Multiconfiguration Pair-Density Functional Theory for Iron Porphyrin with CAS, RAS, and DMRG Active Spaces

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ABSTRACT: Porphyrins are present in many metalloproteins, and they are also important components of a variety of nonbiological functional materials. Furthermore, they are representative of the kind of large, strongly correlated system that is especially difficult for accurate calculations. For example, predicting the order of their spin states has been challenging. Here we study the energetic order of four states (one singlet, two triplets, and one quintet) of iron porphyrin, FeP, by the multiconfiguration pair-density functional theory (MC-PDFT). Five active space prescriptions, namely, CAS(8,6), CAS(8,11), CAS(16,15), RAS(34,2,2,13,6,16), and DMRG(34,35), are used to obtain the kinetic energy, density, and on-top density. Although the prediction of which spin state of FeP is the ground state depends on the selection of the active space when one uses multireference second-order perturbation theory and such calculations lead incorrectly to a quintet ground state with the largest studied active space, all five active spaces correctly lead to a triplet ground state when one uses MC-PDFT. We conclude that the (34,35) active space is large enough to give a qualitatively correct description of the orbital space and configuration space such that one obtains the correct spin state prediction when the external correlation energy is added accurately in a post-SCF step. We also conclude that MC-PDFT can provide an efficient and accurate approach to treat the electron correlation in large, strongly correlated systems with the complexity of iron porphyrin.

INTRODUCTION

Iron porphyrin is a prosthetic group in many metalloporphyrins, and it plays an important role in a large number of biological reactions.12 Fe(II) porphyrin complexes have been widely studied by both experiment and theory.11–26

Experiments, including magnetic,8 Mössbauer,3,7 proton NMR measurements,3,7 and Raman spectra,5 reveal that both tetraphenylporphathionor (FeTPP) and octaethylporphathion (FeOEP) molecules possess a triplet ground state. However, determining the correct ground state of even the smaller unsubstituted iron porphyrin (FeP) continues to be challenging.11

The FeP molecule has three low-lying spin states, singlet, triplet, and quintet. It has been reported that the ground state is triplet and that correct energetic ordering of the spin states can be obtained with single-reference CCSD(T)12 and Kohn–Sham density functional theory,24 although in general one prefers multireference methods for reliable calculations of transition metal spin splittings.11 The reference wave function for many multireference calculations is complete-active-space self-consistent field theory (CASSCF),35 in which the orbitals are optimized for a configuration interaction wave function containing all possible configuration state functions (CSFs) in a chosen active space. For accurate work, external correlation energy, i.e., correlation energy represented by excitations out of the active space, must also be included. However, it has been found when adding external correlation energy by second-order perturbation theory that the energetic order of the low-lying spin states of FeP depends on the choice of active space with the CASPT2 method. One possible conventional active space would be 8 active electrons in 11 active orbitals, including five valence d orbitals (“3d orbitals”) and five correlating d orbitals (“3d orbitals to describe the double shell35 effect”) on the iron atom and one Fe−N σ bonding orbital. Unfortunately, CASPT2 calculations with this choice of active space predict the quintet state to be the ground state by a large margin (0.71 eV).11

Such an active space could be enlarged by adding π orbitals of the porphyrin ring, but this leads to 34 active electrons on 35 active orbitals, which puts it beyond the feasibility limit of...
conventional CASSCF and CASPT2 calculations. Restricted active-space self-consistent field theory (RASSCF)\textsuperscript{38} is an alternative way to treat larger active spaces than are practical with CASSCF calculations. By dividing the active space into RAS1, RAS2, and RAS3 subspaces and restricting the occupation patterns in each subspace, the computational costs can be greatly reduced and larger active spaces become affordable.

Multiconfiguration pair-density functional theory (MC-PDFT)\textsuperscript{39,40} is an efficient multiconfiguration method that includes both internal and external correlation energy without calculating them separately. In MC-PDFT, the energy is evaluated using the kinetic energy, the density, and the on-top density of a multiconfiguration wave function. MC-PDFT has been applied to varieties of molecular systems and usually shows high accuracy.\textsuperscript{41–44} Recently, we have shown\textsuperscript{45} that MC-PDFT may be combined with very large MCSCF wave functions obtained by the density matrix renormalization group (DMRG), which allows for much larger active spaces than conventional CASSCF and RASSCF algorithms. The combinations of CASSCF, RASSCF, and DMRG with PDFT will be, respectively, called CAS-PDFT, RASS-PDFT, and DMRG-PDFT. We note that another approach to combining the advantages of DMRG and DFT is the work of Hedegård et al.,\textsuperscript{32} who combined single-configuration DFT for short interelectronic separations with complete-active-space DMRG for long interelectronic separations.

Here we present a study of FeP with CAS-PDFT, RASS-PDFT, and DMRG-PDFT to show that DMRG-PDFT calculations allow the use of large active spaces and to see the computational performance of MC-PDFT with different sizes of active spaces. In particular, we study the performance of various methods with respect to whether they get the state ordering correct.

## COMPUTATIONAL DETAILS

The FeP molecule has $D_{4h}$ symmetry, and we study the $^1A_{1g}$, $^3A_{2g}$, $^3E_g$, and $^5A_{1g}$ states with $D_{2h}$ symmetry enforced. The states are identified in Table 1 by the occupations of the iron 3d orbitals. The available experiments show that the ground state is a triplet,\textsuperscript{3–5} but they are not definitive as to whether it is $^3A_{2g}$ or $^3E_g$.

### Table 1. Electron Configurations of the Spin States of FeP

<table>
<thead>
<tr>
<th>state</th>
<th>configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1A_{1g}$</td>
<td>$(d_{x^2-y^2})^0(d_{xy})^6(d_{xz},d_{yz})^4$</td>
</tr>
<tr>
<td>$^3A_{2g}$</td>
<td>$(d_{x^2-y^2})^2(d_{xy})^4(d_{xz},d_{yz})^2$</td>
</tr>
<tr>
<td>$^3E_g$</td>
<td>$(d_{x^2-y^2})^0(d_{xy})^2(d_{xz},d_{yz})^2(d_{dz^2})^2$</td>
</tr>
<tr>
<td>$^5A_{1g}$</td>
<td>$(d_{x^2-y^2})^0(d_{xy})^2(d_{xz},d_{yz})^2(d_{dz^2})^4$</td>
</tr>
</tbody>
</table>

We use the ANO-RCC basis\textsuperscript{53,54} contracted to [7s6p5f3d2g1h] for Fe, to [4s3p2d1f] for C and N, and to [3s1p] for H. (This is the same basis set as used in refs 11 and 21.) We use the second-order Douglas–Kroll–Hess (DKH)\textsuperscript{55} approximation to include scalar relativistic effects. Geometries for each of the states are taken from ref 21. Cholesky decomposition is used to decompose two-electron integrals with a threshold of $10^{-6}$.

All of the calculations are performed with the OpenMolcas 8.3 software package\textsuperscript{56,57} with our previously reported implementation\textsuperscript{45} of DMRG-PDFT. The DMRG calculations are performed with the QCMAquis software suite\textsuperscript{29–61} in OpenMolcas 8.3. The calculations are steered by the settings and warm-up procedures of the QCMAquis program.\textsuperscript{62}

### RESULTS AND DISCUSSION

The active spaces considered, namely, CAS(8, 6), CAS(8, 11), CAS(16, 15), RAS(34, 22; 13, 6, 16), and DMRG(34, 35), are listed in the first column of Table 2. The smallest active space (8, 6) is obtained by removing the five 3d\textsuperscript{2} orbitals of the (8, 11) active space (which is explained above). The active space with 16 electrons on 15 orbitals is an extension of the (8, 11) space by adding all subvalence 3s and 3p orbitals of the iron atom. The (34, 35) active orbital space used in the RAS(34, 22; 13, 6, 16) and DMRG(34, 35) calculations is explained above.

The RAS(34, 22; 13, 6, 16) active space has the same active orbital space as a CAS(34, 35) active space, but it differs in which configuration state functions are included. The configurations included are determined by the partition into RAS1, RAS2, and RAS3 subspaces. The selection of RAS1, RAS2, and RAS3 subspaces made here is the same as in ref 11. Following the notation in ref 11, RAS($n_1,m_1,j_1,k_1$) active space contains $n$ active electrons and allows a maximum number of $l$ holes in RAS1 and a maximum of $m$ particles in RAS3. The numbers of orbitals in RAS1, RAS2, and RAS3 are $i$, $j$, and $k$, respectively. Note that the 3s and 3p orbitals of the iron atom are not included in the RAS(34, 22; 13, 6, 16) or DMRG(34, 35) calculations. The RASSCF calculations involved 849 651, 1 525 365, 1 543 829, and 876 568 CSFs for the $^1A_{1g}$, $^3A_{2g}$, $^3E_g$, and $^5A_{1g}$ states in $D_{2h}$ symmetry.

The (34, 35) active orbital space is also studied with the DMRG method. The DMRG wave function is expanded in matrix product states (MPS),\textsuperscript{63} and the expansion is limited by the bond dimension $M$. The exact CASSCF energy for a given active space would be reached if DMRG were applied with a large enough (i.e., converged) bond dimension, and we therefore tested three values of $M$ to show convergence. The configuration interaction dynamically extended active space (CI-DEAS) protocol\textsuperscript{64–65} is used to produce the initial MPS for the DMRG calculation.

The active orbitals of the four states are similar for each active space, and they are illustrated in the Supporting Information by giving the active orbitals of the $^3A_{2g}$ state from the RAS(34, 35) calculation and those of $^5A_{1g}$ state from CAS(8, 11) calculation. The Supporting Information also gives two sample input files, including geometries and guess orbitals.

MC-PDFT calculations require the choice of an on-top density functional, and we present results for two choices: TPBE\textsuperscript{39} and HTPBE.\textsuperscript{42}

Table 2 gives the calculated energies for the four considered states with various methods that we have explained, with both on-top density functionals and with three values of bond dimension in the case of the DMRG calculations. Table 2 also compares the MC-PDFT results to calculations with multireference perturbation theory.

RASPT2 calculations with the (34,35) active space, like the CASPT2 calculations with an (8,11) active space, incorrectly predict a quintet ground state.\textsuperscript{11} The only CASPT2 or RASPT2 calculations that predict a triplet ground state are those performed with an active space of 16 electrons on 15 orbitals.\textsuperscript{11}

Although the small (8,6) active space contains no double-shell effect, CAS-PDFT calculations with the (8,6) active space
Table 2. Relative Energies (in eV) of the Four States of FeP\textsuperscript{a,b}

<table>
<thead>
<tr>
<th>active space</th>
<th>method</th>
<th>(1^3\text{A}_\text{g})</th>
<th>(2^3\text{A}_\text{g})</th>
<th>(3^3\text{E}_\text{g})</th>
<th>(1^1\text{A}_\text{g})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8, 6)</td>
<td>CASSCF</td>
<td>2.41</td>
<td>1.18</td>
<td>1.21</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>CAS-PDFT-tPBE</td>
<td>1.25</td>
<td>0.00</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>CAS-PDFT-fPBE</td>
<td>0.99</td>
<td>0.03</td>
<td>0.00</td>
<td>0.36</td>
</tr>
<tr>
<td>(8, 11)</td>
<td>CASSCF</td>
<td>2.05</td>
<td>0.71</td>
<td>0.80</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>CAS-PDFT-tPBE</td>
<td>1.73</td>
<td>0.21</td>
<td>0.30</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>CAS-PDFT-fPBE</td>
<td>1.03</td>
<td>0.00</td>
<td>0.05</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>CAS-PDFT-hPBE</td>
<td>0.82</td>
<td>0.00</td>
<td>0.01</td>
<td>0.62</td>
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<tr>
<td>(16, 15)</td>
<td>CASSCF</td>
<td>1.56</td>
<td>0.54</td>
<td>0.64</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>CASPT2</td>
<td>1.30</td>
<td>0.00</td>
<td>0.10</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>CAS-PDFT-tPBE</td>
<td>1.26</td>
<td>0.00</td>
<td>0.03</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>CAS-PDFT-fPBE</td>
<td>1.17</td>
<td>0.00</td>
<td>0.00</td>
<td>0.56</td>
</tr>
<tr>
<td>(34,2,2; 13,6,16)\textsuperscript{c}</td>
<td>RASSCF</td>
<td>2.18</td>
<td>0.87</td>
<td>0.93</td>
<td>0.00</td>
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<td></td>
<td>RASPT2</td>
<td>1.50</td>
<td>0.20</td>
<td>0.29</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>RAS-PDFT-tPBE</td>
<td>1.31</td>
<td>0.00</td>
<td>0.08</td>
<td>0.37</td>
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<tr>
<td></td>
<td>RAS-PDFT-hPBE</td>
<td>1.12</td>
<td>0.00</td>
<td>0.04</td>
<td>0.63</td>
</tr>
<tr>
<td>(34,35)</td>
<td>DMRG(M = 100)</td>
<td>1.99</td>
<td>0.83</td>
<td>0.95</td>
<td>0.00</td>
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<tr>
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<td>DMRG(M = 200)</td>
<td>1.91</td>
<td>0.80</td>
<td>0.90</td>
<td>0.00</td>
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<tr>
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<td>DMRG(M = 300)</td>
<td>1.92</td>
<td>0.78</td>
<td>0.87</td>
<td>0.00</td>
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<tr>
<td></td>
<td>DMRG-PDFT-tPBE (M = 100)</td>
<td>1.22</td>
<td>0.00</td>
<td>0.01</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>DMRG-PDFT-tPBE (M = 200)</td>
<td>1.26</td>
<td>0.00</td>
<td>0.02</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>DMRG-PDFT-tPBE (M = 300)</td>
<td>1.22</td>
<td>0.00</td>
<td>0.02</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>DMRG-PDFT-hPBE (M = 100)</td>
<td>1.04</td>
<td>0.03</td>
<td>0.00</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>DMRG-PDFT-hPBE (M = 200)</td>
<td>1.07</td>
<td>0.02</td>
<td>0.00</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>DMRG-PDFT-hPBE (M = 300)</td>
<td>1.04</td>
<td>0.02</td>
<td>0.00</td>
<td>0.70</td>
</tr>
</tbody>
</table>

\textsuperscript{a}All energies are relative to energy of the lowest energy state (which is in bold font) for that line of the table. \textsuperscript{b}The CASPT2 and RASPT2 results are taken from ref 11. \textsuperscript{c}The (34,2,2; 13,6,16) notation is explained in the text; note that this is a restricted active space with the same number of electrons (34) and orbitals (35) as the CAS (34,35) case, but the intersubspace excitations are restricted to a maximum of two holes in RAS1 and a maximum of two particles in RAS3.

Table 3. DMRG-SCF, DMRG-cu(4)-CASPT2, and DMRG-PDFT Energy (in kcal/mol) for the Four States of FeP with Various Values of M When Using the (8,11) Active Space\textsuperscript{a}

<table>
<thead>
<tr>
<th>M</th>
<th>(1^3\text{A}_\text{g})</th>
<th>(1^3\text{A}_\text{g})</th>
<th>(1^3\text{E}_\text{g})</th>
<th>(3^3\text{A}_\text{g})</th>
<th>(1^3\text{A}_\text{g})</th>
<th>(1^3\text{A}_\text{g})</th>
<th>(3^3\text{E}_\text{g})</th>
<th>(3^3\text{E}_\text{g})</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>48.9</td>
<td>14.8</td>
<td>42.1</td>
<td>17.3</td>
<td>-9.0</td>
<td>6.3</td>
<td>20.1</td>
<td>-6.7</td>
</tr>
<tr>
<td>20</td>
<td>47.5</td>
<td>15.3</td>
<td>41.2</td>
<td>16.8</td>
<td>-8.6</td>
<td>5.2</td>
<td>19.0</td>
<td>-7.2</td>
</tr>
<tr>
<td>30</td>
<td>47.3</td>
<td>15.4</td>
<td>40.0</td>
<td>16.5</td>
<td>-8.4</td>
<td>5.1</td>
<td>18.7</td>
<td>-7.2</td>
</tr>
<tr>
<td>40</td>
<td>47.3</td>
<td>15.4</td>
<td>39.8</td>
<td>16.5</td>
<td>-8.3</td>
<td>5.1</td>
<td>18.6</td>
<td>-7.1</td>
</tr>
<tr>
<td>50</td>
<td>47.3</td>
<td>15.4</td>
<td>39.8</td>
<td>16.5</td>
<td>-8.4</td>
<td>5.0</td>
<td>18.6</td>
<td>-7.2</td>
</tr>
<tr>
<td>100</td>
<td>47.2</td>
<td>15.4</td>
<td>39.8</td>
<td>16.4</td>
<td>-8.3</td>
<td>5.0</td>
<td>18.5</td>
<td>-7.1</td>
</tr>
<tr>
<td>400</td>
<td>47.2</td>
<td>15.4</td>
<td>39.8</td>
<td>16.4</td>
<td>-8.3</td>
<td>5.0</td>
<td>18.5</td>
<td>-7.1</td>
</tr>
<tr>
<td>1000</td>
<td>47.2</td>
<td>15.4</td>
<td>39.8</td>
<td>16.4</td>
<td>-8.3</td>
<td>5.0</td>
<td>18.5</td>
<td>-7.1</td>
</tr>
</tbody>
</table>

\textsuperscript{a}The energy for each state is compared with that of the \(1^3\text{A}_\text{g}\) state calculated with the same method and M. The tPBE functional is used for DMRG-PDFT. The DMRG-CASPT2 results are taken from ref 21.
as shown in Table 2. However, by using the same RAS space in MC-PDFT, the triplet is correctly predicted to be the ground state. The energy of the $^3\text{A}_g$ state is 0.37 and 0.63 eV lower than the $^3\text{E}_g$ one for the tPBE and tPBE functionals, respectively; these values are quite close to the gaps in (8, 11) and (16, 15) active space.

To further test the performance of MC-PDFT with the (34, 35) active orbital space, a DMRG-CASSCF wave function was calculated. This is the same active orbital space as used in the RASSCF calculations. Because the DMRG calculation is done in the CASSCF formalism, it removes the interspace restrictions of the RAS calculations. We used values of 100, 200, and 300 for bond dimension $M$ in this study, and we found that the results for $M = 100$ agree with those for $M = 200$ and 300 to within 0.01−0.04 eV, depending on the case considered.

In many cases, DMRG-CASPT2 are carried out with a large value for $M$ to ensure accuracy; for example, ref 21 used values up to $M = 1000$. Reference 21 also tested an approximation to DMRG-CASPT2 called DMRG-cu(4)-CASPT2. In this method, the 4-particle reduced density matrix is not computed directly but rather is estimated from the 3-particle reduced density matrix to reduce the cost scaling from $a^4$ to $a^3$, where $a$ is the number of active orbitals. In order to provide an indication of the relative convergence rates of PDFT and PT2 with respect to $M$, Table 3 compares our DMRG-PDFT energy for the studied four states of FeP with various values of $M$ in (8,11) active space to the DMRG-cu(4)-CASPT2$^{21}$ calculations of ref 21; the table also shows the DMRG-SCF results without external correlation. The table shows that the energy gaps between different spin states converge more rapidly for DMRG-PDFT than for DMRG-cu(4)-CASPT2; in particular, the DMRG-PDFT gaps are within 0.4−0.7 kcal/mol of the converged ones for $M = 10$, where the gaps for DMRG-SCF are unconverged by 0.9−1.7 kcal/mol and those for DMRG-cu(4)-CASPT2 are unconverged by 1.4−2.3 kcal/mol. The stability of the PDTF results in the 100−1000 range of $M$ is a further indication that the bond dimension used in this study is sufficiently large for FeP.

The DMRG-PDFT calculations show that all values used for $M$ in this study yield the triplet as the ground state. When tPBE is used, the $^3\text{A}_g$ state is the ground state, while the $^3\text{E}_g$ state is predicted as the ground state with the tPBE functional. Although different triplet states are obtained for the two types of functionals, the energies of $^3\text{A}_g$ and $^3\text{E}_g$ states are quite close (within 0.01−0.03 eV, depending on the case considered).

**CONCLUSIONS**

There are three main conclusions.

1. The use of DMRG-CASSCF allows a large complete active space for FeP. To perform a conventional CASSCF calculation with this active space, the number of CSFs would be $2.2 \times 10^{13}$, $5.2 \times 10^{10}$, and $5.7 \times 10^{10}$ for the singlet, triplet, and quintet, respectively (without using spatial symmetry).

2. Even with this large complete active space, external correlation energy is needed to get quantitative results. DMRG is a powerful method, but for large molecules it is not sufficient on its own. When DMRG is used to approximate CASSCF wave functions with larger active spaces than can be handled by conventional methods, the active space might be too large to perform DMRG-CASPT2$^{21,69}$ or DMRG-NEVPT2$^{59,67,68}$. Using DMRG in conjunction with PDFT is, however, a powerful way to compute energies that approximate the full correlation energy, not just the internal correlation contained in the active space.

3. MC-PDFT yields a triplet ground state for all five choices of active space: CAS(8,6), CAS(8,11), CAS(16,15), and RAS (34,2,2;13,6,16) and the CAS(34,35) active space used for DMRG. The quintet—triplet energy gap has only a small variation between the four largest active spaces, which demonstrates the stability of MC-PDFT. The CASPT2 and RASPT2 calculations for FeP depend on the active space, and they correctly predict a ground state of triplet for only one of the active spaces employed (16 active electrons on 15 active orbitals). Thus, we see that the DMRG-PDFT calculations are more accurate than CASPT2, which furthermore is not affordable for the largest active space.

The finding that MC-PDFT provides an efficient and accurate approach to treat electron correlation in the iron porphyrin systems is very encouraging for large transition metal complexes in general.

**REFERENCES**


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**ASSOCIATED CONTENT**

Supporting Information

The Supporting Information is available free of charge on the ACS Publications website at DOI: 10.1021/acs.jpca.8b12479.

Absolute energy in hartrees of MSCSF and MC-PDFT calculations; sample input files and active orbitals (including geometry and guess orbitals) (PDF)

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Notes

The authors declare no competing financial interest.

**ACKNOWLEDGMENTS**

This work was supported in part by the National Science Foundation under grant no. CHE-1464536.


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