2018 IEEE Conference on Decision and Control (CDC)
Miami Beach, FL, USA, Dec. 17-19, 2018

A Moments Based Approach to Designing MIMO Data Driven Controllers for Switched Systems

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***Abstract*— The goal of this paper is to develop a computationally tractable framework for data driven control of switched linear MIMO systems. Given a model structure and experimental data collected at different operating points, we seek to directly design a controller that stabilizes all plants compatible with this information, without an explicit plant identification. The main result of the paper shows that this problem can be recast into a polynomial optimization form and efficiently solved, leading to a robust controller with guaranteed ℓ^∞ worse-case performance for any switching amongst all plants that could have generated the observed experimental data. The effectiveness of the proposed technique is illustrated with a numerical example.**

I. Introduction

Due to a large research effort undertaken during the past decade, the problem of designing controllers for switched linear systems has been, to a large extent, solved (see for instance [11], [18], [1], [17], [9] and references therein)). However, all of these approaches rely on the availability of a model of the system to be controlled. Thus, in practical cases, designing controllers for switched systems usually entails first identifying a plant model along with worst-case identification error bounds that can then be used in conjunction with existing controller design techniques. However, the process of identifying models for switched systems and obtaining error bounds by validating these models against additional data is far from trivial. Indeed, in its most general form, this identification/(in)validation step is known to be NP-hard (see for instance [20], [19]). Note that this two step approach is conservative, even in the LTI case, since typically the error bounds provided by the identification/(in)validation steps are not tight.

Data driven control methods seek to circumvent this conservatism by directly synthesizing a controller from the experimental data, without identifying the plant first. A large portion of these methods accomplish this by finding a controller that minimizes a suitable performance index. Tuning based data driven approaches include iterative feedback [13], frequency domain, [14], correlation based [15], and virtual reference feedback tuning [5], [10], [4]. An alternative, robust optimization based approach was proposed in [7]. While successful, these techniques are restricted to time invariant plants. Indeed, to the best of the authors' knowledge, the only existing data-driven control method capable of handling switched systems is [8], albeit with the caveat imposing superstability, rather than stability, a much stronger requirement. Motivated by the issues noted above, this paper seeks to develop a switched DDC framework, capable of handling finite, noisy data records, while guaranteeing closed loop stability of all plants in the consistency set. Contrary to [8], here we impose stability, rather than superstability, leading to a (non-convex) polynomial optimization problem. However, as we show in the paper, this problem can be efficiently solved by exploiting recent advances in semi-algebraic optimization. When compared against [8], the main trade-off is computational complexity versus conservatism. Indeed as we illustrate with an example, the propose technique can find stabilizing controllers in cases where the superstability based approach fails to do so.

The paper is organized as follows: section II introduces the notation, reviews some background results and formally states the problem under consideration. Section III exploits ideas from robust optimization, polyhedral Lyapunov functions and polynomial optimization to reduce the problem to a tractable convex optimization. Section IV illustrates these ideas with some examples and shows the advantages of the proposed approach vis-a-vis the one introduced in [8] . Finally, Section V presents some conclusions and discusses open issues.

This work was supported in part by NSF grants CNS–1646121, CMMI–1638234 and ECCS–1808381; AFOSR grant FA9550-15-1-0392; and the Alert DHS Center of Excellence under Award Number 2013-ST-061-ED0001 The authors are with the ECE Department, Northeastern University, Boston, MA 02115. emails: dai.ti@husky.neu.edu, msznaier@coe.neu.edu

II. Preliminaries

A. Notation and background results

$\mathbf{x}, \mathbf{X}$	a vector in $\mathbb{R}^n$, a matrix in $\mathbb{R}^{m\times n}$
$\mathbf{X} \geq 0$	$\mathbf{X}$ is element-wise non-negative (e.g. $\mathbf{X}(i,j) \geq 0$)
$\mathbf{X} \succeq 0$	$\mathbf{X}$ is positive semi-definite
$\|\mathbf{X}\|_\infty$	ℓ^∞ induced-norm of $\mathbf{X}$

$$\|\mathbf{X}\|_\infty \doteq \sup_i \sum_{j=1}^{n} |\mathbf{X}(i,j)|$$

$\|\mathbf{x}\|_\infty$	ℓ^∞-norm of $\mathbf{x}$: $\|\mathbf{x}\|_\infty \doteq \sup_i |\mathbf{x}(i)|$
$\otimes$	Kronecker product
$\mathbf{vec}(\mathbf{X})$	matrix vectorizing operation

$$\mathbf{vec}(\mathbf{X}) = \left[(\mathbf{X}(:,1))^T \dots (\mathbf{X}(:,n))^T\right]^T$$

$\mathbf{mat}(\mathbf{x})$	vector to matrix operation
$\mathbf{diag}(\mathbf{s})$	create a diagonal matrix from the vector $\mathbf{s}$.

B. Polynomial optimization problems

In this paper, we will reduce the data driven control problem to a (non-convex) quadratically constrained quadratic problem (QCQP) of the form:

$$\min_{\mathbf{x}} \mathbf{v_x}^T \mathbf{Q}_o \mathbf{v_x} \; s.t. \; \mathbf{v_x}^T \mathbf{Q}_k \mathbf{v_x} \geq 0, k = 1, \dots, N \quad (1)$$

for some symmetric matrices $\mathbf{Q}_i, i = 0, ..,$ where $\mathbf{v_x}^T = \left[1, x_1 \; x_2 \; \dots, x_n\right]$. These problems are a special case of general polynomial optimization problems of the form:

$$p^* = \min_{\boldsymbol{x} \in \mathcal{K}} \quad p(\boldsymbol{x}) = \sum_{\boldsymbol{\alpha}} p_{\boldsymbol{\alpha}} \boldsymbol{x}^{\boldsymbol{\alpha}} \quad (2)$$

where $\boldsymbol{\alpha} \doteq [\alpha_1, \dots, \alpha_n]$, $\boldsymbol{x}^{\boldsymbol{\alpha}} = \prod_{i=1}^n x_i^{\alpha_i}$ and the set $\mathcal{K} = \{\boldsymbol{x} \in \mathbb{R}^n : g_k(\boldsymbol{x}) \geq 0, \; k = 1, \cdots, N\}$ is defined by a collection of polynomial constraints of the form $g_k(\boldsymbol{x}) = \sum_\alpha g_{k,\alpha} \boldsymbol{x}^\alpha \geq 0$.

It can be shown [16], that problem (2) is equivalent to the following optimization problem over the set $\mathcal{P}(\mathcal{K})$ of probability measures μ supported on $\mathcal{K}$:

$$\begin{aligned} p^* &= \min_{\mu \in \mathcal{P}(\mathcal{K})} \int p(\boldsymbol{x}) \mu(d\boldsymbol{x}) = \min_{\mu} \sum_{\boldsymbol{\alpha}} p_{\boldsymbol{\alpha}} \boldsymbol{m}_{\boldsymbol{\alpha}} \\ &\text{subject to } \boldsymbol{m}_{\boldsymbol{\alpha}} \doteq \int_K \boldsymbol{x}^{\boldsymbol{\alpha}} \mu(d\boldsymbol{x}) \end{aligned} \quad (3)$$

where $\boldsymbol{m}_{\boldsymbol{\alpha}}$,denotes the $\boldsymbol{\alpha}^{\text{th}}$ moment with respect to μ. Problem (3) is convex since the objective function is affine in $\boldsymbol{m}_{\boldsymbol{\alpha}}$, while the constraints are convex, albeit infinite dimensional. As shown in [16] a (convergent) sequence of finite dimensional convex relaxations with cost $p^d_{\boldsymbol{m}} \uparrow p^*$ can be obtained by replacing the constraints in (3) by semidefinite constraints of the form:

$$\begin{aligned} \boldsymbol{M}_d(\boldsymbol{m})_{i,j} &= \boldsymbol{m}_{\boldsymbol{\alpha}(i)+\boldsymbol{\alpha}(j)} \succeq 0, \; \forall i,j \leq S_d \\ \boldsymbol{L}_d(g_k \boldsymbol{m})(i,j) &= \sum_{\boldsymbol{\beta}} g_{k,\boldsymbol{\beta}} \boldsymbol{m}_{\boldsymbol{\beta}+\boldsymbol{\alpha}^{(i)}+\boldsymbol{\alpha}^{(j)}} \succeq 0 \\ & \qquad \forall i,j \leq S_{d-\lceil \frac{\deg(g_k(\boldsymbol{x}))}{2} \rceil} \end{aligned} \quad (4)$$

where $\boldsymbol{M}$ and $\boldsymbol{L}$, the truncated moment and localizing matrices, contain moments of order up to $2d$ and $S_d \doteq \binom{d+n}{n}$, leading to a semi-definite program of the form

$$p^d_{\boldsymbol{m}} \doteq \min_{\boldsymbol{m}} \sum_{\boldsymbol{\alpha}} p_{\boldsymbol{\alpha}} \boldsymbol{m}_{\boldsymbol{\alpha}} \text{ subject to (4)} \quad (5)$$

If for some d the solution to the problem above satisfies

$$\text{rank}[\mathbf{M}_d(\mathbf{m})] = \text{rank}\left(\mathbf{M}_{d-\max(\deg(g_k(\boldsymbol{x}))}\right) \quad (6)$$

then the relaxation is exact, that is $p^d_{\mathbf{m}} = p^*$.

Remark 1: In the case of QCQP of the form (1), the lowest order relaxation of (3) corresponds to $d = 1$, with objective and localizing matrices given by Trace$(\mathbf{Q}_o\mathbf{M}_1)$ and Trace$(\mathbf{Q}_k\mathbf{M}_1)$ respectively. If the solution to this relaxation satisfies rank$(\mathbf{M}_1)= 1$, it can be easily shown that it is indeed exact. We will exploit this property in Section III to obtain a computationally tractable algorithm to synthesize data driven controllers.

C. Stability of switched systems

Consider a switched discrete time linear system:

$$\boldsymbol{x}(t+1) = \boldsymbol{A}_i \boldsymbol{x}(t), i \in \{1, 2, ..n_s\} \quad (7)$$

As shown in Lemma 4.1 in [1], (7) is asymptotically stable under arbitrary switching if and only if there exists a full column rank matrix $\mathbf{V}$ and n_s matrices $\mathbf{H}_i$, $\|\mathbf{H}_i\|_\infty < 1$ such that

$$\mathbf{V}\mathbf{A}_i = \mathbf{H}_i \mathbf{V}, \; i = 1, \dots, n_s \quad (8)$$

In this case the function $\mathcal{V}(\boldsymbol{x}) \doteq \|\mathbf{V}\mathbf{x}\|_\infty$ is a polyhedral Lyapunov function for (7).

D. Statement of the Problem

Consider the setup shown in Figure 1, where each node represents an active subsystem. Our goal is to design a switched state feedback controller that stabilizes, under arbitrary switching, all possible plants compatible with the observed experimental input/output data and some minimal a-priori information about the system. Formally, this problem can be stated as:

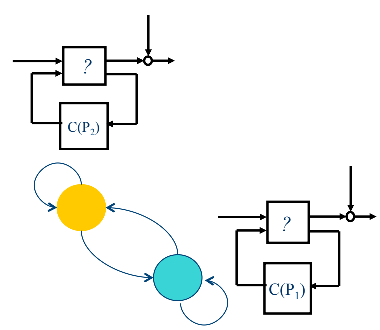

Fig. 1: Setup for Switched Data Driven Control Synthesis.

Problem 1: Consider a switched system composed of n_s LTI subsystems of the form:

$$\mathbf{x}_{k+1} \quad = \quad \mathbf{A}_i \mathbf{x}_k + \mathbf{B}_i \mathbf{u}_k + \mathbf{w}_k \quad (9)$$

where $\mathbf{x}_k \in \mathbb{R}^n$, $\mathbf{u}_k \in \mathbb{R}^m$, and $\mathbf{w}_k \in \mathbb{R}^n$, denote the state, input and process noise, and where i denotes the active sub-system at time k. Given experimental data $\{\mathbf{u}_k, \mathbf{x}_k, \mathbf{x}_{k+1}\}_{k=0}^T$, collected from an experiment where each subsystem is sufficiently excited, find a switched state feedback controller $\mathbf{F}_i$, $i = 1, \ldots, n_s$ such that $\mathbf{x} = 0$ is an asymptotically stable equilibrium point of the resulting closed loop system

$$\mathbf{x}_{k+1} = (\mathbf{A}_{\sigma_t} + \mathbf{B}_{\sigma_t}\mathbf{F}_{\sigma_t})\mathbf{x}_k \tag{10}$$

for any switching sequence $\sigma_t \in \{1, \ldots, n_s\}$ for all pairs $(\mathbf{A}_{\sigma_t}, \mathbf{B}_{\sigma_t})$ consistent with the experimental data.

In the next section, we will show that, for the case of ℓ^∞ bounded noise, the problem above can be recast as a polynomial optimization problem, which in turn can be relaxed to a semi-definite program.

III. Main Results

In this section we present the main result of the paper: a convex reformulation of Problem 1. Given a bound ϵ on the ℓ^∞ norm of the process noise (e.g. $\|\mathbf{w}\|_\infty \leq \epsilon$), define the consistency set $\mathcal{P}$ as the set of all pairs $(\mathbf{A}_j, \mathbf{B}_j)$ compatible with this bound and the experimental data. It can be easily seen that $\mathcal{P} = \cup \mathcal{P}^{(i)}$, where each of the $\mathcal{P}^{(i)}$ is a polytope of the form:

$$\mathcal{P}^{(i)} \doteq \{\mathbf{a}_j^{(i)}, \mathbf{b}_j^{(i)} \colon \| \left(\mathbf{x}_{t_1^{(i)}}^T \otimes \mathbf{I}\right) \mathbf{a}_j^{(i)} + \left(\mathbf{u}_{t_1^{(i)}}^T \otimes \mathbf{I}\right) \mathbf{b}_j^{(i)} - \mathbf{x}_{t_1^{(i)}+1} \|_\infty \leq \epsilon\} \tag{11}$$

where $(\mathbf{A}_j^{(i)}, \mathbf{B}_j^{(i)})$ denotes a generic pair in $\mathcal{P}^{(i)}$, $\mathbf{a}_j^{(i)} \doteq \mathbf{vec}(\mathbf{A}_j^{(i)})$, $\mathbf{b}_j^{(i)} \doteq \mathbf{vec}(\mathbf{B}_j^{(i)})$ and where $t_k^{(i)}$, $k = 1, \ldots, m_i$ denotes the times at which the i^{th} system is active. In this context, Problem 1 is equivalent to:

Problem 2: Find a full column rank matrix $\mathbf{V}$, and n_s matrices $\mathbf{F}_i$ such that, for all pairs $(\mathbf{A}_j^{(i)}, \mathbf{B}_j^{(i)}) \in \mathcal{P}^{(i)}$, there exist a matrix $\mathbf{H}_j^{(i)}$ with $\|\mathbf{H}_j^{(i)}\|_\infty < 1$, such that

$$\mathbf{V}\left[\mathbf{A}_j^{(i)} + \mathbf{B}_j^{(i)}\mathbf{F}_i\right] = \mathbf{H}_j^{(i)}\mathbf{V} \tag{12}$$

for $i = 1, \ldots, n_s$.
Note that in principle, the number of rows of the matrix $\mathbf{V}$ is not bounded a priori. This fact, combined with the bilinear dependence of (12) renders Problem 2 extremely challenging. Thus, in the sequel, we will consider a relaxation where we seek solutions where the matrix $\mathbf{V} \in \mathbb{R}^{n\times n}$. This allows for recasting Problem 2 into the following robust optimization form:

Problem 3: Find a full rank matrix $\mathbf{V} \in \mathbb{R}^{n\times n}$, a switched feedback gain $\mathbf{F}_i$ and matrices $\mathbf{H}_j^{(i)}$ such that

$$\mathbf{V}(\mathbf{A_j^{(i)}} + \mathbf{B_j^{(i)}}\mathbf{F_i}) = \mathbf{H}_j^{(i)}\mathbf{V} \text{ and } \|\mathbf{H}_j^{(i)}\|_\infty \leq d < 1 \tag{13}$$

for all pairs $(\mathbf{A}_j^{(i)}, \mathbf{B}_j^{(i)}) \in \mathcal{P}^{(i)}, i = 1, \ldots, n_s$.

While in principle this provides only sufficient conditions for the existence of a switched gain $\mathbf{F}_i$ that solves the DDC problem, this relaxation can be reformulated as a polynomial optimization problem and solved using the techniques outlined in Section II.

Theorem 1: Denote by $t_k^{(i)}$, $k = 1, \ldots, m_i$ the time instants where the i^{th} sub-system is active. Let:

$$\mathcal{X}^{(i)} \doteq \begin{bmatrix} \mathbf{x}_{t_1^{(i)}}^T \otimes \mathbf{I} \\ \vdots \\ \mathbf{x}_{t_{m_i}^{(i)}}^T \otimes \mathbf{I} \end{bmatrix}, \mathcal{U}^{(i)} \doteq \begin{bmatrix} \mathbf{u}_{t_1^{(i)}}^T \otimes \mathbf{I} \\ \vdots \\ \mathbf{u}_{t_{m_i}^{(i)}}^T \otimes \mathbf{I} \end{bmatrix}, \boldsymbol{\xi}^{(i)} \doteq \begin{bmatrix} \mathbf{x}_{t_k^{(i)}+1} \\ \vdots \\ \mathbf{x}_{t_{m_i}^{(i)}+1} \end{bmatrix}$$

Given a (full rank) matrix $\mathbf{V} \in \mathbb{R}^{n\times n}$, a matrix $\mathbf{S} \in \mathbb{R}^{r\times n^2}$ and a non-negative vector $\boldsymbol{\lambda} \in \mathbb{R}^r$ there exist switched feedback gains $\mathbf{F}_i$ and matrices $\mathbf{H}_j^{(i)}$ such that

$$\mathbf{V}(\mathbf{A_j^{(i)}} + \mathbf{B_j^{(i)}}\mathbf{F_i}) = \mathbf{H}_j^{(i)}\mathbf{V} \text{ and } \mathbf{S}\text{vec}(\mathbf{H}_j^{(i)}) \leq \boldsymbol{\lambda} \tag{14}$$

for all pairs $(\mathbf{A}_j^{(i)}, \mathbf{B}_j^{(i)}) \in \mathcal{P}^{(i)}$, $i = 1, \ldots, n_s$ if and only if there exist n_s matrices $\mathbf{Y}^{(i)} \in \mathbb{R}^{r\times 2nm_i}$, $\mathbf{Y}^{(i)} \geq 0$ and $\mathcal{F}_i \in \mathbb{R}^{m\times n}$ such that

$$\begin{aligned} &\mathbf{Y}^{(i)} \begin{bmatrix} \mathcal{X}^{(i)} & \mathcal{U}^{(i)} \\ -\mathcal{X}^{(i)} & -\mathcal{U}^{(i)} \end{bmatrix} = \begin{bmatrix}(\mathbf{S}(\mathbf{V}^{-T} \otimes \mathbf{V}) & \mathbf{S}(\mathcal{F}_i^T \otimes \mathbf{V})\end{bmatrix} \\ &\mathbf{Y}^{(i)} \begin{bmatrix} \boldsymbol{\xi}^{(i)} + \epsilon\mathbf{1} \\ -\boldsymbol{\xi}^{(i)} + \epsilon\mathbf{1} \end{bmatrix} \leq \boldsymbol{\lambda} \end{aligned} \tag{15}$$

Proof: (Only a sketch given due to space constraints). Let $\mathbf{F}_i \doteq \mathcal{F}_i\mathbf{V}$ and $\mathbf{H} \doteq \mathbf{VAV}^{-1} + \mathbf{VB}\mathcal{F}_i$. The proof follows by noting that every pair $(\mathbf{A}, \mathbf{B}) \in \mathcal{P}^{(i)}$ satisfies (14) if and only if the polytope:

$$\mathcal{P}_H \doteq \left\{(\mathbf{A}, \mathbf{B}) \colon \mathbf{S}\text{vec}(\mathbf{VAV}^{-1} + \mathbf{VB}\mathcal{F}_i) \leq \boldsymbol{\lambda}\right\}$$

satisfies $\mathcal{P}^{(i)} \subseteq \mathcal{P}_H$ and using the extended Farkas' Lemma [12] to enforce this condition.

■

Corollary 1: Problem 3 is equivalent to the following polynomial feasibility problem: Find n_s matrices $\mathbf{Y}_i \in \mathbb{R}^{n\times 2nm_i} \geq 0, \mathcal{F}_i \in \mathbb{R}^{n\times m}$, a full rank matrix $\mathbf{V} \in \mathbb{R}^{n\times n}$ and a non-negative vector $\boldsymbol{\lambda}$ with elements $\lambda_i \leq d < 1$ such that (15) holds for all matrices $\mathbf{S} \in \mathbb{R}^{n\times n^2}$ of the form:

$$\mathbf{S} = \begin{bmatrix}\text{diag}(\mathbf{s}_1) & \text{diag}(\mathbf{s}_2) & \ldots\text{diag}(\mathbf{s}_n)\end{bmatrix} \tag{16}$$

where $\mathbf{s}_i \in \mathbb{R}^n$ is a vector with elements $s_{i,j} = \pm 1$.

Remark 2: Note that by defining $\mathbf{Z} = \mathbf{V}^{-1}$ and imposing the additional constraint $\mathbf{VZ} = \mathbf{I}$, the problem above reduces to a (non-convex) quadratic program. In principle, this problem can be reduced to a sequence of SDPs or to a rank-constrained LMI using the techniques outlined in Section II-B (see Remark 1). However, this problem has a large number of constraints due to the need to consider 2^{n^2} matrices $\mathbf{S}$ with all possible sign vectors in $\mathbb{R}^{n^2}$. Thus, while the Corollary above is of theoretical interest, from a practical standpoint, its use is limited to relatively low order systems.

To address the computational complexity noted above, in the next result we introduce a relaxation of Problem 3, that, albeit potentially conservative, has substantially lower computational complexity. In addition, consistent numerical experience shows that this relaxation works well in situations were the noise level ϵ is small.

Corollary 2: Problem 3 is solvable if there exists a matrix $\mathbf{\Lambda} \in \mathbb{R}^{n \times n}$, $\mathbf{\Lambda} \geq 0$ and a scalar $d < 1$ such that $\mathbf{\Lambda 1} \leq d\mathbf{1}$ and the conditions in Theorem 1 hold with

$$\mathbf{S} = \begin{bmatrix} \mathbf{I}_{n^2 \times n^2} \\ -\mathbf{I}_{n^2 \times n^2} \end{bmatrix} \text{ and } \boldsymbol{\lambda} = \begin{bmatrix} \text{vec}(\mathbf{\Lambda}) \\ \text{vec}(\mathbf{\Lambda}) \end{bmatrix}$$

As before, the problem above reduces to a polynomial optimization problem. To obtain a computationally efficient algorithm, in this paper we will consider the first order relaxation, and impose the additional constraint $\text{rank}(\mathbf{M}) = 1$. Finally, using a (re-weighted) nuclear norm as surrogate for rank leads to the algorithm outlined in Algorithm 1.

Algorithm 1 Reweighted $\|.\|_*$ based DDC design

Initialize: $iter = 0, \mathbf{W}^{(0)} = I$, $d < 1$

repeat

Solve

$$\begin{aligned} \min_m \quad & \text{Trace}(\mathbf{W}^{(iter)}\mathbf{M}) \\ & \text{subject to:} \\ & \boldsymbol{\lambda} \geq 0 \\ & \mathbf{M}(m) \succeq 0 \\ & \mathbf{M}(1,1) = 1 \\ & \mathbf{VZ} = \mathbf{I} \\ & \mathbf{mat}(\boldsymbol{\lambda})\mathbf{1} \leq d\mathbf{1} \\ & \text{and, for all } i = 1, \ldots, n_s \\ & \mathbf{Y}^{(i)} \geq 0 \\ & \mathbf{Y}^{(i)} \begin{bmatrix} \mathcal{X}^{(i)} & \mathcal{U}^{(i)} \\ -\mathcal{X}^{(i)} & -\mathcal{U}^{(i)} \end{bmatrix} = \begin{bmatrix} \mathcal{K} & \mathcal{N}^{(i)} \\ -\mathcal{K} & -\mathcal{N}^{(i)} \end{bmatrix} \\ & \mathbf{Y}^{(i)} \begin{bmatrix} \boldsymbol{\xi}^{(i)} + \epsilon\mathbf{1} \\ -\boldsymbol{\xi}^{(i)} + \epsilon\mathbf{1} \end{bmatrix} \leq \boldsymbol{\lambda} \end{aligned}$$

where $\quad \mathcal{K} = \mathbf{Z}^T \otimes \mathbf{V}, \mathcal{N}^{(i)} = \mathcal{F}_i^T \otimes \mathbf{V}$

$\mathbf{M}$ represents the moment matrix given by:

$$\mathbf{M} = [1, \mathbf{vec}(\mathbf{V})^T, \mathbf{vec}(\mathbf{Z})^T, \mathbf{vec}(\mathcal{F}_i)^T]^T \times [1, \mathbf{vec}(\mathbf{V})^T, \mathbf{vec}(\mathbf{Z})^T, \mathbf{vec}(\mathcal{F}_i)^T]$$

Update

$$\mathbf{W}^{(iter+1)} = (\mathbf{M}^{(iter)} + \sigma_2(\mathbf{M}^{(iter)})I)^{-1}$$
$$iter = iter + 1$$

until $rank\{\mathbf{M}\} = 1$.

IV. Illustrative Example

In this section, we apply the proposed approach with a simple academic example and illustrate its advantages when compared against the approach proposed in [8]. Consider a MIMO LTI system that arbitrarily switches between two modes, one stable and the other unstable. The dynamics of the two sub-systems are given by:

$$\mathbf{A}_1 = \begin{bmatrix} 0.61 & 0.8 & 0 \\ -0.8 & 0.61 & 0 \\ 0 & 0 & 0.2 \end{bmatrix} \mathbf{B}_1 = \begin{bmatrix} -0.1503 & 0.4128 \\ -0.7616 & -0.5129 \\ -0.0099 & 0.5701 \end{bmatrix} \quad \text{(System 1)}$$

$$\mathbf{A}_2 = \begin{bmatrix} 0.2 & 0.4 & 0 \\ -0.4 & 0.2 & 0 \\ 0 & 0 & 0.6 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} -0.8518 & -0.5586 \\ -0.2122 & -0.9974 \\ -0.9932 & -0.6216 \end{bmatrix} \quad \text{(System 2)}$$

The experimental data was generated by applying a random input u, with $\|u\|_\infty \leq 1$, and the output was corrupted with ℓ^∞ bounded random noise. The noise bound for both systems is $\|\mathbf{w}\|_\infty \leq 0.2$. The corresponding trajectories are shown in Fig. 2.

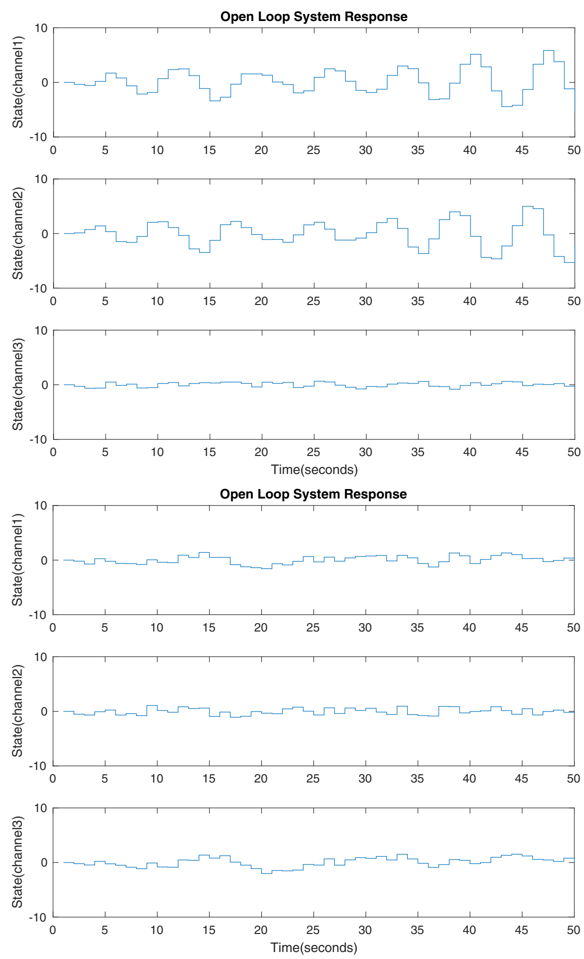

Fig. 2: (Simulated) experimental data. Top: System 1. Bottom: System 2.

For this example, the convex algorithm proposed in [8] fails to produce a stabilizing controller (the algorithm yields an optimal value $d > 1$, which certifies that the switched system cannot be superstabilized). On the other hand, Algorithm 1 led to a value $d = 0.99 < 1$, with the corresponding controller switching between the following gains:

$$\mathbf{F}_1 = \begin{bmatrix} -0.0769 & 0.9355 & 0.0736 \\ -0.4922 & -0.2585 & -0.1192 \end{bmatrix} \quad \text{(Gain 1)}$$

$$\mathbf{F}_2 = \begin{bmatrix} 0.1669 & 0.1345 & 0.4496 \\ -0.1915 & 0.0670 & -0.1117 \end{bmatrix} \quad \text{(Gain 2)}$$

with

$$\mathbf{V} = \begin{bmatrix} -0.3367 & -0.6297 & -1.0102 \\ -0.0512 & 1.1437 & -0.4968 \\ -0.8773 & -0.1640 & 0.1145 \end{bmatrix}$$

and

$$\mathbf{Z} = \begin{bmatrix} -0.0376 & -0.1806 & -1.1149 \\ -0.3354 & 0.7023 & 0.0877 \\ -0.7683 & -0.3776 & 0.3169 \end{bmatrix}$$

For reference purposes, we note that for the ground truth data we have $\|\mathbf{H}_1\|_\infty = \|\mathbf{V}(\mathbf{A}_1 + \mathbf{B}_1\mathbf{F}_1)\mathbf{Z}\|_\infty = 0.7873$ and $\|\mathbf{H}_2\|_\infty = \|\mathbf{V}(\mathbf{A}_2 + \mathbf{B}_2\mathbf{F}_2)\mathbf{Z}\|_\infty = 0.6146$. As expected, both of these values are smaller than d, which is the worst case value over all possible switching sequences and all plants in the consistency set.

The trajectories of each individual closed loop system starting from a random initial condition (with no input and without switching) are shown in Fig. 3. Similarly, Fig. 4 shows the trajectories corresponding to a random initial condition with $\|x_0\|_\infty \leq 10$, driven by the switching sequence $\sigma = [21122211211122222111]$. Note that, as expected, in all cases the states converge to zero exponentially, with convergence rate better than d^k.

Finally, we briefly illustrate the disturbance rejection properties of the resulting controllers. Assume that the closed loop system is affected by an ℓ^∞ bounded disturbance $\mathbf{w}$, that is:

$$\mathbf{x}_{k+1} = (\mathbf{A}_\sigma + \mathbf{B}_\sigma \mathbf{F}_\sigma)\mathbf{x}_k + \mathbf{w}_k$$

From (14) it can be shown that the set

$$\mathcal{S} \doteq \left\{ \mathbf{x} \colon \|\mathbf{V}\mathbf{x}\|_\infty \leq \mu \doteq \frac{\|\mathbf{V}\mathbf{w}\|_\infty}{1-d} \right\}$$

is positively invariant. Thus, any trajectory starting in $\mathcal{S}$ is uniformly bounded (over all possible switching sequences) by $\|\mathbf{x}\|_\infty \leq M$, where $M = \max \|\mathbf{x}\|_\infty$ subject to $\|\mathbf{V}\mathbf{x}\|_\infty \leq \mu$. This noise rejection property is illustrated in Fig. 5 showing the response of the switched closed loop system to a random disturbance $\mathbf{w}$ with components ± 1.

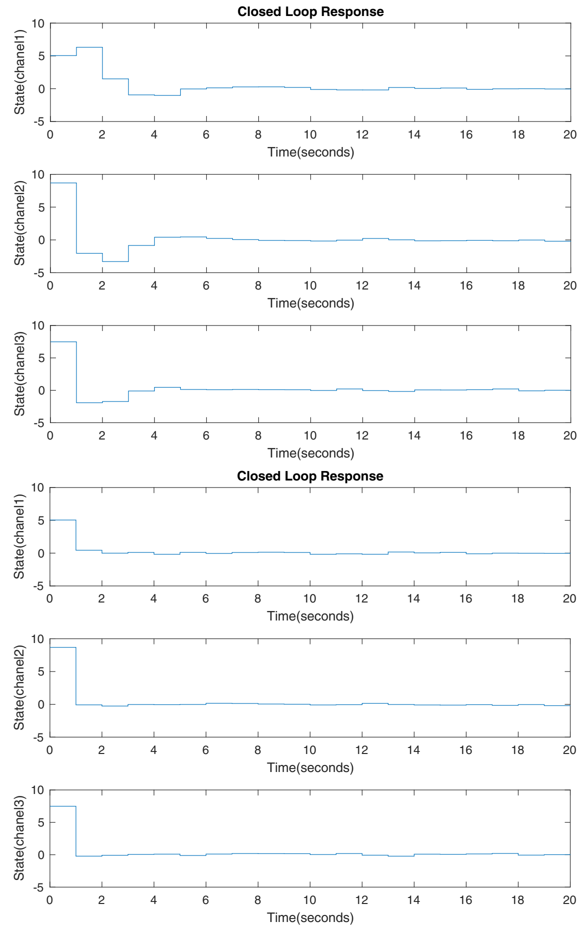

Fig. 3: Individual system's closed loop response to a random initial condition showing exponential convergence to 0 faster than d^k. Top: System 1. Bottom: System 2.

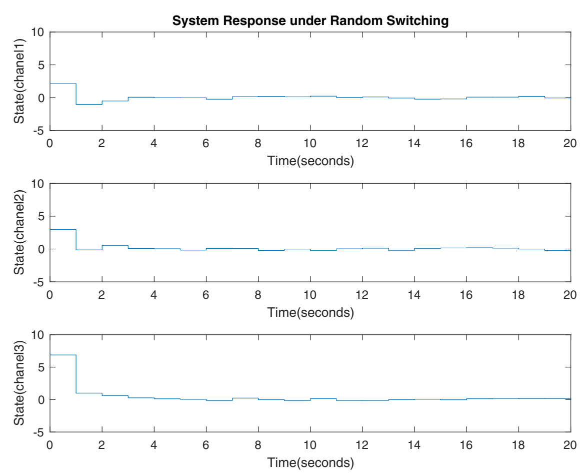

Fig. 4: State trajectories corresponding to a random initial condition and random switching. Again, as expected, convergence rate to 0 is faster than d^k.

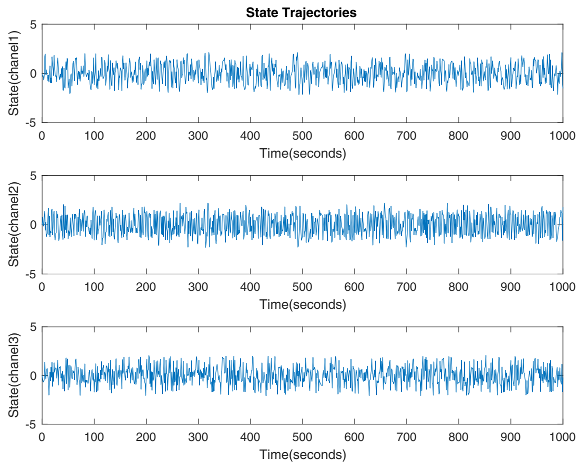

Fig. 5: State trajectories corresponding to a random ℓ^∞ bounded disturbance.

V. Conclusions

In this paper we presented a framework for synthesizing data driven switched state feedback controllers for switched discrete time systems. The key idea is to exploit necessary and sufficient conditions for stability, given in terms of the existence of a common polyhedral Lyapunov function. While in principle this leads to a very challenging non-convex optimization problem, the main result of the paper shows that, if this polyhedral function is limited to have at most $2n$ faces, then the problem can be reduced, via Farkas' Lemma, to a polynomial optimization. In turn, by exploiting tools from the theory of moments, this problem can be reduced to a rank-constrained SDP for which efficient convex relaxations are readily available. The resulting controller is guaranteed to exponentially stabilize (with convergence rate of at least d^k) all plants in the consistency set. When compared against the technique proposed in [8], the approach proposed here leads to less conservative results since it only enforces closed loop stability (rather than imposing super-stability, a much stronger, coordinate dependent concept). This was illustrated with a simple example, where the proposed approach led to a stabilizing controller while the approach in [8] failed to do so. On the other hand, this reduced conservatism comes at the price of a heavier computational burden. For instance, for the simple example in Section IV, Algorithm 1 took typically around 20 to 30 seconds, while the approach in [8] was able to certify infeasibility of superstabilization in only 2.2 seconds. Efforts are currently under way to reduce the computational complexity of Algorithm 1, by exploiting the chordal structure of the problem and developing custom made first order methods based on randomized linear algebra.

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