

# Improving Resiliency of Power Grids during Extreme Events

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**Abstract**—This paper presents results of an investigation concerning the resilience of power grids during extreme events such as storms or hurricanes. It considers availability of forecasts which will allow approximate estimation of paths where physical damage to equipment and facilities will be likely. Based on the identified lines which are most likely to be damaged in the next period, an optimal combination of load shedding and line switching actions are determined while taking into account the critical loads that need to be served with highest priority. Simulation results are presented using the IEEE 118-Bus system to illustrate the proposed strategy.

**Keywords**—Load Shedding, Mixed Integer Programming (MIP), Power Transfer Distribution Factor (PTDF), Resilient Power Grids.

## I. INTRODUCTION

Extreme events that are caused by natural disasters like hurricanes, thunderstorms, tornadoes etc. can cause havoc and lead to significant financial losses as well loss of lives. While it is almost impossible to prevent natural disasters, their impact may be controlled or minimized by taking timely and strategic actions. In this paper, power grids will be considered as one of the major infrastructures which not only are most impacted by such disasters but also have significant control tools to improve resilience under such conditions. Typical examples of physical damage experienced by power grids are those causing major generator and/or line outages.

The primary goal of the system operator under such extreme events is to prevent a partial or complete blackout possibly due to cascaded outages. Moreover, it is crucial that those "must serve" or critical loads are given the highest priority in order to maintain uninterrupted service to those customers. These may include public safety organizations such as fire and rescue services, ambulance and emergency medical services, offices of emergency services, airports, fire stations, elderly homes, as well as important communication facilities, cell towers, etc. Therefore, considerations cannot be strictly based on technical priorities dictated by the system topology and loading but they should also account for society's health care and emergency service requirements.

There is a rich literature on the studies conducted in the general area of resilient power grids. Several methods are presented for equipment failures, man-made attacks and natural disasters [1]–[5]. On the other hand, there are also a

large and equally rich volume of papers involving optimization methods based on topology control [6]–[10] which primarily aim to address issues in the operation and optimization of power markets. One recent study proposed a solution that combined load shedding and topology control algorithms to find the best preventive action for the worst case scenario [11]. While providing a very useful contribution, this approach does not address the issues related to moving event window, where one needs to continuously update the control actions to maintain resiliency under changing conditions imposed by the extreme event during its active time frame. Depending on the type of event, this period may extend from hours to days. The proposed approach in this paper considers such a dynamic scenario, where loads may be changing due to evacuations or congregations during the natural disaster and also assumes availability of feedback from emergency services in order to adjust load shedding priorities based on changing needs and conditions. Hence, the proposed strategy attempts to improve resiliency of the power grid by solution of an optimization problem that combines load shedding and topology control at desired intervals during the extreme event. Optimization problem uses the present load priority data as well as the amount of must-serve loads at various substations as inputs. In this work, it is assumed that such data and information will be received from health and emergency services as well as from mobile communication companies.

The first step of the proposed strategy involves finding the probable generator and line outages in the next forecast period as the extreme event continues to remain active. Next step is updating the system topology which includes bus, generator and branch data with respect to the expected line and generator outages. Third step is adding "virtual" generators, assigning them appropriate costs and specifying their operating limits followed by the solution of the base case power flow problem. Finally, optimal topology changes are determined [7] - [8] with the help of shift factors and power transfer distribution factors (PTDF). This last step provides a new topology for the power grid indicating the breakers to be switched on or off. Such intentional line switchings can be considered as preemptive actions to eliminate or minimize the need to shed load at one or more locations.

## II. PROBLEM FORMULATION

Consider a power system having  $n_B$  buses,  $n_L$  branches,  $n_G$  generators and  $n_D$  loads. During an extreme event or natural disaster it is assumed that a certain number of lines and generators will be lost, creating violations of operational limits on certain lines and transformers. The objective of this work is to determine the optimal generation dispatch and best load shedding strategy as well as line switching scenario that will yield the minimum disruption of service to customers without violating any operational limits. To achieve these objectives, the following optimization problem is formulated.

### A. Objective Function

The objective of the problem considered in this work is to maintain service to the largest number of customers while accounting for priorities which may be dictated by not only technical but also emergency services related concerns. Representation of load shedding is carried out by using "virtual" generators included next to every load that is allowed to be shed. There may be two possible cases: first, load connected to a bus without a generator; second, load connected to a bus where there is also a generator. A virtual generator will be connected to the bus in both cases where the production cost of generation will be assigned a much larger value compared to the cost of generation for the existing actual generators.

Let  $P_G$  represent the vector of all generators including both actual and virtual ones:

$$P_G^T = [P_{GA}^T \quad P_{GV}^T] \quad (1)$$

and the corresponding generation cost vector will then be given by:

$$C_G^T = [C_{GA}^T \quad C_{GV}^T] \quad (2)$$

where:

- $P_{GA}^T$  is the vector of actual generator production,
- $P_{GV}^T$  is the vector of virtual generator production,
- $C_{GA}^T$  is actual generation cost vector,
- $C_{GV}^T$  is virtual generation cost vector.

Objective function will thus take the form:

$$\min C_G^T P_G \quad (3)$$

where the costs are assigned such that:

$$\min(C_{GV}) \geq \max(C_{GA}) \quad (4)$$

Artificially assigning a set of linear monotonically increasing costs will provide a simple way to align the load shedding solution with the given or assumed ordered priority list of loads to be shed. Consider that there are  $n_D$  loads in a power system. Then, the cost of virtual generators can be assigned as follows.

$$\begin{bmatrix} C_{GV,1} \\ C_{GV,2} \\ C_{GV,3} \\ \vdots \\ C_{GV,n_D} \end{bmatrix} = \begin{bmatrix} C_{GV,1} \\ C_{GV,1} + k \\ C_{GV,1} + 2k \\ \vdots \\ C_{GV,1} + (n_D - 1)k \end{bmatrix} \quad (5)$$

where,  $k$  is a arbitrarily chosen positive number. This assignment will ensure that virtual generator 1 will be activated first and virtual generator  $n_D$  will be used last.

### B. Constraints

There will be three sets of constraints associated with this problem formulation. These will be described in detail below.

#### 1) Power Balance Equations:

$$\sum_{i=1}^{n_B} (P_G(i) - L(i)) - \sum_{j=1}^{n_L} I_j^2 R_j = 0 \quad (6)$$

where,

$n_B$  and  $n_L$  are the number of buses and branches respectively,  $I_j^2 R_j$  is the loss associated with branch  $j$ ,

$I_j$  is current on branch  $j$ ,

$R_j$  is the resistance of branch  $j$ ,

$L$  is the bus load vector.

The first order Taylor approximation is used to express line losses where all voltage magnitudes are set equal to 1 p.u. Thus,  $I_j$  is replaced by  $f_j$  above in Eq. 6 where  $f_j$  represents the power flow along branch  $j$ :

$$\sum_{i=1}^{n_B} (P_G(i) - L(i)) - \sum_{j=1}^{n_L} f_j^2 R_j = 0 \quad (7)$$

#### 2) Generator Limits:

$$0 \leq P_G \leq \bar{P} \quad (8)$$

$\bar{P}$  is the vector of power injection upper limits for both actual and virtual generators.

$$\bar{P}^T = [\bar{P}_{GA}^T \quad \bar{P}_{GV}^T] \quad (9)$$

Note that, while  $\bar{P}_{GA}^T$  is limited by generation capability of actual generators,  $\bar{P}_{GV}^T$  is limited by the difference between the total load amount and must-serve load amount connected at the same bus. If must-serve load amount is zero, then upper injected power limit of the virtual generator will be equal to total load amount.

#### 3) Switched Lines Modeled as Equivalent Injections:

In addition to load shedding and generator dispatch, a limited number ( $Switch_{max}$ ) of the lines in the system will be allowed to be switched out in order to minimize objective function further. To formulate line switching as an optimization variable, a binary vector "z" will be defined [8] where:

$$z(l) = \begin{cases} 0, & \text{if line } l \text{ is open,} \\ 1, & \text{otherwise.} \end{cases}$$

$$\sum_l (1 - z(l)) \leq Switch_{max} \quad (10)$$

Outage of a line can be modeled by an equivalent pair of power injections at terminal buses of the outaged line as derived in [12]. This derivation will be briefly reviewed here for the outage of a given line  $i - j$ . Consider the pre-outage line flow  $f_{po}$  on line  $i - j$ . It can be shown that [12]:

$$\Delta P_i = -\Delta P_j = \frac{f_{po}}{1 - PTDF_{ij}^{mn}} \quad (11)$$

where,

- $\Delta P_i$  is the change in net injection at bus  $i$ ,
- $\Delta P_j$  is the change in net injection at bus  $j$ .

$PTDF_{ij}^{mn}$  is the Power Transfer Distribution Factor (PTDF) of line  $i - j$  for a power transfer between buses  $m$  and  $n$ , and it can be calculated as:

$$PTDF_{ij}^{mn} = SF_{ij}^m - SF_{ij}^n \quad (12)$$

where, the shift factor ( $SF_{ij}^k$ ) represents the sensitivity of flow on branch  $i - j$  to the net power injection at bus  $k$ . Considering a system with  $n_B$  buses and  $n_L$  branches, the corresponding  $n_L \times (n_B-1)$  SF matrix can be formed as follows:

$$SF = \tilde{B}AB' \quad (13)$$

where,

- $A$  is  $n_L \times (n_B-1)$  branch incidence matrix,
- $B'$  is  $(n_B-1) \times (n_B-1)$  submatrix obtained by eliminating the slack bus row/column of the imaginary part of the bus admittance matrix.
- $\tilde{B}$  is  $n_L \times n_L$  primitive line admittance matrix.

The following constraint is used to ensure that the changes in terminal bus injections  $\Delta P$  for the closed lines ( $z(l) = 1$ ) will remain zero, and for opened lines they will assume appropriate non-zero values [8]:

$$-K(1-z) \leq \Delta P \leq K(1-z) \quad (14)$$

where,  $K$  is a large number.

It is assumed that a subset of lines are designated as switchable and that this information is available as input. In the case of line switching, the resulting changes in the flows of other lines can be found using the above defined sensitivities. These can be formulated as the following two inequality constraints for both switchable and monitored lines:

$$\begin{aligned} \underline{f}^M &\leq SF^M(P_G - L - D \cdot \sum_{j=1}^{n_L} f_j^2 R_j) + \\ &PTDF^{MS} \Delta P \leq \bar{f}^M \end{aligned} \quad (15)$$

$$\begin{aligned} \tilde{F}^S z &\leq SF^S(P_G - L - D \cdot \sum_{j=1}^{n_L} f_j^2 R_j) + \\ &(PTDF^{SS} - I) \Delta P \leq \tilde{F}^S z \end{aligned} \quad (16)$$

where,

$\underline{f}$  and  $\bar{f}$  are vectors of lower and upper limits of transmission line flows respectively,

$\tilde{F}$  and  $\bar{F}$  are diagonal matrices with  $\underline{f}$  and  $\bar{f}$  as their diagonal entries, respectively,

Superscripts  $S$  and  $M$  refer to the switchable and monitored lines respectively,

$D$  is normalized vector which distributes total loss to buses proportional to the bus loads as suggested in [13]:

$$D = \frac{L}{\sum_{i=1}^{n_B} L_i} \quad (17)$$

### C. Load Shedding by Mixed Integer Programming (MIP)

Using the objective function and constraints presented above in Sections II-A and II-B, the following optimization problem can be solved to determine the best generation dispatch, load shedding as well as line switching strategy within the given constraints:

$$\min_{P_G, \Delta P, z} C_G^T P_G \quad (18)$$

subject to:

$$\begin{aligned} \sum_{i=1}^{n_B} (P_G(i) - L(i)) - \sum_{j=1}^{n_L} f_j^2 R_j &= 0 \\ 0 \leq P_G &\leq \bar{P} \\ \underline{f}^M &\leq SF^M(P_G - L - D \cdot \sum_{j=1}^{n_L} f_j^2 R_j) + \\ &PTDF^{MS} \Delta P \leq \bar{f}^M \\ \tilde{F}^S z &\leq SF^S(P_G - L - D \cdot \sum_{j=1}^{n_L} f_j^2 R_j) + \\ &(PTDF^{SS} - I) \Delta P \leq \tilde{F}^S z \\ -K(1-z) &\leq \Delta P \leq K(1-z) \\ \sum_{l=1}^{n_L} (1 - z(l)) &\leq Switch_{max} \\ z(l) \in \{0, 1\} &\quad if \quad l \in S \\ z(l) = 1 &\quad if \quad l \in M \end{aligned}$$

## III. IMPLEMENTATION AND TESTING

This section contains the results of solving the above described optimization problem using the IEEE-118 Bus Test System in MATLAB environment.

### A. Implementation

Consider a scenario likely to occur in case of a hurricane, where several branches and generators may be taken out of service due to physical damage. Such a scenario can be simulated by assigning probability of outages to branches and generators and letting these probabilities be updated by an independent forecaster. This forecaster may be using not only weather forecasts but also expert knowledge about structural vulnerabilities of transmission towers, overhead lines,

substations etc. Those lines and generators whose outage probabilities exceed a set threshold will be assumed to be taken out in the next optimization cycle. The period of this optimization cycle will depend on the frequency of the outage probability updates received from the independent forecaster. In this paper, the updated outage probability is assumed to be available on an hourly basis. Therefore, the topology of the IEEE-118 Bus System, branch data and bus data are modified at each hour.

As explained in section II, virtual generators are added to the system. Moreover, their costs are determined using load shedding priority information which is expected to be provided and periodically updated by emergency, health care as well as communication network services which all require power in order to maintain their operations. Also, upper generation limits of virtual generators will be assigned using must-serve load data and load amounts. Next, the shift factors and PTDFs are calculated for the given topology. Note that, to be able to find new power flows each time the topology changes, shift factors and PTDFs need to be used. However, since shift factors and PTDFs would only provide incremental changes to power flows, a power flow solution needs to be found before using shift factors and PTDF matrices.

Finally, using the cost of generators, bus and branch data reflecting the topology, and the switchable line data, a Mixed Integer Programming (MIP) problem is solved. The result of MIP problem will yield the status (on/off) of each switchable line. Furthermore, it will also provide the net injected power at each of the network buses for the new topology. Hence, optimal load shedding amounts can be recovered from the solution obtained for the virtual generators. Moreover, load amounts will be updated based on the available hourly load data [14] in each optimization cycle. A flow chart of the overall implementation is shown in Fig. 1.

## B. Test Results

This section experimentally illustrates the benefits of using the proposed load shedding and line switching strategy. This is accomplished by comparatively solving the load shedding optimization with and without employing the MIP formulation described in this paper. In order to keep the scenario simple, tests are performed assuming that only a single line can be switched in each cycle. This assumption can be relaxed without loss of generality of the problem formulation.

The algorithm is executed assuming that load amounts are changing according to the given hourly load data. Also, it is assumed that the line/generator outages initiated by the extreme event occur hourly if their outage probabilities are higher than a pre-set threshold. Therefore, at each optimization cycle, a subset of lines may be taken out of service for the next hour. Probabilities of line/generator outages caused by the extreme event (in this case a hurricane) are assumed to depend on the direction and speed of the eye of the hurricane. The assumed hurricane direction is indicated by the red arrow in Fig. 2 [15], which also highlights those lines with high

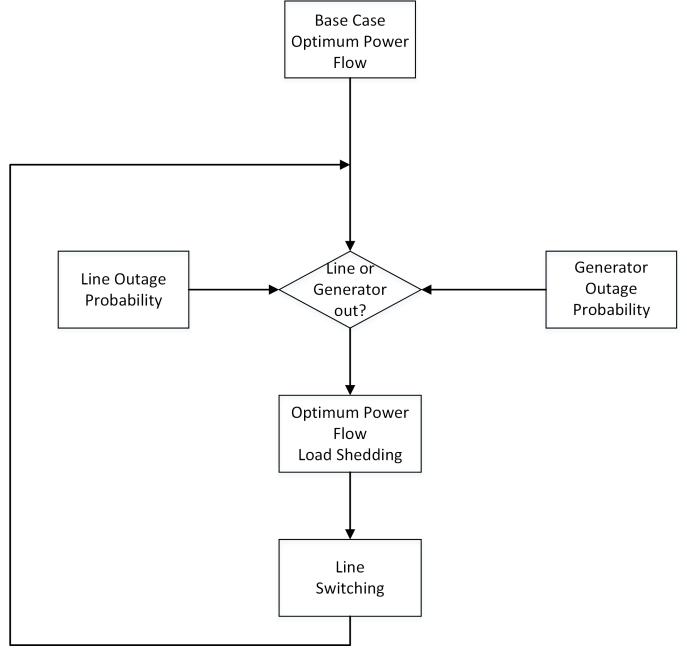


Fig. 1. Implementation Flowchart

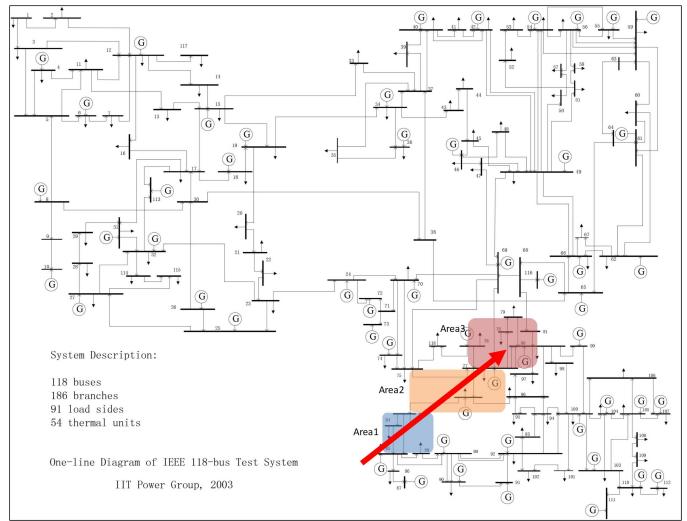


Fig. 2. Assumed trajectory of the hurricane in 118-Bus System

probability of outage. In this scenario, Area-1, Area-2 and Area-3 include 3, 4 and 9 such lines respectively.

Note that all loads are assumed to be ordered according to a load shedding priority list. The cost of virtual generators increase starting with the last one to the first one in this ordered priority List. The comparison of total load shedding amounts for the two cases, with and without employing strategic line switching is given in Fig. 3. It is evident from Fig. 3 that the load shedding amounts are smaller when line switching is employed along with optimal dispatch.

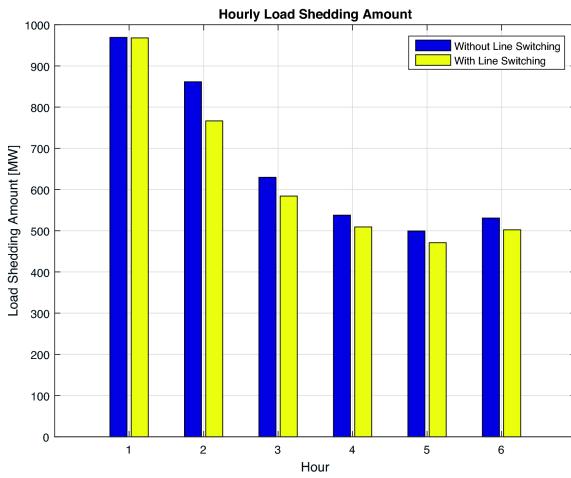


Fig. 3. Computed Load Shedding Amounts with/without Line Switching

#### IV. CONCLUSIONS

This paper considers an extreme natural event such as a hurricane and attempts to minimize its impact on the population by minimizing load shedding. Instead of considering the situation at a given point in time, it proposes a periodic set of optimization actions as the conditions change during the active period of the extreme event. An independent forecaster of probability of outages for physical structures such as transmission towers, substation equipment, etc. in the system is assumed to exist. Data and information received from such an entity will be used to take the optimal action for the next hour of the extreme event. Simulation results obtained using a hurricane scenario are provided to illustrate the potential utilization and benefits of the proposed approach.

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#### REFERENCES

- [1] C. Abbey, D. Cornforth, N. Hatziaargyriou, K. Hirose, A. Kwasinski, E. Kyriakides, G. Platt, L. Reyes, and S. Suryanarayanan, "Powering through the storm: Microgrids operation for more efficient disaster recovery," *IEEE power and energy magazine*, vol. 12, no. 3, pp. 67–76, 2014.
- [2] C. Chen, J. Wang, F. Qiu, and D. Zhao, "Resilient distribution system by microgrids formation after natural disasters," *IEEE Transactions on smart grid*, vol. 7, no. 2, pp. 958–966, 2016.
- [3] Z. Bie, Y. Lin, G. Li, and F. Li, "Battling the extreme: A study on the power system resilience," *Proceedings of the IEEE*, 2017.
- [4] B. Li, R. Roche, and A. Miraoui, "A temporal-spatial natural disaster model for power system resilience improvement using dg and lines hardening," in *PowerTech, 2017 IEEE Manchester*. IEEE, 2017, pp. 1–6.
- [5] E. Venizelos, D. Trakas, and N. Hatziaargyriou, "Distribution system resilience enhancement under disastrous conditions by splitting into self-sufficient microgrids," in *PowerTech, 2017 IEEE Manchester*. IEEE, 2017, pp. 1–6.
- [6] E. B. Fisher, R. P. O'Neill, and M. C. Ferris, "Optimal transmission switching," *IEEE Transactions on Power Systems*, vol. 23, no. 3, pp. 1346–1355, 2008.
- [7] E. A. Goldis, M. C. Caramanis, C. R. Philbrick, A. M. Rudkevich, and P. A. Ruiz, "Security-constrained mip formulation of topology control using loss-adjusted shift factors," in *System Sciences (HICSS), 2014 47th Hawaii International Conference on*. IEEE, 2014, pp. 2503–2509.
- [8] P. A. Ruiz, A. Rudkevich, M. C. Caramanis, E. Goldis, E. Ntakou, and C. R. Philbrick, "Reduced mip formulation for transmission topology control," in *Communication, Control, and Computing (Allerton), 2012 50th Annual Allerton Conference on*. IEEE, 2012, pp. 1073–1079.
- [9] P. A. Ruiz, J. M. Foster, A. Rudkevich, and M. C. Caramanis, "Tractable transmission topology control using sensitivity analysis," *IEEE Transactions on Power Systems*, vol. 27, no. 3, pp. 1550–1559, 2012.
- [10] P. A. Ruiz, E. Goldis, A. M. Rudkevich, M. C. Caramanis, C. R. Philbrick, and J. M. Foster, "Security-constrained transmission topology control milp formulation using sensitivity factors," *IEEE Transactions on Power Systems*, vol. 32, no. 2, pp. 1597–1605, 2017.
- [11] G. Huang, J. Wang, C. Chen, J. Qi, and C. Guo, "Integration of preventive and emergency responses for power grid resilience enhancement," *IEEE Transactions on Power Systems*, 2017.
- [12] A. J. Wood and B. F. Wollenberg, *Power generation, operation, and control*. John Wiley & Sons, 2012.
- [13] E. Litvinov, T. Zheng, G. Rosenwald, and P. Shamsollahi, "Marginal loss modeling in Imp calculation," *IEEE Transactions on Power Systems*, vol. 19, no. 2, pp. 880–888, 2004.
- [14] ERCOT. (2017) ERCOT hourly load data archive. [Online]. Available: [http://www.ercot.com/gridinfo/load/load\\_hist](http://www.ercot.com/gridinfo/load/load_hist)
- [15] I. I. of Technology: Electrical and C. E. Department. (2003) One line diagram of 118 bus test system. [Online]. Available: [http://motor.ece.iit.edu/data/IEEE118bus\\_inf](http://motor.ece.iit.edu/data/IEEE118bus_inf)