

Learn Low Wavenumber Information in FWI via Deep Inception Based Convolutional Networks

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SUMMARY

In this paper, we will explore the possibility of synthesizing the low-frequency data from the high-frequency data. The synthesized low-frequency data are used to improve the full-waveform inversion (FWI). Unlike all previously methods, to the best of our knowledge, this is the first attempt to utilize a data driven approach to solve the problem. We propose to learn the low wavenumber information in FWI via the Deep Inception based Convolutional Networks. Once the deep learning network is sufficiently trained, the network can be used to predicted the low-frequency data with high accuracy on a completely different unknown velocity model. In the end, we validate the quality of the predicted low-frequency data and the robustness of this deep learning approach.

INTRODUCTION

In current seismic data acquisition environments, reliable low-frequency components below 5 Hz do not practically exist in typical seismic data sets. This will affect the performance of FWI, which takes into account the full waveform information of seismic data to reconstruct an earth model and is expected to generate the maps of geophysical properties continuously covering a broadband wavenumber spectrum. Without low-wavenumber components of the velocity model, FWI may converge to a local minimum when the inversion starts at high frequencies or the starting velocity model is not sufficiently close to the true model. Computationally synthesizing the low frequencies from the high-frequency data has attracted much attention recently. However, it is a very challenging signal-processing problem and is a highly nonlinear and nondeterministic operation. This paper will be kept brief with regard to the treatment of related research works. Readers who are interested in more technical aspects of this topic can refer to articles such as Ma and Hale (2013), Baek et al. (2014), Warner and Guasch (2016), Van Leeuwen and Herrmann (2013), Tang et al. (2013), Alkhalifah (2014), and Li and Demanet (2016). The review article published by Hu et al. (2018) provides a comprehensive overview of latest research advances.

Following a phase modulation/demodulation concept commonly used by musicians for instrument tuning, Hu (2014) proposed beat tone FWI to suppress the cycle-skipping phenomenon. This method utilizes two seismic data sets at two slightly different high frequencies to retrieve the buried low wavenumber information. However, although the beat tone strategy successfully amplifies the low wavenumber information, it fails to completely exclude the information of the high wavenumber. Figure 1 show the comparison between ground-truth 3 [Hz] data and the beat tone data extracted from the 8 [Hz] and 11 [Hz] data.

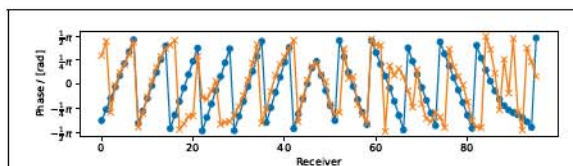


Figure 1: An example of comparing the beat tone data to the true low frequency data. The blue line represents the true 3 [Hz] data. The orange line indicates beat tone data extracted from the 8 [Hz] and 11 [Hz] data.

By observing the behavior of the beat tone data, we found that the naive regression algorithm is unable to find a reliable method that could synthesize the data of low frequencies from the high-frequency data. However, first, a nonlinear and nondeterministic mapping function exists between the phase information of the beat tone data extracted from high frequency components and that of the true low frequency data. Second, we could find a robust mapping function independent of subsurface structures by mining the relationship via a pure data-driven approach. In this paper, we propose a novel data driven framework and verify its feasibility in synthesizing the low-frequency data from the high-frequency data. Our method is built upon a Deep Inception Based Convolutional Network (Szegedy et al. (2015)) and its structural detail will be explained in the following sections.

A DATA DRIVEN FRAMEWORK FOR LEARNING LOW-FREQUENCY DATA FROM HIGH-FREQUENCY DATA

Figure 2 shows that our framework is divided into two phases. In the first phase, we preprocess the high-frequency data in the complex domain. In the second phase, the processed data will be fed into the deep neural network to predict the corresponding low-frequency data.

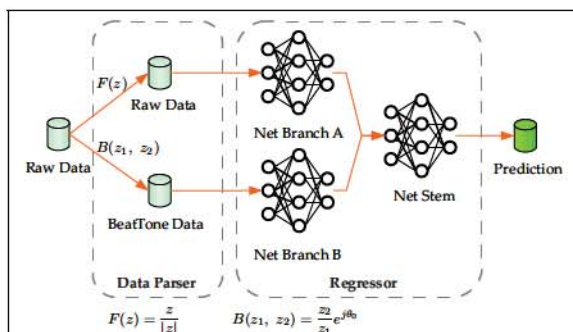


Figure 2: The structure of the data driven frame for learning the low-frequency data.

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Data Preprocessing

The model will be trained with two kinds of data:

- (1) **Normalized raw data:** The amplitude of the raw data is normalized in the complex domain, this operator can be described as $F(z) = \frac{z}{|z|}$;
- (2) **Beat tone data:** The beat tone data is generated according to Hu (2014). Denote two high frequency-data z_1 and z_2 with frequencies respectively at f_1 and f_2 ($f_2 > f_1$), the beat tone data is generated as $B(z_1, z_2) = \frac{z_2}{z_1} e^{j\theta_0}$.

The real part and the imaginary part of the data at different frequencies will be fed into different channels of the convolutional network. For data collected at consecutive frequencies $\{f_l, f_l + \Delta f, f_l + 2\Delta f, \dots, f_h\}$, the number of channels required for the normalized raw data is:

$$C_r = 2 \cdot \left(\frac{f_h - f_l}{\Delta f} + 1 \right). \quad (1)$$

To generate the beat tone data for a specified low-frequency at f_o , we need two data sets of high-frequencies at $(f, f + f_o)$. The network can be trained to synthesize data at multiple low-frequencies. Using N as the number of total low-frequency data sets the network can produce, the number of input channels required by the beat tone data is:

$$C_b = 2 \cdot \sum_{i=0}^{N-1} \left(\frac{f_h - f_o^{(i)} - f_l}{\Delta f} + 1 \right). \quad (2)$$

Then the number of channels of the network output is:

$$C_o = 2N. \quad (3)$$

In the experiments, we use $8 \leq f \leq 18$ [Hz] data to predict $\{1.5, 3, 5\}$ [Hz] data and $10 \leq f \leq 18$ [Hz] data to predict $\{1.5, 3, 5, 7\}$ [Hz] data. The channel numbers in our experiment are summarized in Table 1 and $\Delta f = 0.5$ [Hz].

Input Band	C_r	C_b	C_o
≥ 8 [Hz]	42	88	6
≥ 10 [Hz]	34	70	8

Table 1: Channel numbers in experiments

Deep Inception based Convolutional Networks

As shown in Figure 2, any arbitrary neural network could be used in the framework. We have designed and tested with different network structures. The framework learns to synthesize the low-frequency data by using both the normalized high-frequency data and the beat tone data. Therefore, the network is set to learn a model that will amplify the low wavenumber information and exclude the high wavenumber information at the same time. The branch A and B share the same structure. Their

outputs would be merged together by the channel concatenation. The merged layer would be fed into a stem network with mixed features from both branches and the output is mapped to the low-frequency data.

Among all networks we have tested so far, the inception based convolutional network (Szegedy et al. (2017)) is most effective in solving the problem. Figure 3a and Figure 3b shows the two basic units of the inception model. Figure 3a shows the convolutional block which is composed of $D+2$ parallel branches. Denote the depth as d , then:

- (1) If $d = -1$, this branch has a pooling layer and a $\times 1$ convolutional layer (which is also called projection).
- (2) If $d = 0$, this branch only has a projection layer, which means it is just a combination of the original features.
- (3) If $d > 0$, there would be d convolutional layers after the projection. The projection will produce an output with the length of $2 \times (d-1) \times \lfloor \frac{N}{2} \rfloor + N$.

The cup sign ($\cup$) in Figure 3a represents concatenating all channels from each branches. Since there are $D+2$ branches, if each branch has C_o output channels, the number of output channels of the whole block will be $(D+2)C_o$. The other basic unit is the deconvolutional block which is illustrated in Figure 3b. It is designed by adding a deconvolutional layer before the convolutional inception block. The deconvolutional layer will up-sample the input layers by the stride of s . In the regression model we may need to reduce the length of the sample during the convolution and use this operation to recover the original length so that the features would be easier to be mapped to the target space.

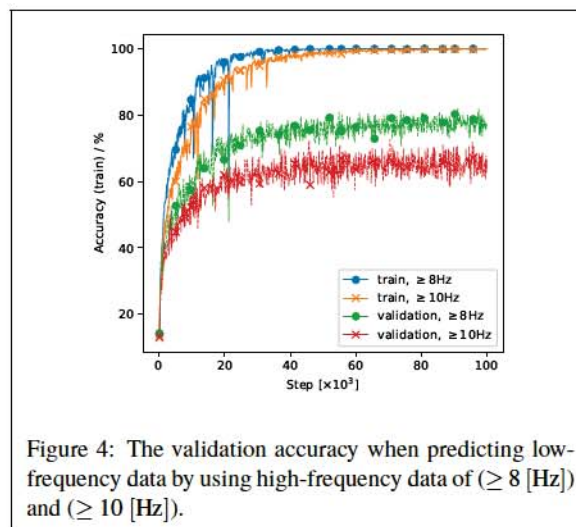
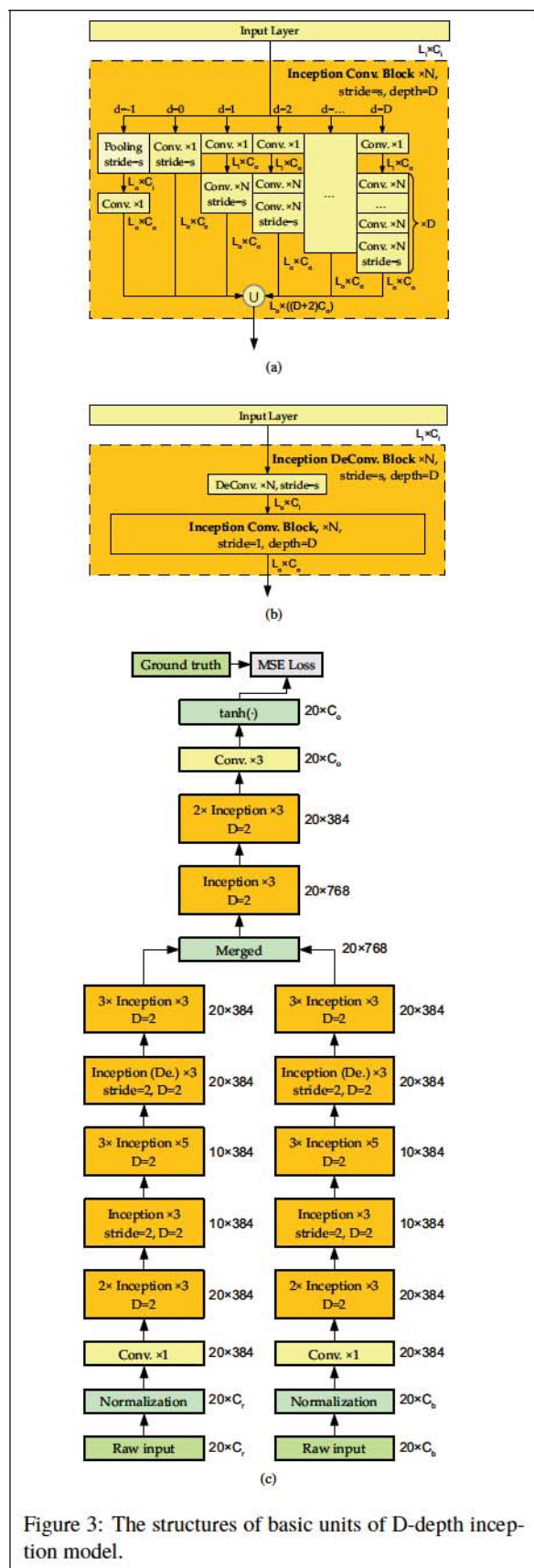
Figure 3c shows the structure of the regressor. We crop several samples with a length of 20 from the original 96-length samples randomly. Then the samples could be pre-processed for the raw data and beat tone data respectively. In each branch network we use 10 inception block and concatenate the output channels from 2 branch networks. In Figure 3c, all depths (denoted as D) of every individual inception block is set to be 2. The depth of the whole network is 41. We use $\tanh$ function to map the output to the range of $(-1, 1)$. The objective of the learning process is to minimize the MSE loss against the ground truth. The network can be represented as a function $\mathcal{F}(\mathbf{x}, \Theta)$, where $\mathbf{x}$ represents the input, Θ are all parameters of the network. Given the ground truth $\mathbf{y}$, the optimization can be described as:

$$\min_{\Theta} \sum_{\mathbf{x}, \mathbf{y}} \|\mathcal{F}(\mathbf{x}, \Theta) - \mathbf{y}\|_2^2. \quad (4)$$

FEASIBILITY STUDY

In order to verify the robustness of the model for adapting to different subsurface structures, *the Deep Inception based Convolutional Networks are trained with SEG Model 1997 2.5D and tested against the Marmousi Model*. Figure 4 shows the training and validation accuracy when using different sets of high-frequency data. It indicates that as the frequency gets

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higher, the accuracy for predicting low-frequency data is getting lower, possibly due to the increased level of nonlinearity of the underlying mapping operator between beat tone data and true low frequency data. Introducing more training data with more varieties may improve the prediction accuracy and further enhance the robustness of this approach.

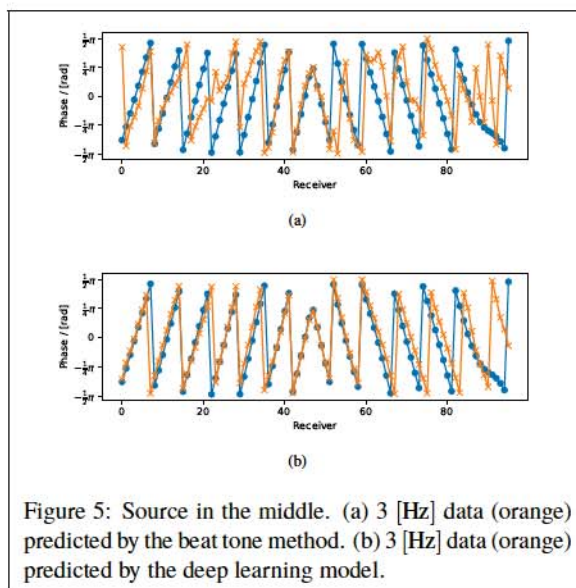
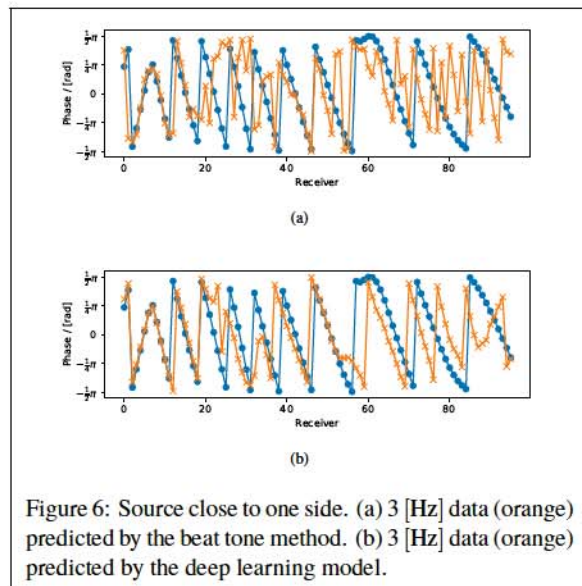


Figure 5a show the true 3 [Hz] Marmousi data (blue) and the beat tone 3 [Hz] data (orange) extracted from 13 [Hz] and 10 [Hz] data. As a comparison, Figure 5b show the 3 [Hz] data (orange) predicted by the deep learning network with both normalized raw data and beat tone data of high-frequency (≥ 10 [Hz]). The proposed deep learning network successfully suppresses much of the high wavenumber information. Figure 6 compares the beat tone result and the deep learning prediction for 3 [Hz] on the same data set. However, since the source is moved away from the center, signals received far away from the source are difficult to tune. Leveraged by the data driven approach, the

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deep learning model demonstrates far more superior prediction performance in adapting to different scenarios.

FWI Inversion

In this section, we compare the velocity model reconstructed by the FWI engine by using the 3 [Hz] data predicted by the beat tone and our proposed deep learning network. The 3 [Hz] predicted by the deep learning network is input into the *conventional* FWI engine for the inversion instead of the beat tone FWI engine used in Hu (2014). The starting model is a simple 1D linear model.

The FWI results of various scenarios are shown in Figure 7. Figure 7a shows the conventional FWI inversion result by using the true 3 [Hz] data. The conventional FWI inversion result of beat tone 3 [Hz] (from 10 [Hz] and 13 [Hz]) is displayed in Figure 7b. Strong artifacts can be observed in both shallow and deep parts. Figure 7c demonstrates the conventional FWI inversion result of 3 [Hz] predicted from the high frequency data (≥ 10 [Hz]) by the deep learning network. Compared to Figure 7a, there is high level of similarity at the same spatial resolution, which validates the quality of the predicated low-frequency data and the robustness of this deep learning network.

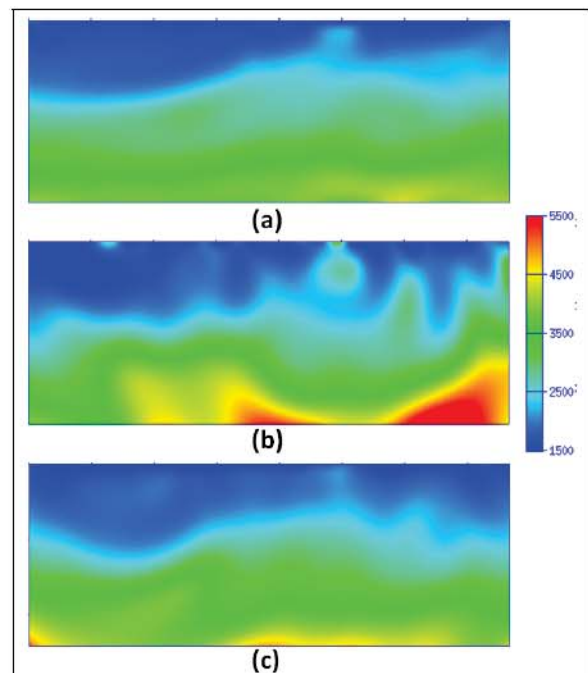
CONCLUSION

Starting the frequency continuation using the low-frequency data will help FWI to avoid convergence to local minima. However, it is a very challenging task to computationally synthesizing the low-frequency data from the high-frequency data. As the analytical solutions to the bandwidth extension problem often fail to adapt to different scenarios, we proposed a data driven approach leveraged by the deep learning network. The numerical example proves that the predicted low frequency

data are very similar to the true low frequency data. By introducing more training models and training datasets with different varieties, the quality of the low frequency data prediction is expected to be significantly improved.

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