A Convex Optimization Approach for Distributed Energy Trading of Interconnected Microgrids

Tamal Roy, Zhen Ni
Department of Electrical Engineering and Computer Science
South Dakota State University
Brookings, SD, USA 57007
Email: {tamal.roy, zhen.ni}@sdstate.edu

Marco Ciarcià
Department of Mechanical Engineering
South Dakota State Univeristy
Brookings, SD, USA 57007
Email: {marco.ciarcia}@sdstate.edu

***Abstract*—A distributed convex optimization framework for energy trading of interconnected microgrids is investigated to improve the economy and reliability of system operation. In this work, a distributed energy trading approach for interconnected operation of islanded microgrids is studied. Specifically, the system includes several islanded microgrids that can trade energy in a given topology. A distributed iterative deep cut ellipsoid (DCE) algorithm is implemented with limited information exchange. This approach will address the scalability issue and also secure local information on cost functions. During the iterative process, the information exchange among interconnected microgrids is restricted to electricity prices and expected trading energy. Numerical results are presented in terms of the convergent rate of the algorithm for different topologies, and the performance of the DCE algorithm is compared with sub-gradient algorithm.**

***Index Terms*—Distributed energy resources, sub-gradient method, deep cut ellipsoid algorithm, Lagrange multiplier, and interconnected microgrids.**

I. Introduction

In traditional power systems, energy is generated by large generation plants in centralized fashion. In centralized systems, the energy needs to be transported over long distance and through complex transportation meshes to the end users. Complicated, inflexible structures can create a burden to the whole power system and are susceptible to outages [1]. The smart grid aims to improve the traditional power grid by introducing the interconnected microgrids system (IMS) in a distributed way.

The distributed microgrid system allows the energy exchange with several micro-grids which are islanded from the utility grid. By using IMS, it is easy to ensure the full utilization of local energy resources, reduce the energy operating cost and achieve reliability of power delivery [2]. Interconnection of microgrids can provide improved electric service reliability and better power quality for the end users. From the aspect of energy trading game, the microgrids (MGs) can act as players from cooperative perspective. During different time periods, MGs can act as seller and buyer based on their respective load demand and aim at maximizing their individual benefits. Therefore, distributed energy trading is necessary to meet the global operation goal of an interconnected micro-grid system that preserves scalability and privacy issues. Recent studies focus on the energy optimization strategy of smart grid. The energy optimization can be divided into two types: centralized optimization and distributed optimization. Normally, if all the MGs share information on their respective load, generation and grid condition, the system could be easily implemented based on classical optimization such as optimal power flow.

For instance, in [3], [4], the authors consider a method of joint and distributed control of IMS and residential community. Alternatively, a method of Newton-like descent is proposed in [5] to solve the three-phase optimal power flow problems. From the security perspective, these centralized solutions may undergo privacy issues [5] that encouraged the authors of [6] to deploy distributed optimal power flow (OPF) in the power system in [7], [8]. However, the OPF problem is non-convex, and the solution is too complicated to compute since it has multiple local optimal points.

In this work, we will focus on the trading mechanism of interconnected micro-grids rather than the electrical operation of the utility grid. In the context of energy trading, distributed energy resources can convert the current oligopolistic market into a flexible one [9]. For instance, the authors in [10] proposed a game theoretic approach to trade the stored energy with other elements of the grid. In terms of demand response, the authors in [11] studied a generalized Nash equilibrium problem that considered demand response where aggregators and micro-grids are formulated as a non-cooperative game. However, the majority of existing works focus on an energy trading mechanism based on an architectural framework [12]–[14].

Motivated by aforementioned works, we studied the energy trading mechanism between the islanded MGs without the need of a central coordinator. Each MG buys/sells energy from/to adjacent MGs without sharing the local cost information. The objective of this work is to minimize the global operation cost (generation plus transmission costs) by preserving the local information. Compared with the previous works (e.g., [14], [15]), the main contributions of this work include: (i) A distributed iterative algorithm based on the deep cut ellipsoid (DCE) method is proposed for energy trading between isolated MGs. Different from prior works, this work analyzes the comparative study between two distributed energy trading approaches using different topologies (e.g., Full, Line, Ring, and Star). (ii) The performance of two distributed algorithms is compared with different case studies.

The remainder of this paper is structured as follows. Section II represents the model of the energy exchange network. Section III illustrates the distributed model and algorithm of the interconnected microgrids system. Section IV presents the simulation results and discussion. Section V concludes this paper.

II. System Model

A system of N = 4 interconnected MGs is considered through a power interconnection infrastructure and a communication network as shown in Figure 1.

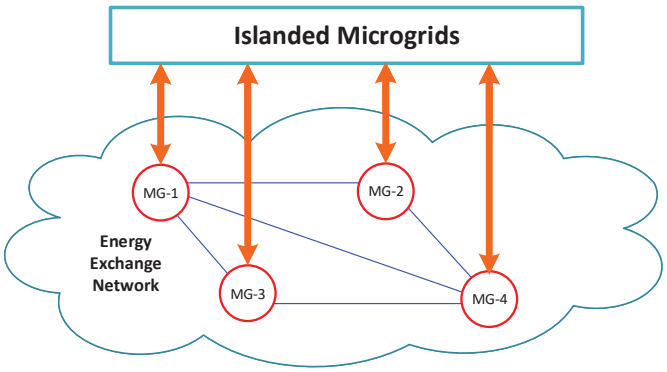


Fig. 1: A network of multiple interconnected MGs, distribution power line and communication network.

During each scheduling time, $E_i^{(g)}$ and $E_i^{(c)}$ are the generation and consumption of MG i respectively. Moreover, MG i is allowed to sell energy $E_{i,j}$ to MG j, $j \neq i$, and to buy energy $E_{k,i}$ from MG k, $k \neq i$. The power balance within the MG requires

$$E_i^{(g)} + e_i^T \mathbf{A}^T \mathbf{E}_i^{(b)} = E_i^{(c)} + e_i^T \mathbf{A} \mathbf{E}_i^{(s)} \tag{1}$$

where the two N-dimensional column vectors are defined as:

$$\mathbf{E_i^{(b)}} = \begin{bmatrix} E_{1,i} \\ . \\ . \\ . \\ E_{N,i} \end{bmatrix} \quad \text{and} \quad \mathbf{E_i^{(s)}} = \begin{bmatrix} E_{1,i} \\ . \\ . \\ . \\ E_{N,i} \end{bmatrix}$$

where $\mathbf{E_i^{(b)}}$ and $\mathbf{E_i^{(s)}}$ are the energy purchasing and selling vector of MG i. In order to introduce the connection between MGs, an adjacency matrix $\mathbf{A} = [a_{i,j}]_{N \times N}$ is defined. If there exists a connection between MG i to MG j, element $a_{i,j}$ is set as 1 and 0 otherwise. Note that $\mathbf{A}$ may be nonsymmetric, meaning that at least two MGs are allowed to share energy in one direction only. Moreover, two MGs without connection defines $a_{i,j} = 0 \rightarrow E_{i,j} = 0$ for all $i, j = 1..., ..N$.

The objective of this problem is to minimize the total operating cost of interconnected microgrid system, consisting of power generation and transmission cost. The energy exchanged by interconnected MGs form the equilibrium point of the following minimization problem:

$$\min_{E_{i,j}} \quad \sum_{i=1}^{N=4} C_i(E_i^{(g)}) + \sum_{i=1}^{N=4} e_i^T \mathbf{A}^T \beta(\mathbf{E_i^{(b)}}) \tag{2}$$

$$\text{subject to} \quad E_{i,j} \geq 0, \forall i, j \tag{3}$$

$$E_i^{(c)} + e_i^T (\mathbf{A E_i^{(s)}} - \mathbf{A^T E_i^{(b)}}) \geq \mathbf{0}, \forall \mathbf{i} \tag{4}$$

where $C_i(E_i^{(g)})$ is defined as the cost of generating $E_i^{(g)}$ units of energy at MG i; $\beta(E_i^{(b)})$ is the cost of transferring $E_{i,j}$ units of the energy between MG i and MG j; $\mathbf{e}_i$ is the ith column of the $N \times N$ identity matrix; $E_i^{(b)}$ is the vector composed of the energy bought from other MGs by MG i;

The multiple MGs in one interconnected microgrid system, which has their set of strategies, should be coordinated in order to achieve the global objective of the system and meet power demands.

In the system model mentioned above, two cost functions have been introduced, namely, cost function $C_i(E_i^{(g)})$ is the cost of MG i spend to generate the energy $E_i^{(g)}$, and the cost function $\beta(E_i^{(b)})$ is the cost of transferring energy between MG i to MG j. Both cost functions are positive valued, monotonically increasing, convex and twice differentiable. Each MG is capable of producing extra energy by using a diesel generator at an "extra cost". The cost function $C_i(E_i^{(g)})$ of a diesel generator (DG) is modeled as a quadratic polynomial. The fuel cost is represented as: $C_{DGi}(E_i^{(g)}) = a_i + b_i P_{DGi} + c_i P_{DGi}^2$ where a_i, b_i and c_i are the fuel cost coefficients of DG; and P_{DGi} is the output power of DG i.

The total operation cost $C_i(E_i^{(g)})$ includes the cost of all DG units of MG i, $C_i(E_i^{(g)}) = \sum_{N=1}^{N=4} C_{DGi}$. For the transportation cost, many factors may have an influence on the model, i.e, the investment and construction cost of the network, etc. For simplicity, we imagine the cost of all connection topologies of the system is the same. The transmission cost is a second-order quadratic polynomial.

III. Distributed Model and Algorithm

1) Distributed optimal scheduling model: When considering the minimization problem (2), one can readily identify that the objective function is strictly convex. Moreover, a centralized unit needs a control unit that is aware of all system information. This fact implies a considerable amount of data traffic to gather all the information and can miss some annoying privacy issues. In this regard, we propose a distributed iterative approach by decomposing the problem N local subproblems, which can be implemented by the MGs in an autonomous and cooperative manner.

By utilizing the Lagrangian method and duality theorem, a multiplier strategy is introduced as the exchanged information between MGs to solve the subproblem for each MG. Thus, the distributed iterative solution (2) can be rewritten as:

$$C^* = \min_{\varepsilon_i^{(s)}, E_{i,j}} \quad \sum_{i=1}^{N=4} C_i(E_i^{(g)}) + \sum_{i=1}^{N=4} e_i^T \mathbf{A}^T \beta(E_i^{(b)}) \tag{5}$$

$$\text{subject to} \quad \text{constraints } (3), (4), \text{and}$$

$$\varepsilon_i^{(s)} = e_i^T \mathbf{A} E_i^{(s)}, \forall i \tag{6}$$

The only difference with respect to (2) is the introduction of new variable $\varepsilon_i^{(s)}$ to represent the energy sold by MG i and later it will be equal to all the energy bought by other MGs from MG i. The coupling constraint can be represented as $\varepsilon_i^{(s)} = e_i^T \mathbf{A} E_i^{(s)}$.

Due to the decomposition theory [16] of the primal-dual problem (2), Lagrange multipliers are introduced to relax the coupling constraints for solving the dual problem (7)

$$C^* = \max_{\lambda} \quad C(\lambda) \tag{7}$$

where, $C(\lambda) = \sum_{i=1}^{N} C_i^l(\lambda)$

$$C_i^l(\lambda) = \min_{\varepsilon_i^{(s)}, E_i^{(b)}} \quad C_i(\varepsilon_i^{(s)}, E_i^{(b)}, \lambda)$$
$$\text{subject to} \quad \text{constraints } (3), (4), \text{ and } (6) \tag{8}$$

For each MG, we have:

$$C_i(\varepsilon_i^{(s)}, E_i^{(b)}, \lambda) = C_i(E_i^{(g)}) + e_i^T \mathbf{A}^T \beta(\mathbf{E}_i^{(b)}) + e_i^T \mathbf{A}^T diag\lambda \mathbf{E}_i^{(b)} - \lambda_i \varepsilon_i^{(s)} \tag{9}$$

that is the contribution of MG i to the Lagrangian function relative to (2). The parameter λ gathers all the Lagrange multipliers λ_i corresponding to coupling constraints $\varepsilon_i^{(s)} = e_i^T \mathbf{A} E_i^{(s)}$,

respectively and for all $i = 1,....,N$. Based on above analysis, each Lagrange multiplier λ_i can be defined as the marginal cost of MG i, namely the selling price of a unit of power to neighboring MGs. Thus, Lagrange function can be seen as net expenditure. The net expenditure of each MG has four parts: (i) $C_i(E_i^{(g)})$ is the generation unit cost function; (ii) $e_i^T \mathbf{A}^T \beta(\mathbf{E}_i^{(b)})$ is the transmission network cost resulted from transferring energy bought from other MGs; (iii) $e_i^T \mathbf{A}^T diag\lambda \mathbf{E}_i^{(b)}$ is the cost due to buying energy; and (iv) $\lambda_i \varepsilon_i^{(s)}$ is the income by selling energy.

2) Distributed algorithm: The problem can be transformed to maximum dual problem. To this end, the optimal Lagrangian multiplier converge to the optimal point of dual problem (7), $\lambda^* = argmax_\lambda C(\lambda)$. More specifically, at each point $\lambda[k]$, each MG minimizes its corresponding contribution to the Lagrange function by solving the local subproblem (8) and determining the minimum point $(\varepsilon_i^{(s)}[k], \mathbf{E}_i^{(b)}[k]) = (\varepsilon_i^{(s)}(\lambda[k]), \mathbf{E}_i^{(b)}(\lambda[k]))$.

In the previous work [14], the Sub-Gradient Algorithm is used to solve the optimization problem. In particular, the Lagrange multiplier is updated according to

$$\lambda_i[k+1] = \lambda_i[k] + \alpha[k] \begin{bmatrix} e_1^T A E_1^{(s)}[k] - \varepsilon_1^{(s)}[k] \\ . \\ . \\ . \\ e_N^T A E_N^{(s)}[k] - \varepsilon_N^{(s)}[k] \end{bmatrix} \tag{10}$$

where, $\alpha[k]$ is a positive step factor. However, the Sub-Gradient Algorithm needs the initial guess of price (λ) and step size (α). The initial assumption is restrictive in the Sub-Gradient Algorithm to find an optimal solution set. This initial assumption often makes the algorithm slower. A faster algorithm is needed to improve the system performance.

The approach proposed in this work is based on the Deep Cut Ellipsoid Algorithm. According to [17], the *DCE* is used to determine the feasibility of a system of linear inequalities. This algorithm generates a "decreasing" sequence of ellipsoids that contain a minimizing point. The update of the dual variables has been done in this algorithm. The idea of choosing initial ellipsoid is to localize the set of candidate λ's within a closed and bounded set. Therefore, this algorithm releases the users to initialize the price values (λ) at the first iteration and from choosing the step size (α).

The size and boundary of the ellipsoid can be represented as λ and matrix P respectively. The sub-gradient of $C(\lambda)$ in $\lambda = \lambda[k]$ need to be computed from $k-th$ can be described as

$$\varsigma[k] = [e_N^T A E_N^{(s)}[k] - \varepsilon_N^{(s)}[k]]_{N\times 1}, \forall\lambda \tag{11}$$

Then we have, $C(\lambda) \leq C(\lambda[k]) + \varsigma_T(\lambda - \lambda[k]), \forall\lambda$ and the sub-gradient needs to be normalized as,

$$\upsilon[k] = \frac{\varsigma[k]}{\sqrt{\varsigma^T \times P[k] \times \varsigma[k]}} \tag{12}$$

First, the Lagrange multiplier (λ) can be represented as,

$$\lambda_i[k+1] = \lambda_i[k] + \frac{1+N\times\alpha}{N+1} \times P[k] \times \upsilon[k] \tag{13}$$

Second, the boundary (matrix P) of the ellipsoid can be updated as:

$$P[k+1] = \frac{N^2}{N^2-1} \times (1-\alpha^2) \times (P[k] - \frac{2(1+N\alpha)}{(N+1)(1+\alpha)} \times P[k] \times \upsilon[k] \times (\upsilon[k])^T \times P[k]) \tag{14}$$

where, α is a positive step factor, $P[k]$ is the boundary of solution space and k is the iteration number.

Next, the updated Lagrange multiplier (λ) will check the original bounds. If it is within the bound, then it has converged otherwise it will take next iteration according to (11), (12), (13), and (14).

Algorithm 1 summarizes are the steps of the proposed distributed iterative algorithm.

Algorithm 1 Distributed optimal scheduling algorithm

1: Initialize λ_{min}, λ_{max}, $\lambda_i[0]$, $P[0]$, $N = 4$, $\alpha = 0$, k=0
2: At k^{th} iteration
3: At any MG i
4: Compute the sub-gradient $\varsigma[k] = [e_N^T A E_N^{(s)}[k] - \varepsilon_N^{(s)}[k]]_{N\times 1}, \forall\lambda$
5: Normalize the sub-gradient $\upsilon[k] = \frac{\varsigma[k]}{\sqrt{\varsigma^T \times P[k] \times \varsigma[k]}}$
6: MGs exchange $\lambda_i[k]$ with neighboring MG
7: MG i computes $\varepsilon_i^{(s)}[k]$ and $E_i^{(b)}[K]$ using (5) with $\lambda[k]$.
8: MG i informs MG $j(j \neq i)$ the energy it expects to buy namely $E_{j,i}[k]$, at the given price $\lambda_j[k]$.
9: According to the expected purchasing energy $E_{j,i}[k]$ from other MGs, MG i obtains
10: $E_i^{(s)}[k] \Rightarrow [E_{i1}[k].....E_{iN}[k]]^T$
11: MG i updates according to step 12 and 13
12: $\lambda_i[k+1] = \lambda_i[k] + \frac{1+N\times\alpha}{N+1} \times P[k] \times \upsilon[k]$
13: $P[k+1] = \frac{N^2}{N^2-1} \times (1-\alpha^2) \times (P[k] - \frac{2(1+N\alpha)}{(N+1)(1+\alpha)} \times P[k] \times \upsilon[k] \times (\upsilon[k])^T \times P[k])$
14: At any MG i
15: If $\lambda_i < \lambda_{min}$
16: $\varsigma[k] = -1$, $\upsilon[k] = \frac{\varsigma[k]}{\sqrt{P[k]}}, \alpha = \frac{(\lambda_{min} - \lambda_i)}{\sqrt{P[k]}}$
17: Then, MG i updates according to step 18 and 19
18: $\lambda_i[k+1] = \lambda_i[k] + \frac{1+N\times\alpha}{N+1} \times P[k] \times \upsilon[k]$
19: $P[k+1] = \frac{N^2}{N^2-1} \times (1-\alpha^2) \times (P[k] - \frac{2(1+N\alpha)}{(N+1)(1+\alpha)} \times P[k] \times \upsilon[k] \times (\upsilon[k])^T \times P[k])$
20: $k = k+1$
21: Until stopping criteria is met.

To solve the dual problem (7), each MG should be aware of $\varepsilon_i^{(s)}[k]$ and $E_i^{(b)}[K]$, namely the total energy it sold and the vector composed of energy bought from other MGs, respectively. Moreover, we can compute $E_i^{(s)}$ from $E_i^{(b)}$. Combined with Algorithm 1, the Lagrangian multipliers can be updated. Therefore, all necessary data can be computed by each MG without a centralized controller. Also, the information shared between MGs is bound to Lagrange multipliers λ_i and the expected buying energy $E_{j,i}$. Hence, the privacy of MGs can be secured. According to Algorithm 1, each Lagrange multiplier λ_i can be interpreted as the price per energy unit requested by MG i to sell energy to its neighboring MGs. Using the Lagrangian function (8), each MG pays for generating energy, for purchasing energy and for transferring the energy it purchases. On the other hand, the MG is paid for the energy it sells.

By solving the problem (8), the MG maximizes profit for some given selling ($\lambda_i[k]$) and buying ($\lambda_j, j \neq i$) prices per unit energy. Based on the Algorithm 1, the price λ_i would be modified constantly until the energy demand matches energy offer. As reported by (13), if the energy offered by MG i is less than the requested energy from other MGs, the price must be increased as the demand exceeds the supply. Conversely, when the demand by MG i is less than the supply, the price will be decreased. However, the price does not change when the supply and demand are at equilibrium.

3) Solution of The Local Subproblem: In this section, the solution of the local subproblem is reported to support the global minimization problem of the system. The minimization subproblem (8) at MG i behaves according to six possible cases. Table I expresses these six cases to support the local subproblem.

TABLE I: Possible Cases of Local Subproblem of MG i.

Cases	Generation	Buy	Sell
1	✓	–	–
2	–	✓	–
3	✓	✓	–
4	✓	–	✓
5	–	✓	✓
6	✓	✓	✓

The intention of MG i is to minimize local cost or, equivalently, to maximize net profit, when λ_is are interpreted as exchanging prices per energy unit. In the first case, the MG i is generating all and only the energy it consumes, that is $\varepsilon_i^{(s)} = 0$ and $E_i^{(g)} = E_i^{(c)}$. The MG i is not interested to sell energy since the selling price is lower than marginal generation cost. Indeed, the income will be lower than the extra production cost. In addition, purchasing is not beneficial either since the purchasing price is higher than the marginal production cost. Therefore, as for case 1, the MG i should remain self-constrained. However, MG i is always willing to trade energy since their local cost $(C_i(E_i^{(c)}) + \beta(0))$ is higher than the net payment. This scenario holds only in case 6. Similar considerations hold for other cases.

IV. Result and Discussion

Several case studies have been considered based on proposed energy trading mechanism. An interconnected test system consisting of four different MGs, including DG units only. The interconnection topology of interconnected microgrid system is represented in Figure 2.

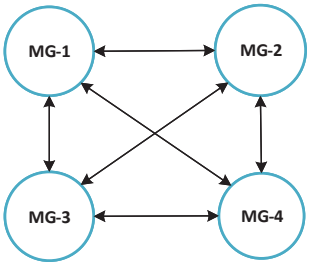


Fig. 2: A fully interconnected microgrid system.

The fuel coefficients of DG (a = 86.39 \$, b = 56.56 $\$/MW$ and c = 0.33 $\$/(MW)^2$) are considered from [18]. The coefficients of transfer cost function are p = 0, q = 0, r = 3.68. The cable capacity assumes 100 MW. We have introduced a soft upper bound E_{max}= 5 MW The transfer cost function is set without the upper bound. All simulation studies have been done on a single machine with an i7 dual-core processor.

1) Trading prices: Figure 3 represents the iterative process of electricity price of each MG.

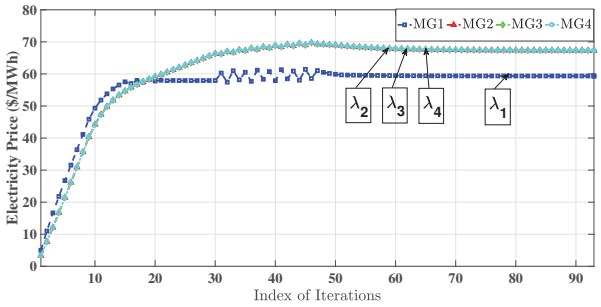


Fig. 3: Iterative process of the electricity price of each MG.

The curves refer to a fully connected system, where microgrid loads are $E^{(c)}$ = [1,6,6,6] MW and each MG generation capacity is E_{max} = 5 MW. The result shows that the DCE algorithm converges after 58 iterations. The prices of MG1, MG2, MG3 and MG4 are 59.35 $\$/MWh$, 67.32 $\$/MWh$, 67.32 $\$/MWh$ and 67.32 $\$/MWh$, respectively. However, the electricity prices of MGs converge to different values with same initial prices. Besides, Figure 3 depicts the final selling prices of MGs which have a direct relationship to their own loads that means the MG that consumes more electricity has a higher selling price after the convergence is achieved. For example, MG1 earns more money by selling energy to the other MGs with a lower price because it has lower power demand.

In fact, the MG1 only generates and sells energy whose local cost function is:

$$C_1 = C_{DG1}(P_{DG1}) - \lambda_1 \varepsilon_1^{(s)} \tag{15}$$

The optimal $\lambda_1 = \lambda_1^*$ can be given in the form of marginal cost:

$$\lambda_1^* = C'(P_{DG1}) \tag{16}$$

On the other hand, MG2, MG3, and MG4 only generate and buy energy from MG 1. They are all buying same amount of energy from MG1, and their local cost functions can be represented as:

$$C_2 = C_{DG2}(P_{DG2}) + \beta(E_{1,2}) + \lambda_2 E_{1,2} \tag{17}$$

$$C_3 = C_{DG3}(P_{DG3}) + \beta(E_{1,3}) + \lambda_3 E_{1,3} \tag{18}$$

$$C_4 = C_{DG4}(P_{DG4}) + \beta(E_{1,4}) + \lambda_4 E_{1,4} \tag{19}$$

Moreover, from the perspective of MG2, MG3, and MG4, λ_2^*, λ_3^* and λ_4^* can be expressed as:

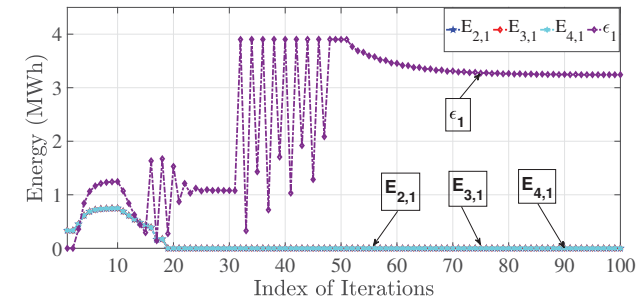
$$\lambda_2^* = C'(P_{DG2}) - \beta'(E_{1,2}) \tag{20}$$

$$\lambda_3^* = C'(P_{DG3}) - \beta'(E_{1,3}) \tag{21}$$

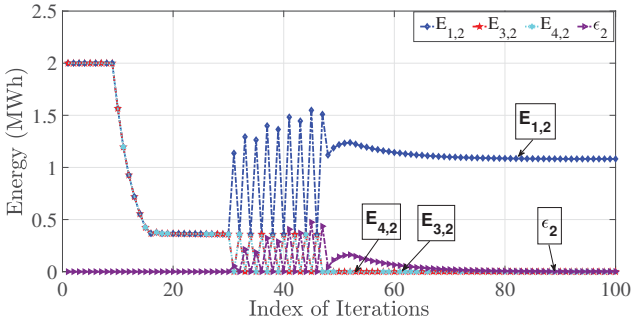
$$\lambda_4^* = C'(P_{DG4}) - \beta'(E_{1,4}) \tag{22}$$

Therefore, MG2, MG3, and MG4 should reduce their net expenditures by purchasing energy from MG1. The price of MG1 after convergence can be calculated according to (16), (20), (21) and (22), which is consistent with the result of Algorithm 1.

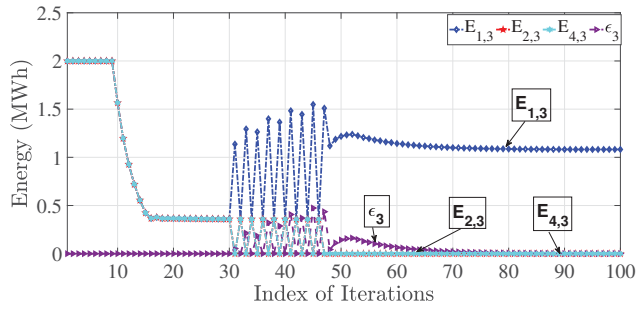
2) Trading energy: The iterative process of the energy trading between MGs is shown in Figure 4.



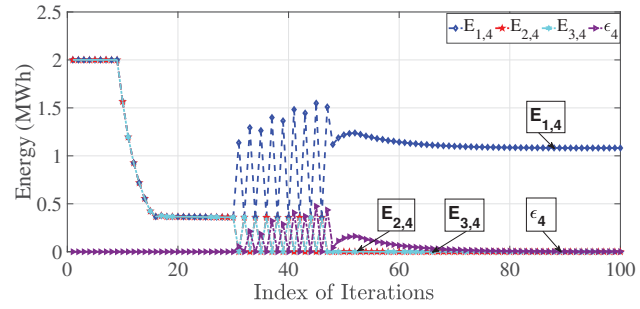
(a) MG1



(b) MG2



(c) MG3



(d) MG4

Fig. 4: Iterative process of the trading energy of MG1, MG2, MG3, and MG4.

The energy trading after convergence at current time slot can be explained as follows: MG2, MG3, and MG4 buy 1.08 MWh energy from MG1 respectively.

However, MG1 has been sold 3.25 MW power to all other MGs. The total energy sold equals the total energy purchased in the system. The coupling constraint $\varepsilon_i^{(s)} = e_i^T A E_i^{(s)}$ is fulfilled after convergence, which show that the algorithm works well. During the optimization, the cost by power transmission between MGs is covered by the electricity buyer. In the current time slot, MG2 buys energy from MG1 to meet its load demand, as the marginal cost of its own generating unit is higher than the sum of the selling price and the transmission cost of MG1. Similarly, the marginal cost of MG3 and MG4 is not economical. It is beneficial to work on lower generation limit.

3) Benefits of interconnection: Given the same setting, each MG can also be operated autonomously. Table II represents the cost comparison of each MG between autonomous and interconnected operation. The results show that energy trading decreases

TABLE II: Cost comparison of each MG between autonomous and interconnected operation.

		Operation	
	MG ID	Autonomous	Interconnected
Cost ($)	MG 1	143.28	332.11
	MG 2	4610.83	377.01
	MG 3	4610.83	376.99
	MG 4	4610.83	376.99
	Total	13975.78	1463.11

the global operation cost and decreases the local expenditure of individual MG that has less generation. Therefore, MG1 gains revenue by selling energy whereas MG2, MG3, and MG4 reduce their costs by purchasing energy.

4) Performance comparison with existing work: To interpret the benefits and advantages of the distributed model and the deep cut ellipsoid algorithm, the results are compared with the existing work [14] in terms of exchanged information, the number of MGs, solution algorithm, and performance. The studied algorithm features advantages in several aspects, especially in algorithm performance. The CDE algorithm has shown a better convergence performance compared to the Sub-gradient algorithm proposed in [14]. In this algorithm, the information shared among MGs is limited to Lagrange multipliers and the expected buying energy quantities, which are only communicated with trading MGs. As for the convergence performance of algorithms, the simulation results show the proposed method improves performance over the distributed sub-gradient algorithm of [14]. The detail iterative process comparison of price in MG3 between this work and the work in [14] based on same test cases is shown in Figure 5.

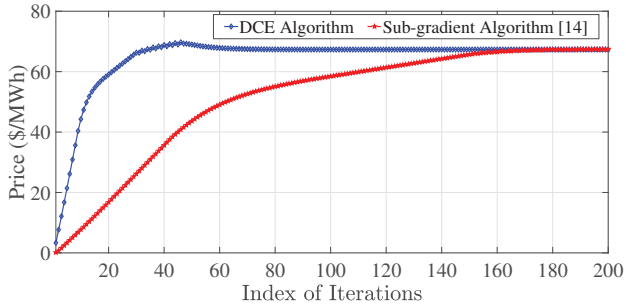


Fig. 5: Iterative process comparison of price in MG 3 between this work and that in [14].

The proposed approach releases the system to make the restrictive assumption, which reflects in faster convergence. The deep cut ellipsoid algorithm has a faster iteration speed due to faster shrinking. First, the initial assumption of price is made based on the total cost function. Then, the price needs to be maintained within the bounded limit, which reduces the solution space and speeds up the system. Finally, the prices of MG3 in this work and the work in [14] converge to the same value. For energy exchange network, four different topologies (e.g., Full, Ring, Line, and Star) are considered as in Figure 2 and Figure 6.

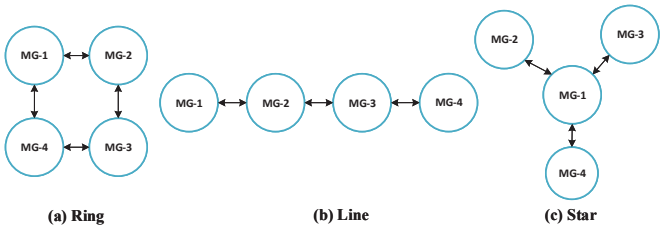


Fig. 6: Three different connection topologies.

The performance comparison for sub-gradient algorithm and DCE algorithm has also been done for four topologies as in Figure 7.

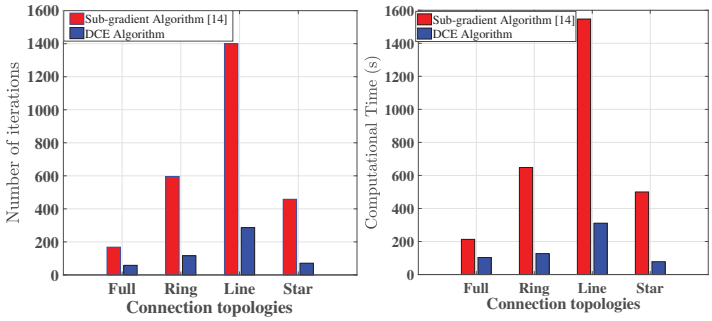


Fig. 7: Comparison between Sub-gradient algorithm and DCE Algorithm with the different topology in terms of iteration and time.

The four topologies are compared in terms of iteration and time to show the performance improvement of the proposed method. As shown in Figure 7, the fully interconnected topology gives the best performance. While for other three topologies, the star-connected topology has advantages over other two since it improves the income for MG1, which achieves the highest cost reduction although it has worst cost reduction performance for MG2, MG3, and MG4. The proposed method performs better than the slow sub-gradient algorithm in terms of iteration and time. Therefore, the MGs should operate in a distributed manner which lowers the interaction time with fewer data exchanges.

With more insight into the results, the search routines of the sub-gradient algorithm seems zigzag shaped. The sub-gradient algorithm is the fastest direction for the increasing of objective function value. Therefore, it could be a good choice to search on the sub-gradient direction in the local space.

However, this algorithm's convergence speed is slowed in the global space due to its zigzag-shaped search direction. For this drawback, the deepest cut ellipsoid algorithm with a sequence of shrinking ellipsoids that is polynomial in time is studied in this work. During each iteration, the sequence of each ellipsoid is smaller in volume than its predecessor due to its deepest cut. After that, it is easy to find a feasible point within this smallest global space. The proposed method has addressed the problem by the zigzag typed searching direction and eventually quicken the convergence.

V. Conclusion

In this paper, a distributed energy trading approach was investigated under a distribution network. The problem is formulated as an energy management problem to minimize the total system cost. An hour-ahead optimization model is constructed and the objective function includes the operation of DGs and network tariff. A distributed iterative algorithm was studied based on deep cut ellipsoid method considering descent search direction. The convergence of the proposed method was proved and verified with numerical results. Moreover, the results show that each MG can adjust generation of DGs or trade with other MGs with an extensive consideration of generation cost, trading price, and load characteristics. The distributed energy trading based on the DCE algorithm was applied to four topologies and found that certain topologies were more beneficial than others. Compared with the existing work [14], the proposed approach shows the advantageous features of modeling and performance.

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