldsymbol{\theta}$ can be obtained by minimizing the following negative log-likelihood function $l(\boldsymbol{\theta})$ [30], [31]:

$$\begin{aligned} \hat{\boldsymbol{\theta}} &= \arg \min_{\boldsymbol{\theta}} l(\boldsymbol{\theta}) = \arg \min_{\boldsymbol{\theta}} \sum_{n=0}^{N-1} \\ &\quad -\log \left[\Phi\left(y_n \frac{\left(\sum_{k=1}^K a_k \sin(\omega_k n) + b_k \cos(\omega_k n)\right) - h_n}{\sigma}\right) \right]. \end{aligned} \quad (5)$$

Let $\boldsymbol{\omega} = [\omega_1, \omega_2, \dots, \omega_K]^T$ be a vector composed of the frequencies of $s_t(\boldsymbol{\theta})$. For given $\boldsymbol{\omega}$, the above optimization problem is convex in $\{a_k\}_{k=1}^K$ and $\{b_k\}_{k=1}^K$, and therefore can be solved by any of several global methods [30], [31]. With this fact in mind, the ML estimator can be directly implemented by performing a K -dimensional search of $\boldsymbol{\omega}$ on the feasible space of frequencies $[0, \pi]^K$. Supposing that there are L points to search in each dimension, the $2K$ -dimensional convex problem must be solved $O(L^K)$ times. As the number of sinusoids K increases, the search over the high-dimensional frequency space becomes computationally prohibitive and more efficient algorithms must be considered [29], [31].

C. 1bRELAX

Inspired by the infinite precision RELAX algorithm [34], the 1bRELAX algorithm is proposed as a relaxation-based approach to iteratively maximize the likelihood function [31]. The steps of 1bRELAX can be summarized as follows:

Step 1: Assume $K = 1$. Obtain $\{\hat{a}_1, \hat{b}_1, \hat{\omega}_1\}$ by solving (5) via the exhaustive search (over ω_1) approach.

Step 2: Assume $K = 2$. Obtain $\{\hat{a}_2, \hat{b}_2, \hat{\omega}_2\}$ by solving (5) via the exhaustive search approach with $\{a_1, b_1, \omega_1\}$ replaced by their most recent estimates $\{\hat{a}_1, \hat{b}_1, \hat{\omega}_1\}$.

Next, redetermine $\{\hat{a}_1, \hat{b}_1, \hat{\omega}_1\}$ by solving (5) via the exhaustive search approach with $\{a_2, b_2, \omega_2\}$ replaced by their most recent estimates $\{\hat{a}_2, \hat{b}_2, \hat{\omega}_2\}$.

Iterate the previous two steps until practical convergence, i.e. the relative change of cost function is small enough.

Step 3: Assume $K = 3$. Obtain $\{\hat{a}_3, \hat{b}_3, \hat{\omega}_3\}$ by solving (5) via the exhaustive search approach with $\{a_i, b_i, \omega_i\}_{i=1}^2$ replaced by their most recent estimates $\{\hat{a}_i, \hat{b}_i, \hat{\omega}_i\}_{i=1}^2$.

Next, redetermine $\{\hat{a}_1, \hat{b}_1, \hat{\omega}_1\}$ by solving (5) via the exhaustive search approach with $\{a_i, b_i, \omega_i\}_{i=2}^3$ replaced by their most recent estimates $\{\hat{a}_i, \hat{b}_i, \hat{\omega}_i\}_{i=2}^3$. Then, update $\{\hat{a}_2, \hat{b}_2, \hat{\omega}_2\}$ by solving (5) via the exhaustive search approach with $\{a_i, b_i, \omega_i\}_{i=1,3}$ replaced by their most recent estimates $\{\hat{a}_i, \hat{b}_i, \hat{\omega}_i\}_{i=1,3}$.

Iterate the previous three steps until practical convergence.

Remaining steps: Continue until the desired or estimated model order is reached.

In each step, 1bRELAX estimates the parameters of a new sinusoid based on the sinusoids obtained in the previous steps and then updates the parameters of each sinusoid iteratively. Since the update procedure is implemented by means of an exhaustive search, 1bRELAX is rather time-consuming.

Note that if we do not update the parameter estimates of each sinusoid iteratively in each step, then the 1bRELAX algorithm becomes a CLEAN-like algorithm [34]–[36] hereafter, which we refer to as 1bCLEAN. Compared with 1bRELAX, the 1bCLEAN algorithm has a lower computational complexity, but provides poorer resolution and estimation accuracy.

III. 1BMMRELAX

In this section, we derive a majorization-minimization (MM, see [32], [33]) based 1bRELAX algorithm, referred to as 1bMMRELAX, to reduce the computational cost of 1bRELAX.

A. The MM Approach

We first derive a computationally efficient MM approach to solve the optimization problem in (5). Let us define an auxiliary vector $\mathbf{x}(\boldsymbol{\theta}) = [x_0(\boldsymbol{\theta}), \dots, x_{N-1}(\boldsymbol{\theta})]^T$, with

$$x_n(\boldsymbol{\theta}) = y_n \left(\frac{s_n(\boldsymbol{\theta}) - h_n}{\sigma} \right), n = 0, 1, \dots, N-1. \quad (6)$$

The cost function in (5) can be rewritten as

$$l(\mathbf{x}(\boldsymbol{\theta})) = - \sum_{n=0}^{N-1} \log \Phi(x_n(\boldsymbol{\theta})) = \sum_{n=0}^{N-1} f(x_n(\boldsymbol{\theta})), \quad (7)$$

where $f(x) \triangleq -\log\Phi(x)$. At any given point $\hat{x}$, we have $\eta \in [x, \hat{x}]$ such that:

$$f(x) = f(\hat{x}) + f'(\hat{x})(x - \hat{x}) + \frac{f''(\eta)}{2}(x - \hat{x})^2. \quad (8)$$

Note that for any given x [37], [38],

$$0 < f''(x) < 1. \quad (9)$$

We can obtain the following upper bound for $f(x)$:

$$f(x) \leq f(\hat{x}) + f'(\hat{x})(x - \hat{x}) + \frac{1}{2}(x - \hat{x})^2, \quad (10)$$

which becomes an equality for $x = \hat{x}$.

Inserting (10) into (7), we can find a majorizing function for $l(\mathbf{x}(\boldsymbol{\theta}))$:

$$G[\mathbf{x}(\boldsymbol{\theta})|\hat{\mathbf{x}}] = \sum_{n=0}^{N-1} f(\hat{x}_n) + f'(\hat{x}_n)[x_n(\boldsymbol{\theta}) - \hat{x}_n] + \frac{1}{2}[x_n(\boldsymbol{\theta}) - \hat{x}_n]^2. \quad (11)$$

With this observation in mind, the updating formula at the $(i+1)$ th iteration of the MM approach is given by:

$$\begin{aligned} \min_{\boldsymbol{\theta}} G[\mathbf{x}(\boldsymbol{\theta})|\mathbf{x}^i] \\ = \min_{\boldsymbol{\theta}} \sum_{n=0}^{N-1} [x_n(\boldsymbol{\theta}) - (x_n^i - f'(x_n^i))]^2 \end{aligned} \quad (12)$$

where the index i is the iteration number and $\mathbf{x}^i = \mathbf{x}(\boldsymbol{\theta}^i)$ is the estimate obtained at the i th iteration.

Inserting (6) into (12), we note that the above update formula is equivalent to

$$\min_{\boldsymbol{\theta}} \sum_{n=0}^{N-1} [s_n(\boldsymbol{\theta}) - z_n(\boldsymbol{\theta}^i)]^2, \quad (13)$$

where

$$z_n(\boldsymbol{\theta}) = \sigma y_n[x_n(\boldsymbol{\theta}) - f'(x_n(\boldsymbol{\theta}))]. \quad (14)$$

Note that the optimization problem (13) can be interpreted as a sinusoidal parameter estimation problem for infinite precision data. Viewing $\{z_n(\boldsymbol{\theta}^i)\}_{n=0}^{N-1}$ as the input signal, the infinite precision RELAX algorithm [34] provides a computationally simple and efficient approach, due to using FFT operations, to decrease the objective function in (13).

B. 1bMMRELAX

We now present a new method to efficiently estimate the sinusoidal parameters from signed measurements based on the aforementioned MM technique. This method, referred to as 1bMMRELAX, speeds up the 1bRELAX algorithm by using the MM approach. The 1bMMRELAX algorithm begins by assuming $K = 1$. In the k th step, K is assumed to be k , and we first use the exhaustive search (in the frequency domain) to get the initial parameters of the k th sinusoid making use of the $(k-1)$ sinusoids obtained in the previous $(k-1)$ steps. Next, the algorithm refines the parameter estimates of the k sinusoids by using the MM approach to maximize the

TABLE I
SINUSOIDAL SIGNAL PARAMETERS

Parameter	1	2	3	4	5	6
Amplitude	1	1	0.7	0.8	0.6	0.6
Freq. (rad/s)	0.22π	$0.22\pi + \frac{2\pi}{N}$	0.4π	0.6π	0.74π	0.9π
Phase (rad)	$\frac{2}{3}$	$\frac{5}{3}$	0	$\frac{7}{4}$	$\frac{4}{3}$	$\frac{1}{2}$

likelihood function. Then, we increase the model order k to $k+1$ and the algorithm proceeds until the desired or estimated model order is reached.

The 1bMMRELAX algorithm can be summarized as follows:

Step 1: Assume $K = 1$. Obtain $\{\hat{a}_1, \hat{b}_1, \hat{\omega}_1\}$ by solving (5) via the exhaustive search (in the frequency domain) followed by the MM procedure (see subsection A).

Step 2: a) Assume $K = 2$. Obtain $\{\hat{a}_2, \hat{b}_2, \hat{\omega}_2\}$ by solving (5) via the exhaustive search (in the frequency domain) with $\{a_1, b_1, \omega_1\}$ replaced by their most recent estimates $\{\hat{a}_1, \hat{b}_1, \hat{\omega}_1\}$.

b) Next, redetermine $\{\hat{a}_i, \hat{b}_i, \hat{\omega}_i\}_{i=1}^2$ by solving (5) via the MM approach (see subsection A) with the initialization $\boldsymbol{\theta}^0 = [\hat{a}_1, \hat{b}_1, \hat{\omega}_1, \hat{a}_2, \hat{b}_2, \hat{\omega}_2]$ provided by Step 1 and sub-step a).

Remaining steps: Continue until the desired or estimated model order is reached.

IV. NUMERICAL EXAMPLES

We now present numerical examples to compare the proposed algorithm with 1bRELAX and 1bCLEAN in terms of the estimation accuracy and computational complexity. The case considered is a signal consisting of six sinusoids with different frequencies, amplitudes and phases. The parameters of these sinusoids are shown in Table I. Note that the first two frequencies are closely spaced. The signal length is varied with $N = 128, 256, 512, 1024$ and the signal-to-noise ratios (SNRs) are set to 10 dB and 15 dB, where the SNR is defined as:

$$\text{SNR} = 10 \log_{10} \sum_{k=1}^K \frac{A_k^2}{2\sigma^2}. \quad (15)$$

We generate the signed measurements by comparing the original signal with a time-varying threshold. The threshold $\{h_n\}_{n=1}^N$ is generated randomly from a discrete set of 8 values uniformly distributed in $[-1, 1]$. Two hundred independent Monte Carlo trials are run and each trial has the independent noise and threshold realization. The average mean-squared errors (MSEs) over the six sinusoidal signals are employed to evaluate the performance of the estimators. The maximum number of the MM iterations is set to 50. The simulations are conducted in a PC with 3.10 GHz CPU and 16.00 GB RAM.

Fig. 1 shows the average frequency and amplitude MSEs, the corresponding CRBs, as well as the probabilities of correct frequency estimation P_d . In each trial, the frequency detection is considered correct if the maximum absolute error of the frequency estimate is less than $\frac{2\pi}{N}$. We see that 1bMMRELAX and 1bRELAX provide lower frequency and amplitude MSEs as well as higher P_d values than 1bCLEAN in most

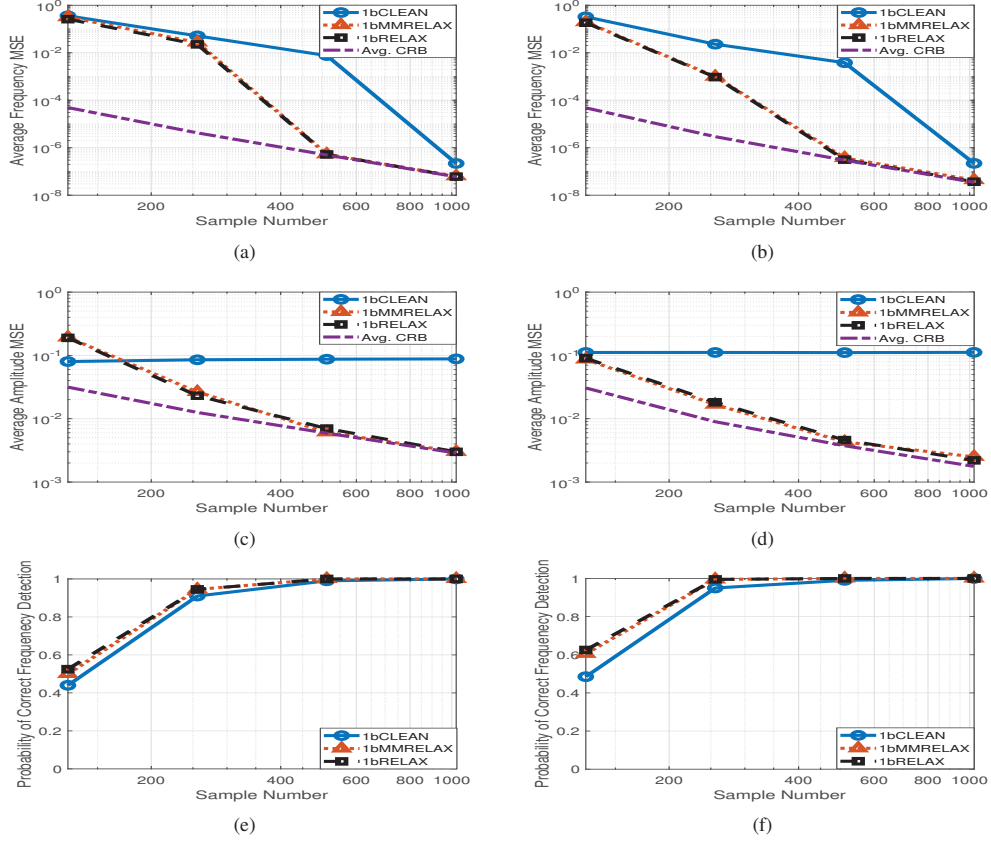


Fig. 1. Sinusoidal parameter estimation performance of 1bCLEAN, 1bMMRELAX, and 1bRELAX for a time-varying threshold: average frequency MSEs vs. N when (a) SNR = 10 dB and (b) SNR = 15 dB, average amplitude MSEs vs. N when (c) SNR = 10 dB and (d) SNR = 15 dB, and probabilities of correct frequency detection vs. N when (e) SNR = 10 dB and (f) SNR = 15 dB.

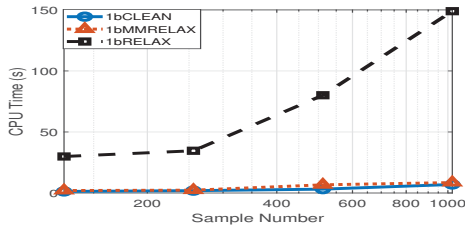


Fig. 2. Average running time vs. N for 1bCLEAN, 1bMMRELAX, and 1bRELAX for SNR=10 dB.

cases. 1bCLEAN cannot achieve a good amplitude estimation performance even when N is large due to the lack of parameter refinement procedure. The MSEs of the estimates obtained by using 1bMMRELAX and 1bRELAX are close to the CRBs when $N \geq 512$. Additionally, as N increases, P_d goes to 1, and the increase of SNR by 5 dB results in an improvement in P_d for all three algorithms, as expected.

The average computational times needed by these three algorithms are recorded in seconds and plotted in Fig. 2 when the SNR is 10 dB. The required times for SNR = 15 dB

are quite similar to those for SNR = 10 dB and omitted. As expected, 1bMMRELAX and 1bCLEAN require similar computational times, and both are much faster than 1bRELAX. This fact makes 1bMMRELAX a desirable algorithm for sinusoidal parameter estimation using signed observations. We also remark that as the number of sinusoids increases, the computational complexity reduction offered by 1bMMRELAX becomes more significant.

V. CONCLUSIONS

In this paper, we have considered the sinusoidal parameter estimation problem for signed measurements obtained via one-bit sampling with time-varying thresholds. Using the MM technique, we have introduced the 1bMMRELAX algorithm to reduce the computational complexity of the 1bRELAX algorithm. The computational cost is reduced via using simple FFT operations rather than the exhaustive searches for frequency estimation. Numerical examples have been presented to demonstrate that the 1bMMRELAX can significantly reduce the computational complexity while maintaining the excellent estimation performance of 1bRELAX.

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