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How to Stop Violence among Homeless: Extension of Voter Model and Intervention Strategies

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***Abstract*—Interventions to reduce violence among homeless youth are difficult to implement due to the complex nature of violence. However, a peer-based intervention approach would likely be a worthy approach as it has been shown that individuals who interact with more violent individuals are more likely to be violent, suggesting a contagious nature of violence. We propose Uncertain Voter Model to represent the complex process of diffusion of violence over a social network, that captures uncertainties in links and time over which the diffusion of violence takes place. Assuming this model, we define Violence Minimization problem where the task is to select a predefined number of individuals for intervention so that the expected number of violent individuals in the network is minimized over a given time-frame. We extend the problem to a probabilistic setting, where the success probability of converting an individual into non-violent is a function of the number of "units" of intervention performed on them. We provide algorithms for finding the optimal intervention strategies for both scenarios. We demonstrate that our algorithms perform significantly better than interventions based on popular centrality measures in terms of reducing violence.**

I. Introduction

Violence perpetuates violence and diffuses through a network like a contagious disease [1]. Cure Violence program[1] is based on a similar idea of treating violence as a contagious disease, and has shown significant reduction in violence. Motivated by the contagious nature, a diffusion model is ideal for modeling spread of violence. Doing so can lead to optimal intervention strategies under certain assumptions. To the best of our knowledge, intervention strategy to reduce violence using diffusion models has received very little attention in the literature [2], [3]. The existing works take a macroscopic approach, disregarding the network structure.

While many diffusion models exist that are variations of Independent Cascade Models, Linear Threshold Model, and Susceptible-infected, they are "progressive" models, i.e., they assume that once activated (or infected), the individuals remain activated. However, in the context of violence, it would mean that a violent person can never become non-violent, which is not applicable. Although some non-progressive extensions do exist, accurate analytical solutions of those models are hard to obtain. A popular model that captures non-progressive diffusion of competing behaviors on social networks is voter model [4]. In voter model individuals are influenced by a randomly selected neighbor[2]. But application of voter model in real-life scenarios such as diffusion of violence has the following drawbacks. (a) There is some uncertainty in the network structure, in the sense that, individuals may forget to mention someone as their peer, and yet be influenced by them [5]. (b) The number of discrete time steps over which the diffusion process unfolds (a parameter required by Voter Model) is often unknown in practice. To deal with these uncertainties, we propose Uncertain Voter Model (UVM) as an extension of Voter Model. Under UVM, we find the optimal intervention strategies to minimize violence. The task is to perform interventions on individuals with constrained "resources" so that they change their state from "violent" to "non-violent" resulting in others adopting "non-violent" state, eventually minimizing violence. We consider two types of interventions: (i) deterministic, where selecting an individual turns them into non-violent, with the constraint being the number of individuals to select; (ii) probabilistic, where an individual's probability of being non-violent is increased based on number of "units" (hours, sessions, etc.) of intervention, with the constraint being the total number of units available.

II. Model

To model the spread of violence we model the network of homeless youth as a graph $G(V, E)$ where every individual is a node which can exist in one of two states: 'violent' or 'non-violent'. We chose to model violence as a non-progressive diffusion process, i.e, a node may switch its state unlike the progressive diffusion where once a node is violent it cannot become non-violent again. Next, we provide a background on Voter Model [4] on which our model is based.

[1]See http://cureviolence.org/

[2]We use the terms "neighbor" and "neighborhood" to refer to the links of a given individual in the network and not their physical neighborhood

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A. Voter Model

In Voter Model [4], at every time step a node u picks an incoming neighbor v at random with a probability $p(v,u)$. The incoming probabilities are normalized such that $\sum_v p(v,u) = 1$. Let $x_{u,t}$ be the probability of node u being violent at time t. According to the model, $x_{u,t} = \sum_v p_{v,u} x_{v,t-1}$. Let $\mathbf{x_t}$ represent the state of all the nodes at time t, with ith element representing the probability that v_i is violent at time t. Suppose matrix M represents the transpose of the adjacency matrix of the weighted network, i.e., $M_{u,v} = p(v,u)$. Then $\mathbf{x_t} = M\mathbf{x_{t-1}}$. It follows that $\mathbf{x_t} = M^t \mathbf{x_0}$. Here $\mathbf{x_0}$ is the initial state of nodes.Define I_X for $X \subseteq V$ as the vector in which the i-th element is 1 if $v_i \in X$. Then the expected number of violent nodes at time t is given by $I_V^T \mathbf{x_t}$.

B. Uncertain Voter Model

A network formed through a survey may have missing edges due to the uncertainty in a person's ability to recall all "friends" they might be influenced by [5]. To capture this aspect, we propose the Uncertain Voter Model (UVM), where we assume that a node which is not directly connected to the node of interest may also influence it. In this model, two mutually exclusive events happen: (i) with probability θ a node randomly selects one incoming neighbor and adopts its state, (ii) with probability $(1-\theta)$ it selects a node that is not its neighbor in the network and adopts its state. We propose two ways of selecting the node form outside the neighborhood: (i) random and (ii) Katz-based.

1) Random: In this case every node which is not a neighbor is equally likely to be selected. Mathematically,

$$x_{u,t} = \theta \sum_{\{v|p(v,u)>0\}} p_{v,u} x_{v,t-1} + (1-\theta) \frac{\sum_{\{v|p(v,u)=0\}} x_{v,t-1}}{|\{v|p_{v,u}=0\}|} \tag{1}$$

If n is the total number of nodes and d_u is the number of incoming neighbors of u, then $|\{v|p_{v,u}=0\}| = n - d_u$. Suppose we define,

$$q_\theta(v,u) = \begin{cases} \theta p_{v,u} & \text{if } p_{v,u} > 0 \\ \frac{1-\theta}{n-d_u} & \text{if } p_{v,u} = 0\,. \end{cases} \tag{2}$$

2) Katz-bazed: We treat the influence from outside the neighborhood as the problem of finding missing edges. A popular method for missing edge detection is using Katz similarity [6], which is based on exponentially weighted number of paths between two nodes, i.e., $K(u,v) = \sum_i \alpha^i |\text{path of length } i \text{ to } u \text{ from } v|$. Since, we are only interested in nodes that are not directly in the neighborhood we take the above summation for $i \geq 2$. The entire similarity matrix is given by $K = \sum_{i\geq 2} \alpha^i M^i = \alpha^2 M^2 (I - \alpha M)^{-1}$,We choose a small value of $\alpha = 0.005$ [6]. We normalize the scores for each node u over all nodes v which are not in its neighborhood, so that the probability of selecting node v is proportional to $K(u,v)$, i.e., $K'(u,v) = K(u,v)/\sum_w K(u,w)$. Katz-based UVM is given by

$$x_{u,t} = \theta \sum_{\{v|p(v,u)>0\}} p_{v,u} x_{v,t-1} + (1-\theta) \sum_{\{v|p(v,u)=0\}} K'_{u,v} x_{v,t-1}\,. \tag{3}$$

Again, we can define

$$q_\theta(v,u) = \begin{cases} \theta p_{v,u} & \text{if } p_{v,u} > 0 \\ (1-\theta)K'(u,v) & \text{if } p_{v,u} = 0\,. \end{cases} \tag{4}$$

From Equations 2 and 4, both random and Katz-based UVM lead to reduction of Equations 1 and Equations 3 to

$$x_{u,t} = \sum_v q(v,u) x_{v,t-1} \text{ or } \mathbf{x_t} = Q_\theta \mathbf{x_{t-1}} \tag{5}$$

where $[Q_\theta]_{u,v} = q_\theta(u,v)$. Now, we define the problem of Violence Minimization as follows.

Problem Definition 1 (Violence Minimization): Given a weighted graph $G(V,E)$, an initial set of violent nodes S, a time frame t, and an integer k, find $T \subseteq S$ such that $|T| = k$, turning the nodes in T into non-violent minimizes the expected number of violent nodes after time t, i.e., $I_V^T \mathbf{x_t}$ under Uncertain Voter Model.

III. Greedy Minimization

Let $\mathbf{x'_0}$ be the vector formed by turning some k nodes into non-violent, resulting in the vector of probabilities $\mathbf{x'_t}$ at time t. Now, minimizing $I_V^T \mathbf{x'_t}$ is equivalent to maximizing $I_V^T(\mathbf{x_t} - \mathbf{x'_t}) = I_V^T Q_\theta^t (\mathbf{x_0} - \mathbf{x'_0})$, i.e., the problem reduces to maximizing

$$I_V^T \Delta\mathbf{x_t} = I_V^T Q_\theta^t \Delta\mathbf{x_0} = \sum_{\{u|\mathbf{\Delta x_0(u)=1}\}} I_V^T Q_\theta^t I_u \tag{6}$$

which can be optimized using greedy strategy [4] as presented in Algorithm 1.

Algorithm 1 Greedy algorithm to minimize violence

function MinViolence(G, S, θ, k, t)
  Compute Q_θ^t for G
  $\forall u \in S$ compute $\sigma(u) = I_V Q_\theta^t I_u$
  Sort $\{\sigma(u)\}$ in descending order and return top k.
end function

A. Uncertainty in Time

Uncertain Voter Model requires t as a parameter which is unknown in real life. While we may have a certain time period (days or weeks) over which we want the intervention to work, finding a relation between that time period and the parameter t is non-trivial as it depends on how often the individuals interact. To capture this uncertainty, we assume that time t takes a value τ with probability $P(t=\tau)$. Now, we wish to minimize $\mathbb{E}(I_V \mathbf{x_t})$ where the expectation is taken over t. Therefore,

$$\mathbb{E}(I_V^T \mathbf{x'_t}) = \sum_\tau P(t=\tau) I_V^T Q_\theta^\tau \mathbf{x'_0} = I_V^T \left(\sum_\tau P(t=\tau) Q_\theta^\tau \right) \mathbf{x'_0}. \tag{7}$$

Notice that a greedy solution like Algorithm 1 still applies.

B. Probabilistic Intervention

In the previous section, we assumed that performing intervention on a "violent" node turns it into "non-violent", i.e., an intervention is always successful. However, in real life this may not be true, and some nodes may require more "units" (hours, sessions, etc.) of intervention than others. Let $s_u(z_u)$ be the probability of success after applying z_u units of intervention to node u. These functions can be different for different nodes, as different individuals may respond differently to interventions. We assume that these functions $\{s_i\}$ are non-decreasing, i.e, adding more units of intervention cannot decrease the probability of success. We also assume that theses functions are concave, i.e., the marginal increase in probability reduces with increasing number of interventions. Such assumptions are similar to those made in immunization literature [7]. Mathematically, if $z' \geq z$, $s_i(z') \geq s_i(z)$, and $s_i(z'+1) - s_i(z') \leq s_i(z+1) - s_i(z), \forall i$. Rewriting Equation 6 for probabilistic intervention, the utility (reduction in violence) obtained by an allocation of $\{z_1, z_2, \ldots, z_n\}, z_i \in \mathbb{N} \cup \{0\}$ is

$$I_V^T Q_\theta^t \Delta \mathbf{x_t} = \sum_u I_V^T Q_\theta^t I_u s_u(z_u) \tag{8}$$

Let $f_u(z_u) = I_V^T Q_\theta^t I_u s_u(z_u)$. This leads to the probabilistic intervention version of Violence Minimization problem, which is equivalent to maximizing $\sum_u f_u(z_u)$, such that $\sum_u z_u = k$. Note that, $I_V^T Q_\theta^t I_u$ is a non-negative constant and $s_u(z_u)$ is non-decreasing concave function, and so, $f_u(z_u)$ is also non-decreasing and concave. Formally, we define this as follows.

Problem Definition 2 (Units Assignment Problem): Given $k \in \mathbb{Z}$ resources and n concave non-decreasing utility functions $f_i : \mathbb{Z} \to \mathbb{R}$, where $f_i(z_i)$ represents the utility of assigning z_i units to function f_i, maximize the total utility $F = \sum_i f_i(z_i)$ subject to $\sum_i z_i = k$.

Algorithm 2 Greedy Maximization using Marginal Returns

function GreedyMax$((f_1, f_2, \ldots, f_n), k)$
  for $i \leftarrow 1 : n$ **do**
    $z_i \leftarrow 0$
  end for
  for $j \leftarrow 1 : k$ **do**
    $idx \leftarrow \arg\max_i (f(z_i + 1) - f(z_i))$
    $z_{idx} \leftarrow z_{idx} + 1$
  end for
  return $(z_1, z_2, \ldots, z_n)$
end function

Lemma 1: For a non-decreasing concave function $f : \mathbb{Z} \to \mathbb{R}$, and $h \geq 1$,

$$f(x+h) - f(x) \leq h(f(x) - f(x-1)) \tag{9}$$
$$f(x) - f(x-h) \geq h(f(x) - f(x-1)) \tag{10}$$

Theorem 1: Algorithm 2 produces the optimal assignment for Units Assignment Problem.
The proof has been omitted for brevity.

IV. Experiments

We have shown that the greedy algorithms described in Algorithms 1 and 2 are optimal under Uncertain Voter Model for deterministic and probabilistic interventions, respectively. However, to study how prominent the difference is from other choices of intervention strategies, we compare it against the following baselines:

- Degree: We define the degree of a node based on the weighted graph as $d_v = \sum_u p_{v,u}$. Then we select top k nodes.
- Betweenness Centrality: Top k nodes are selected based on the betweenness centrality in the graph.

We have performed two sets of experiments:

a) Real-world Homeless Youth (HY) Network: We constructed the network obtained by our surveyed data [9], which consists of 366 nodes and 558 directed edges. Due to the lack of the knowledge of edge-weights, we assume that all incoming links for a node are equally weighted.

b) Synthetic Kronecker graphs: We generated random Kronecker graphs [8] with roughly same number of nodes and edges as the real HY network. The results on synthetic graphs were similar to those obtained on HY network, and have been omitted due to lack of space.

A. Homeless Youth Network

We performed selection and simulated intervention on the same graph, as the network that includes the "forgotten" links is not available. Out of the 366 nodes in the network, 55.01% were "violent" ($x_{u,0} = 1$) and 42.55% are "non-violent" ($x_{u,0} = 0$). Data on the rest of 2.44% are missing and are assumed to be equally likely to be of either state ($x_{u,0} = 0.5$). Based on this "initial state" we run Greedy Minimization for Uncertain Voter Model.

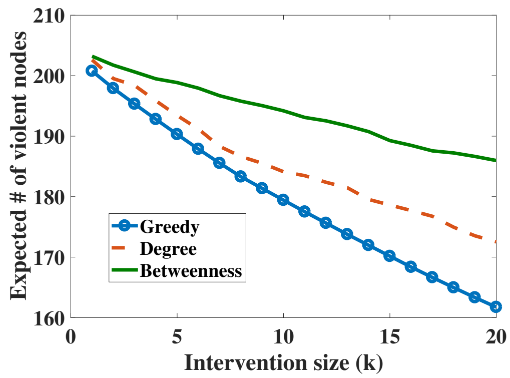
Figure 1(a) show the comparison for expected number of nodes that are violent after $t = 5$ and $t = 10$. Figure 1(b) shows the comparison for probabilistic intervention. The value of θ was set to 1 to generate these plots. Other values for parameters t and θ show similar trends and hence, have been omitted. We observe that the greedy algorithm significantly outperforms both baselines.

The upper limit of number of time steps (t) was chosen to be a small number in our experiments, keeping in mind that homeless youth networks are dynamic, and so in practice, the intervention should be performed in short-term.

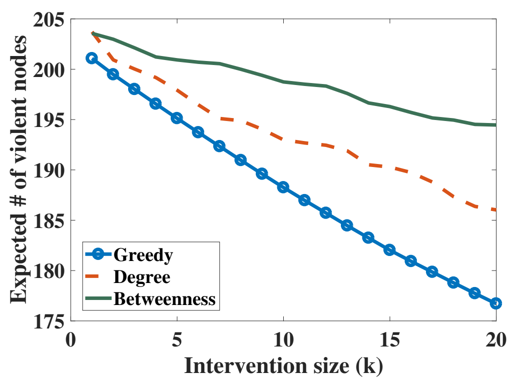
c) Choosing individuals in practice: So far we have presented the comparison of our greedy method against the baseline centrality measures in terms of reduction in violence. Now, we proceed to examine individuals chosen for intervention based on our method. We experimented with different values for parameter $\theta = 1, 0.9, 0.8, 0.7, 0.6$ and 0.5, i.e., increasing edge uncertainty. Table I presents the top 10 nodes (in terms of PID assigned in the survey) chosen for intervention (deterministic). Note that there are many nodes such as PIDs 47, 4, 2086, 2156, and 51, that consistently appear in the top 10, suggesting that the set of chosen individuals is not

TABLE I
TOP 10 SEEDS FOR VARIOUS VALUES OF θ OUTPUT BY GREEDY MINIMIZATION

θ	Selected Seeds										$\mathbb{E}(I_V^T \mathbf{x}'_t)$
1	47	4	2156	51	13	2086	169	2115	2099	2056	179.43
0.9	47	4	2156	2086	51	13	169	2115	2056	2099	183.327
0.8	47	4	2086	2156	51	13	169	2115	2056	89	185.86
0.7	47	4	2086	2156	51	2115	13	169	2056	2125	187.54
0.6	47	4	2086	2115	2156	51	169	13	2056	2125	188.66
0.5	47	4	2086	2115	2156	51	169	13	2056	2125	189.43



(a) Deterministic intervention



(b) Probabilistic intervention

Fig. 1. Comparison of the baseline against the greedy algorithm for varying intervention sizes under UVM.

highly sensitive to the choice of parameters within a sensible range. However, the significant deviation from betweenness and degree centralities (Figure 1(a)) suggests that finding this set is non-trivial. We also varied the value of $t = 2, 4, 6, 8, 10$, and 12. The lists of seeds obtained for different values of t have not been presented for brevity, as they had the same PIDs frequently occurring in the lists. These individuals were selected based on deterministic intervention, which should be applied when the knowledge of personal traits is not available. However, with the availability of personal traits sufficient to model how an individual may respond to intervention ($s_u(z_u)$), probabilistic intervention should be used.

V. CONCLUSIONS

We have proposed Uncertain Voter Model (UVM) to capture the non-progressive diffusion of violence. Under UVM, a node selects one of its neighbors with probability θ or one of the remaining nodes with probability $1 - \theta$, and adopts its state. The model captures uncertainty in network links and time over which the diffusion of violence takes place. We have shown that a greedy algorithm is the optimal intervention strategy to minimize violence under this model. We have extended this deterministic intervention by considering a scenario where the intervention succeeds only with a certain probability as a function of number of resources allocated to the individual. We have also shown that the greedy algorithm maximizing marginal returns forms the optimal intervention strategy. Experiments on synthetic Kronecker graphs suggest that UVM is a better choice than the classic Voter Model, where edges may have been omitted during data collection. Experiments on real-world Homeless Youth network have demonstrated that our intervention strategy significantly outperforms interventions based on popular centrality based measures. We show in our experiments that for sensible choices of parameters the top individuals selected for intervention roughly remain the same. We are in the process of performing more surveys and real-life intervention to verify our model and the approach.

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