Decay of turbulence in a duct with transverse magnetic field

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Abstract: Decay of honeycomb-generated turbulence in a duct with a static transverse magnetic field is studied via high-resolution direct numerical simulations. The simulations follow the experimental study [1], in particular the paradoxical observation of high-amplitude velocity fluctuations, which exist in the downstream portion of the flow when the strong transverse magnetic field is imposed in the entire duct including the honeycomb exit, but not in other configurations. It is shown that the fluctuations are caused by the large-scale quasi-two-dimensional structures forming in the flow at the initial stages of the decay and surviving the magnetic suppression. The study demonstrates that turbulence decay in the presence of a magnetic field is a complex phenomenon critically depending on the state of the flow at the moment the field is introduced.

Key words: Turbulence, Duct flow, Anomalous fluctuations

Introduction This paper addresses decay of turbulence in an electrically conducting fluid in the presence of an imposed static magnetic field. The classical description of this phenomenon (see, e.g. [2]) is based on the two key effects: the suppression of velocity fluctuations via the Joule dissipation of induced currents and the development of anisotropy in the flow. Simple phenomenological modelling suggests that the magnetic field accelerates the decay and leads to faster laminarization of the flow. While correct from the general perspective this picture is challenged by recent results. It has been found that flows with strong magnetic fields often exhibit counterintuitive behaviour characterized by anomalously strong fluctuations of velocity and temperature. One example is the paradoxical experimental observations made in [1] (see Fig. 1). Strong residual velocity fluctuations were detected under conditions in which both the classical theory and pressure drop measurements suggested complete laminarization of the flow.

In the experiment, turbulence was created by a honeycomb in the flow of mercury in a duct of 2x4.8 cm cross-section. A magnetic field of strength up to 1.1 T with the main component parallel to one set of walls was imposed in the test section. The intensity of turbulent fluctuations at a point 43 cm from the honeycomb was measured for flows at the Reynolds number $Re = 7.85 \cdot 10^4$ and Hartmann numbers $Ha \leq 780$. Two distributions of the magnetic field were considered (see Fig. 1). In the case 1, the entire length of the honeycomb was located between the magnet poles, so turbulence was generated and decayed entirely within the practically uniform transverse magnetic field. In the case 2, the magnet poles were shifted downstream, so turbulence was generated at negligible magnetic field and travelled about 5.5 convective times before entering the space between the poles and thus experiencing the full magnetic suppression effect.

Fig. 1 shows that the signals measured in the two cases are about the same for weak magnetic fields. The decrease of the intensity with Ha/Re is, obviously, due to the MHD suppression. For stronger magnetic fields, however, the signals show entirely different trends.

In the case 2, the intensity continues to decrease to about 0.015 at high Ha. In the case 1, the intensity grows rapidly with growing Ha and reaches 0.09 (almost twice the intensity in the flow without magnetic field) at $Ha/Re = 10^{-2}$.

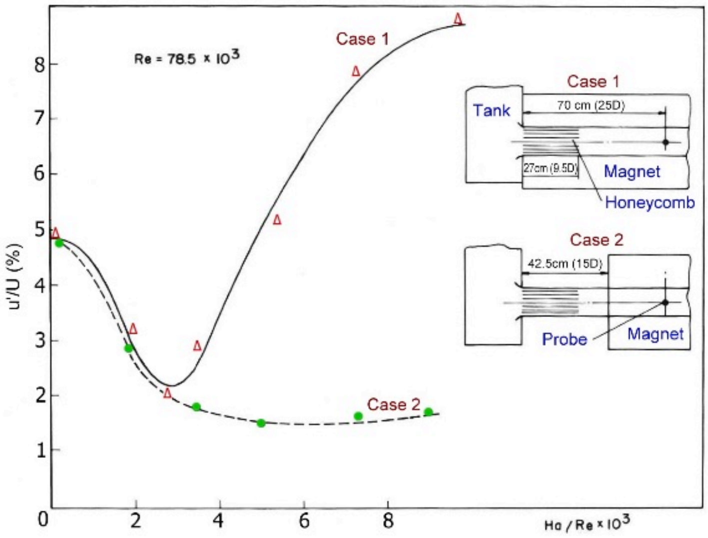

Figure 1. Magnet configuration and key results of [1]. The curves show the turbulence intensity on the duct axis far downstream of the entrance into the magnetic field as a function of Ha/Re.

The rest of this paper presents the results of high-resolution numerical simulations designed to reveal the physical mechanism responsible for the paradoxical behaviour just described. For brevity, two simulations performed at $Ha/Re = 2 \cdot 10^{-3}$ and $Ha/Re = 7 \cdot 10^{-3}$ with the main component of the magnetic field parallel to the longer sides of the duct are shown. Further results and a more detailed discussion can be found in [3].

Physical Model and Numerical Method A flow of an isothermal, incompressible, Newtonian, electrically conducting fluid is considered. The quasi-static (non-inductive) approximation is applied to the MHD interactions. The computational domain includes the test section of the experiment extending by 17.8 hydraulic diameters from the honeycomb's exit. The walls of the duct are no-slip and electrically perfectly insulated. A streamwise velocity distribution approximating the laminar flow within the honeycomb tubes is imposed at the inlet. The boundary conditions at the exit are those of zero streamwise gradient.

The problem is solved numerically using the finite-difference scheme first described in [4] and further developed in the works of the authors of this paper. The scheme is explicit and of second order in time and space. The discretization is on a structured collocated grid built along the lines of the Cartesian coordinate system. The exact conservation of mass, momentum, and electric charge, as well as near-conservation of kinetic energy are achieved by using the velocity and current fluxes obtained by interpolation to staggered grid points. The standard projection technique is applied to compute pressure and enforce incompressibility. The numerical algorithm is parallelized using the hybrid MPI-OpenMP approach. The computational grid is clustered toward the walls of the duct and consists of $3072 \times 512 \times 192$ points. As described in [3], accuracy of the model is confirmed by grid sensitivity studies.

Each simulation is initialized with a laminar state and continued for 100 non-dimensional time units so as to produce a fully developed state of the flow. After that, the simulation is continued for 100 time units. The flow structure is analyzed and turbulence statistics are accumulated and time-averaged during this stage.

Results The results of the computations are in perfect qualitative agreement with the experiment. In particular, in the runs 1 and 2 performed with weak magnetic field ($Ha/Re = 2 \cdot 10^{-3}$) the time-averaged rms fluctuations of the streamwise velocity are found to be about 4.5×10^{-3} in both (case 1 and case 2) configurations. For the runs 3 and 4 performed with a strong magnetic field ($Ha/Re = 7 \cdot 10^{-3}$), the anomalous behavior detected in the experiment is found. The turbulence intensity remains low, about 3.3×10^{-3} in the run 4, when the magnet is shifted downstream of the honeycomb exit (the case 2 configuration). The intensity is much larger, about 1.35×10^{-2} in run 3, when the strong nearly uniform magnetic field covers the entire duct (the case 1 configuration).

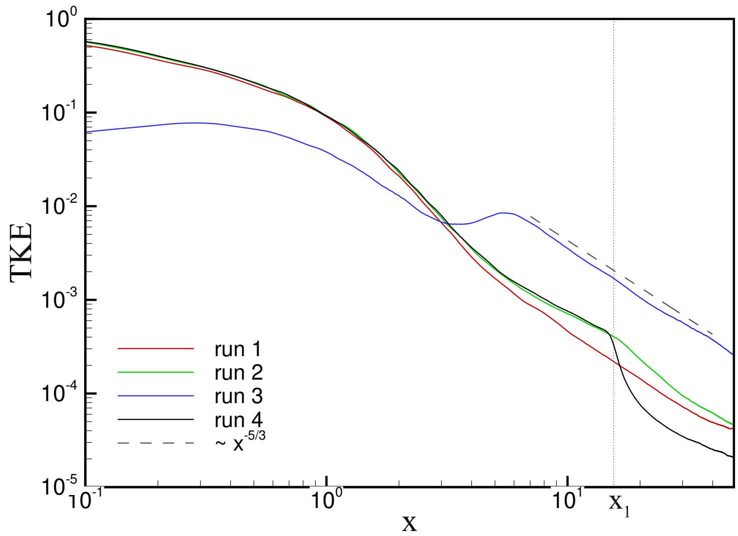

Figure 2. Simulation results. Kinetic energy of velocity fluctuations as a function of position along the centerline of the duct

Fig 2 shows the evolution of the time-averaged kinetic energy of velocity fluctuations along the duct's centerline. The dotted line indicates the location of the corners of the magnet pole-pieces in the runs 2 and 4. The slope line $\sim x^{-5/3}$ is shown purely for comparison. We do not have theoretical arguments either supporting or rejecting such a scaling.

For the runs 1 and 2, the energy decay curves obtained at two locations of the magnet are not very different from each other. For small x, the magnetic damping causes somewhat faster decay in the run 1. The effect of the magnetic field and its configuration is much more pronounced in the flows of runs 3 and 4. The most interesting behavior is in the run 3, where strong magnetic field is present over the entire length of the flow. In the entrance portion of the duct, the generation of turbulence is inhibited and the kinetic energy of fluctuations is an order of magnitude smaller than in the other three cases. The energy grows slightly for $x < 0.3$ and then decays, but much slower than in the other cases. The energy becomes larger than in the other flows at $x > 3$. We see increase of the turbulent kinetic energy in the interval $3 < x < 6$ and slow decay at $x > 6$. At the end of the computational domain, the fluctuation energy in the flow is about an order of magnitude larger than in the other three flows.

The mechanism causing the anomalous behavior in the run 3 is revealed by the structure of the flow shown in Figs 3 and 4. An abbreviated discussion is provided below. A full discussion and further data supporting the mechanisms, such as the two-point correlations of velocity fluctuations and one-dimensional integral length scales, are available in [3].

In the simulations 3 and 4, we have the Reynolds number based on the width of the Hartmann boundary layer $Re/Ha = 143$, which is below the typical laminar-turbulent transition range of the MHD duct flow [5]. Turbulence is therefore suppressed as the fluid moves through the magnetic field. The flows obtained for the two configurations of the magnetic field are, however, clearly different. In the case 2 configuration, there is a distance between the honeycomb and the beginning of the zone of full-amplitude magnetic field. The plot for the run 4 in Fig. 3 clearly shows that the distance is sufficient for the instability and mixing of the jets generated by the honeycomb. Three-dimensional turbulence develops. Upon entering the magnetic field, the turbulent fluctuations are quickly suppressed, which is reflected by the strong reduction of the RMS velocity fluctuations reported above.

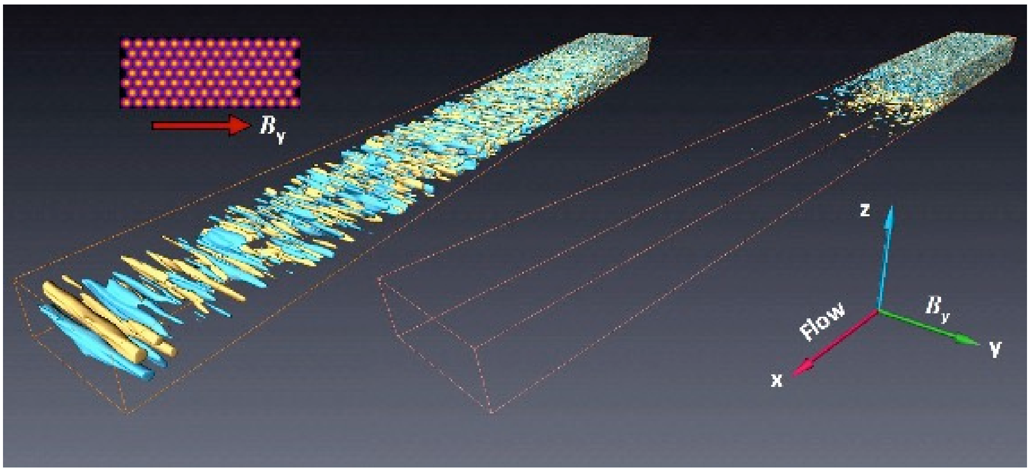

Figure 3. Isosurfaces of the z-component of velocity for the runs 3 (left) and 4 (right). Two isolevels of the same magnitude and opposite signs are visualized. The insert shows the honeycomb pattern and the main component of the magnetic field B_y.

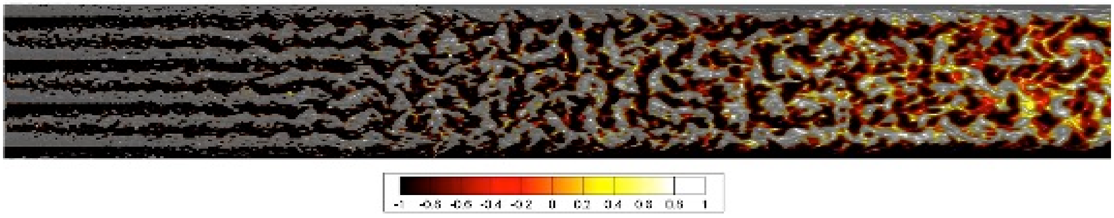

Figure 4. Instantaneous distribution of the vorticity component ω_y parallel to the magnetic field in the (x, z) cross-section through the duct's axis.

In the case 1 configuration, the formation of turbulence near the honeycomb exit occurs in the presence of a full-amplitude magnetic field. The velocity field in the run 3 quickly becomes strongly anisotropic. The instability of the honeycomb jets does not lead to a three-dimensional turbulent state, but to a quasi-two-dimensional flow dominated by structures aligned with the magnetic field. The illustrations in Figs. 3 and 4 and the additional visualizations analyzed in the course of our work (not shown) suggest the following scenario of the evolution of the spatial structure of the flow. In the inlet portion of the duct, approximately at $x < 3$, the dominant feature of the evolution is the transformation of the round jets exiting the honeycomb into quasi-two-dimensional planar jets. Already in the course of this transformation, the jets experience the Kelvin-Helmholtz instability that leads to noticeable waviness at x between 3 and 4 and to roll-up into quasi-two-dimensional vortices at around $x = 5$. The following evolution is characterized by quasi-two-dimensional vortices superimposed on the plug-like profile of the streamwise velocity.

The development of quasi-two-dimensional vortices is our key observation. It provides the basis for the explanation suggested earlier for the anomalously strong velocity fluctuations detected in [1] and, likely, other experiments, in which anomalous residual fluctuations were found. Due to their weak gradients along the field lines, the vortices do not generate strong Joule dissipation. Furthermore, the quasi-two-dimensionality reduces the energy flux from large to small length scales, which implies weaker viscous dissipation. The flow structures are still suppressed by the Joule and viscous dissipation in the boundary layers, but the effect is not strong. The vortices are visible till the end of the flow domain, and are responsible for the generation of high-amplitude velocity fluctuations at far downstream locations.

Conclusions The simulations are in excellent qualitative agreement with the experiments [1]. The anomalous high-amplitude fluctuations are found in the same configuration and at the same values of Ha and Re as in the experiment. The increase of the fluctuation amplitude between the case 1 and case 2 configurations of the magnetic field is of the same order of magnitude in the experiment and the simulations. This allows us to conclude that the physical mechanism described above is responsible for the anomalous fluctuations.

However, the turbulence intensity in the computed flows is about five times lower than measured in the experiment. This is true for both low and high values of Ha and for different structures of the magnetic field. Several possible explanations related to both the numerical and experimental procedures are suggested in [3].

From the viewpoint of the turbulent decay theory, our work provides a good example of non-universality of decay of MHD turbulence. The curves in Fig. 2 and other results presented in [3] show complex behaviour of the fluctuation energy. The decay rate varies with the stage of the process and among the velocity components. The values of the two independent non-dimensional parameters (for example, Ha and Re) do not determine the decay scenario in a unique way. The process is strongly affected by the development, or lack thereof, of quasi-two-dimensional structures. The appearance and nature of such structures is, in turn, determined not just by the strength of the magnetic field, but also by the features of the flow evolution, most importantly, by the state of the flow at the moment the magnetic field is introduced.

Acknowledgments The work is financially supported by the DFG grants KR 4445/2-1 and SCHU 1410/29-1, the Helmholtz Alliance "Liquid metal technologies" (Limtech) and the grants CBET 1435269 and CBET 1803730 from the US NSF. Computations were performed on the parallel supercomputers Jureca of the Forschungszentrum Jülich (NIC) and SuperMUC of the Leibniz Rechenzentrum (LRZ), flow visualization was done at the computing center of TU Ilmenau.

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