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Acceleration-based bridge weigh-in-motion

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Abstract. Bridge Weigh-in-Motion (B-WIM) is the theory of utilizing field measurements to infer the weights of the overhead traffic that passes at full highway speed. There exist a consensus that conventional instrumentation faces substantial practical problems that halts the feasibility of this theory, namely installation time and complexity, especially for high elevation bridges. This article will escort through a new concept by moving from B-WIM system based on strain data to a new B-WIM system based on acceleration records. Kalman-filter-based estimation algorithm is developed to estimate the state vector (displacement and velocities) using limited measured acceleration response. The measured response is transformed to the modal response using the pseudoinverse of the mode shape matrix, which allows utilizing limited measurements number during the estimation process. The estimated state vector is used to feed a moving force identification (MFI) algorithm that shows a good estimating for a quarter-car load.

Keywords: Acceleration, B-WIM, Kalman filter, MFI, strain, measurements, proper, orthogonal, decomposition

1. Introduction

The axle load and gross weight of vehicles are important information for the design of new bridges and pavements, the rating and fatigue life assessments of existing bridges and pavements, design code calibration and the control of overweight vehicles to highway regulations [1]. Therefore, the dynamic moving forces produced by the vehicles on the bridge structure must be determined by adopting the estimation method or measurement techniques. In the late 1970's in the United States, Moses [2] first introduce the Bridge Weigh-In-Motion (B-WIM) system, which is the concept of using measured strains on a bridge to calculate the axle weights as they pass overhead at full highway speed. Then Zhu, and Law extend the theory for multi-span continuous bridge [3]. In more recent years, the field of moving force identification (MFI) has been developed by Chan, Law, and others [4–13]. Law and Fang [8] have applied the dynamic programming method to the MFI problem using zero order regularization. Then González [14] extended the algorithm with first order regularization, which improved the solution accuracy [15, 16].

The main drawback of the B-WIM system is the installation time and cost, especially for the high elevation bridges, which need huge equipment and trained labors to install strain sensors. A wide variety of engineering applications employ acceleration to identify desired information because acceleration sensors are generally cost-effective, convenient to install, have relatively low noise [17], and recently can easily attach to the bridge girders using drones[18]. This paper will focuses on using the acceleration response instead of strains as the main input to the B-WIM system, which becomes as an early step toward Portable B-WIM system.

Most techniques that use acceleration for force estimation are utilizing state vectors estimator, and then apply any other technique (i.e. least square) to estimate the force. Ma et al. [19, 20] proposed a

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Kalman-based method in which a Kalman filter and a least-squares algorithm are employed for the states and input force estimation. This method requires a full state measurement, which not be feasible in the B-WIM system. Then, Ma and Ho [21] extended their previous works to nonlinear systems by using an extended Kalman filter in conjunction with the least-squares estimator. Lourens et al. [22] proposed an augmented Kalman filter (AKF) for dynamic force identification in a combined deterministic-stochastic setting, and applied the method to identify the input forces on a steel I-beam. Azam [23] presented a dual Kalman filter approach for estimating the input and states of a linear state-space model and validated the method based on field measurements from a 39-story tower. Zhi, 2016 [24] presents a Kalman-filter-based estimation algorithm for identification of wind loads on a super-tall building using limited structural responses. The proposed inverse method allows estimating the unknown wind loads and structural responses of a super-tall building using limited acceleration measurements. Further, the target forces in most of this previous works were stationary loads acting at a specific DOF while vehicle loads acting on a bridge structure space, location and time.

In this study, a procedure is developed to estimate moving loads on bridges using limited measured acceleration response, which will be called acceleration based B-WIM system. The system uses kalman-filter-based estimation algorithm to estimate the state vector, and then applying moving force identification algorithm to estimate the moving forces.

2. Structural response estimation using Kalman filter

2.1. State space equation

The force-induced vibration of a bridge can be represented by the following equation of motion:

$$M\ddot{U} + C\dot{U} + KU = F \quad (1)$$

Where U,$\dot{U}$, and $\ddot{U}$ denote the vectors of displacement, velocity, and acceleration, respectively. M, C, and K are the bridge mass matrix, damping matrix, and stiffness matrix, respectively. F is the time history vector of vehicle load. Using the modal response of structure, Equation (1) can be transformed to modal space (Equation(2)).

$$\ddot{Z}_i + 2\xi_i\omega_i\dot{Z}_i + \omega_i^2 Z_i = \Phi_i^T F = f_i \quad (i = 1, 2, \ldots, N) \quad (2)$$

Where Φi denoted the modal shape of the i th mode. $Z_i, \dot{Z}_i, \ddot{Z}_i$, and f_i are the modal displacement, velocity, acceleration and moving force of the *i th* mode, respectively. N is the total number of modes,ξ_i, and ω_i are the damping ration and natural frequency of the *i* the mode. The modal acceleration response can be approximately calculated from limited measured acceleration response using the pseudoinverse of the mode shape matrix (Equation (3)).

$$\ddot{Z}_{qx1} = \left(\Phi_{pxq}\right)^+ \ddot{U}_{px1} \quad (3)$$

Where P denotes the number of measurements, and q is the number of modes considered. The error between the exact and estimated modal acceleration responses can be minimized by choosing the measurements number P exceeding the number of modes governing the structural responses [25].

In the modal space, the state space equation and the modal output (Y) obtained from the acceleration response can be represented using Equation (4).

$$\dot{\lambda}(t) = A_i\lambda_i(t) + B_i f_i \quad (4.1)$$

$$Y_i(t) = H_i\lambda_i(t) + D_i f_i \quad (4.2)$$

Where the system matrix $A_i, \lambda_i, \ f_i$ and B_i

$$A_i = \begin{bmatrix} 0 & 1 \\ -\frac{K_i}{M_i} & -\frac{C_i}{M_i} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\xi_i\omega_i \end{bmatrix},$$

$$\lambda_i = [Z_i \, \dot{Z}_i]^T, \quad f_i = \frac{F_i}{M_i}, \quad B_i = [0\,1]^T$$

For acceleration measurements, H_i and D_i matrices are defined as $\left[-\omega_i^2 \ -2\xi_i\omega_i\right]$, and [1] respectively. Equation (4), discretized over time intervals of length Δt (Equation (5)).

$$\lambda\,(t + 1) = \Psi_i\lambda_i(t) + \Gamma_i f_i(t) \quad (5.1)$$

$$Y_i(t) = H\,\lambda_i(t) + D_i f_i(t) \quad (5.2)$$

Where Ψ_i denotes the state transition matrix and equal to $e^{A_i\,\Delta t}$. Γ_i represents the process noise matrix and can be calculated using Equation (6).

$$\Gamma_i = [\Psi_i - I]\ A_i^{-1}B_i \quad (6)$$

2.2. State vector estimation using Kalman Filter

Based on the Kalman filter for the discrete-time state space system of Equations (5), the state vector $\lambda_i(t)$ can be estimated by the following equations (Equations 7-12) [26–28].

$$\hat{\lambda}(t/t-1) = \Psi\hat{\lambda}(t-1) + J(t-1) \times \left[Y(t-1) - H\hat{\lambda}(t-1)\right] \quad (7)$$

$$\hat{\lambda}(t) = \lambda(t/t-1) + G(t)\left[Y(t) - H\hat{\lambda}(t/t-1)\right] \quad (8)$$

$$J(t-1) = \Gamma Q(t-1)D^T \times \left[D\,Q(t-1)D^T + R(t-1)\right]^{-1} \quad (9)$$

$$P(t/t-1) = [\Psi - J(t-1)H]\,P(t-1) \times [\Psi - J(t-1)H]^T + \Gamma\,Q(t-1)\Gamma^T - J(t-1)\,DQ(t-1)\Gamma^T \quad (10)$$

$$G(t) = P(t/t-1)H\,[HP(t/t-1)H^T + DQ(t)D^T + R(t)]^{-1} \quad (11)$$

$$P(t) = [I - G(t)H]\;P(t/t-1) \quad (12)$$

where $G(t)$ is the Kalman filter gain matrix at time instant t. $P(t)$ denotes the filter's error covariance matrix, $J(t)$ is the a priori gain matrix. The filter is initialized using Equation (13-14):

$$\hat{\lambda}(0) = E[\lambda(0)] \quad (13)$$

$$P(0) = E\left\{\left[\lambda(0) - \hat{\lambda}(0)\right]\left[\lambda(0) - \hat{\lambda}(0)\right]^T\right\} \quad (14)$$

In the estimation process, Kalman filter requires priori knowledge of the covariance matrices of the input force $Q(t)$ and measurement noise $R(t)$. Usually, these covariance matrices can be assumed to be constant matrices [19, 29]. In this study, the following assumptions are adopted.

$$Q(i) = C_Q I \quad (15)$$

$$R(i) = I \quad (16)$$

In which I is an identity matrix, and C_Q is an adjustment factor, it should be large number. The displacement and velocities time history (state vector) are identified as

$$U_{nx1} = \phi_{nxq}\hat{Z}_{qx1} \quad (17)$$

$$\dot{U}_{nx1} = \phi_{nxq}\hat{\dot{Z}}_{qx1} \quad (18)$$

Where n is the number of estimated DOFs.

3. Moving force identification (MFI) algorithm

The MFI algorithm uses inverse dynamics theory to back-calculate a complete time force history for axles or wheels that move on the bridge. The algorithm adopted in this paper is that used by González et al. [14] who improve the work of Law et al. [8] by applying the first-order regularization technique. The first order system is defined by Equations (19-21).

$$\begin{Bmatrix} X \\ g \end{Bmatrix}_{j+1} = \begin{bmatrix} [M] & [P] \\ [0] & [I] \end{bmatrix} \begin{Bmatrix} X \\ g \end{Bmatrix}_j + \begin{Bmatrix} [0] \\ [I] \end{Bmatrix} \{r\}_j, \quad (19)$$

$$[P]_j = \left[[A]^{-1}\;[M - I]\right] \begin{bmatrix} 0 \\ [\Phi]^T[L]_j \end{bmatrix} \quad (20)$$

$$[M] = \exp([A] * h),\; [A] = \begin{bmatrix} 0 & I \\ -[\Omega] & -2\xi[\Omega] \end{bmatrix} \quad (21)$$

Where X is the degree of freedom vector, g is the vector of applied vehicle forces, and $\{r\}_j$ is the increment change in the force between time step j and time step $j+1$. $[\Phi]$ is the modal matrix of normalized eigenvectors, $[L]$ is a time varying location matrix, which defines the load's position at each time step, h is the time step, $[\Omega]$ is a diagonal matrix containing the natural frequencies and ζ is the percentage damping. The force increment $\{r\}_j$ can be define from the following last square minimization with Tikhonov regularization (Equation (22)).

$$\sum_{j=1}^{m}\left(\left\{\{d_{me}\}_i - [Q]\{X\}_j\right\},\;[W]\left\{\{d_{me}\}_j - [Q]\{X\}_j\right\} + \{r\}_j,\;[B]\{r\}_j\right) \quad (22)$$

where d_{me} is the measurement vector (usually strain), [Q] is a vector to relate the measurements to the degree of freedom, (x, y) denotes the vector product of x and y, [W] is an $m \times m$ identity matrix in the least squares error. [B] is a regularized matrix equal to

λ; $[I]$, where λ is the optimum regularization parameter, and its value is usually obtained using the L-curve method [30–33]. It should be mentioned that the MFI algorithm requires a calibrated FE model to extract the required matrices.

4. Computational procedure

The procedure to identify the moving forces on bridge using the new algorithm is summarized as follows:

- Converting limited measured acceleration responses to modal ones using the pseudoinverse of the mode shape matrix (Equation (3)).
- Estimating the unknown modal state vectors from the modal dynamic responses by the Kalman filter equations (Equations (7)–(12)).
- Estimate structural displacement and velocities using Equations (17) and (18).
- Apply MFI algorithm to estimate the moving load (section 3).

5. Numerical simulation

5.1. Quarter car example

As shown in Fig. 1, a simply supported bridge subject to a moving quarter-car model is taken as an example for numerical simulation. The quarter-car travels with constant speed crossing a 20-m approach distance followed by a 15-m simply supported finite element (FE) bridge. The bridge is modeled with 1D Euler–Bernoulli finite beam elements with two degrees of freedom per node, vertical translation, and rotation. The vehicle masses are represented by a sprung mass, m_s, and un-sprung mass, m_a represents the vehicle axle mass and body mass respectively. The Degrees of Freedoms (DOFs) that correspond to the bouncing of the sprung and the axle masses are, u_s, and u_a, respectively. The properties of the quarter-car and the bridge are listed in Table 1 and based upon the work of Cebon [34] and Harris, OBrien [35]. The road surface profile is not considered in this simulation, and its effect has been studied before by Dowling [36] who states that MFI is relatively insensitive to road roughness. The dynamic interaction between the vehicle and the bridge that showing how bridge and vehicle properties affect the response is implemented in MATLAB [37, 38] based on the

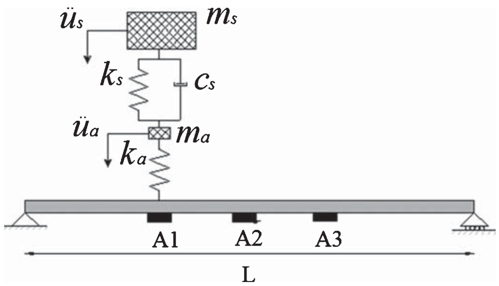

Fig. 1. Theoretical quarter car model on simply supported beam.

Table 1
Bridge properties

Vehicle properties		Bridge properties	
ms	14000 kg	Span	15m
ma	1000 kg	Density	4800 kg/m^3
k_s	2e5 N/m	Width	4.0 m
K_a	2.75e6 N/m	Depth	0.8 m
c_a	1e4 N s/m	Modulus	2.75×10^{10} N/m^2

contact force concept adopted by Yang et al [39]. and Gonzalez [40]. Unless otherwise mentioned, the used scanning frequency is 1000 Hz.

In order to simulate the polluted measurements, white noise is added to the calculated acceleration. The noisy response is calculated as following formula:

$$A_{nosiy} = A_{calculated} + E_p N_{noise}\, \sigma\left(A_{calculated}\right) \tag{23}$$

Where E_p represents noise level choosing as 0.01, 0.05 and 0.10, respectively which represent the noise level in different types of Commercial accelerometers; N_{noise} is a standard normal distribution vector with zero mean value and unit standard deviation. A_{nosiy} is the noisy acceleration, and $\sigma\left(A_{calculated}\right)$ is its standard deviation. The relatively percentage error (RPE) values between the true moving force and the identified force are defined as follow:

$$RPE = \frac{\left\| F_{calculated} - F_{true} \right\|}{\left\| F_{true} \right\|} \times 100 \tag{24}$$

Where $F_{calculated}$, F_{true} are the identified force vector and actual force vector, respectively.

The acceleration measurements has been extracted from the VBI model at three different locations 1/3, 1/2, and 2/3 of the bridge span, which represent A1, A2, and A3 respectively in Fig. 1. The quarter car model crossed the bridge with five different speeds (10, 15, 20, 25 and 30 m/s). Three levels of noise (1%, 5%, and 10%) have been added to each acceleration signal. Figure 2 shows the acceleration records when

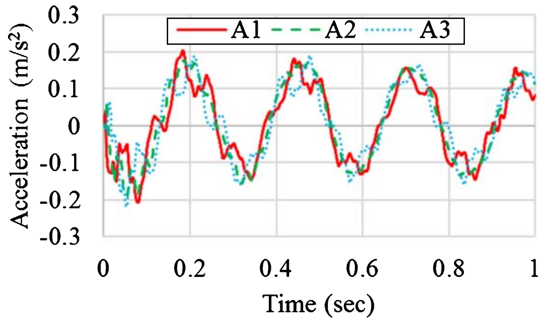

Fig. 2. Acceleration record at the three location (A1 = acceleration at location No. 1.

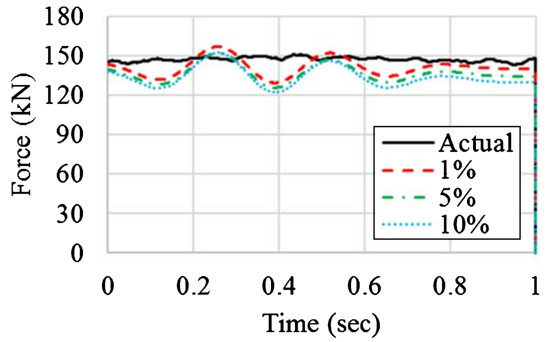

Fig. 4. Force history according to noise level (1%, 5%, and 10%) compaing with the input force (Actual).

the quarter car crossed the bridge at 15 m/s speed. In this study, three acceleration responses at locations A1, A2, and A3 are used for the state vector estimation using KF algorithm. The number of modes governing the structure response has been defined to be one for the simply supported beam. This is based on the proper orthogonal decomposition (POD) [41, 42].

Since there is one moving force only on the bridge, the minimum number of measurements needed for MFI algorithm is defined to be one according to Rowely (Rowely 2007), and in this study the estimated displacement at the mid-span only has been used to estimate the force history.

5.1.1. *Effect of noise*

The noise is added to the simulated acceleration at each 'measurement' location as a white Gaussian noise. In total three levels of noise are analyzed for their effect on the accuracy of the MFI algorithm. Firstly, KF algorithm is applied to estimate the displacement at the same locations of the measurements using the noisy acceleration. Figure 3 shows the actual and the estimated displacement at location A1 and A2 for different levels of noise. It can be seen from these figures that the level of matching between the actual and the estimated displacement is varied according to noise level.

The displacement at the mid-span is used for the MFI work. The middle 60% of the calculated force history (Fig. 4) is averaged to calculate the car load [7, 15]. The error in estimating the GVW according to noise level are –2.8%, –6.0%, and –7.5% for 1%, 5%, and 10% noise respectively.

5.1.2. *Effect of number of modes*

The effect of the mode shapes number that used to estimate the sate vector has been addressed in this section. Three different number of modes have been used to estimate the displacement time history using the three acceleration response. Figure 5-a, and 5-b show the estimated displacement at location A1, and A2 respectively. It can be noted that, the estimated displacement is less accurate when using one mode only while a few enhancement is achieved when the number of modes has increased, which mean that the number of modes do not affect the estimation process as long as it exceed the number of modes governing the structural responses.

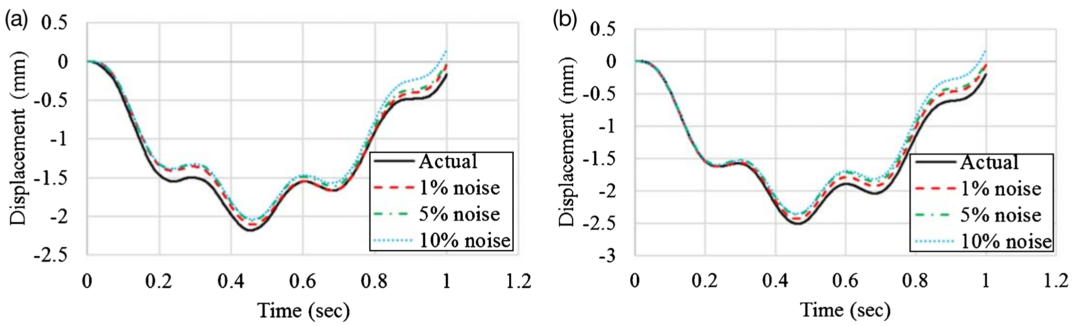

Fig. 3. Estimated displacement with different noise level at locations (a) A1, (b) A2.

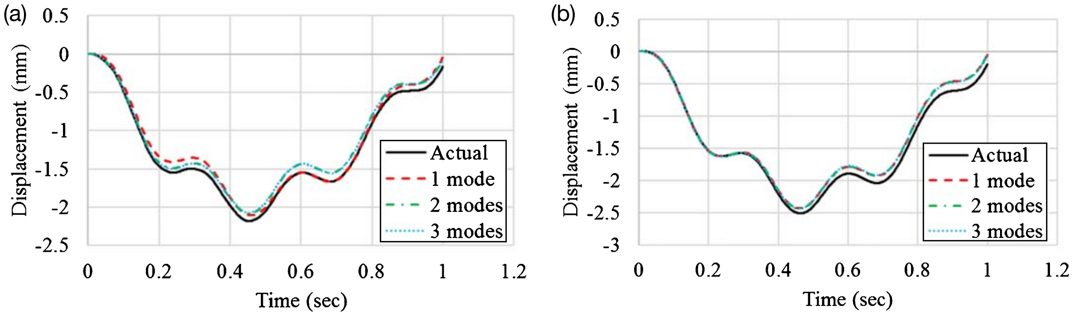

Fig. 5. Estimated displacement using different number of modes (a) Location “A1”, (b) Location “A2”.

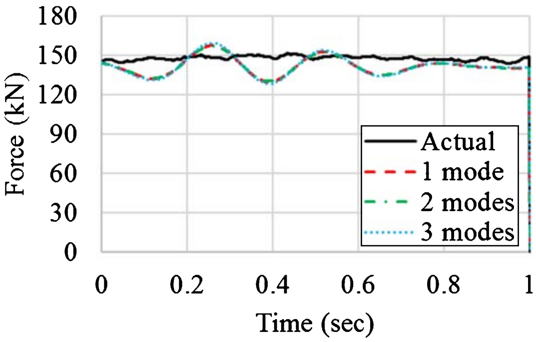

Fig. 6. Force history according to the number of modes compaing with the actual force.

The force history according to the number of modes used is illustrated in Fig. 6. The middle 60% of the calculated force history is averaged to calculate the car load. The error in estimating the GVW according to modes number are –2.8%, –3.9%, and –2.8% for 1, 2, and 3 modes respectively.

5.1.3. *Effect of measurement number*

Three different measurements configuration have been used to estimate the displacement and the force history. Figure 7-a, shows the estimated displacement at one third of the span (location A1), and mid-span (location A2) respectively. It can be noted that, the estimated displacement dose not match the actual one when the acceleration at the mid-span only (1-measurement) has been used. However, when using 2, and 3-measurements (acceleration at one third the span included) the estimated displacement approached the actual one. Also, the estimated displacement does not depend on the measurement location and this is clear in Fig. 7-a when the measurement at location A2 used (1- measurement case) the displacement at location A1 is achieved with the same accuracy as location A2.

The force history according to measurements number that used for displacement estimation is illustrated in Fig. 8. The middle 60% of the calculated force history is averaged to calculate the car load. The error in estimating the GVW according to the number of acceleration measurements used are –7.1%, –3.2%, and –2.8% for 1, 2, and 3 measurements respectively.

5.1.4. *Effect of velocity*

The effect of vehicle velocity on the accuracy of the moving force identification algorithm is analyzed in this section. Again, three acceleration measurements

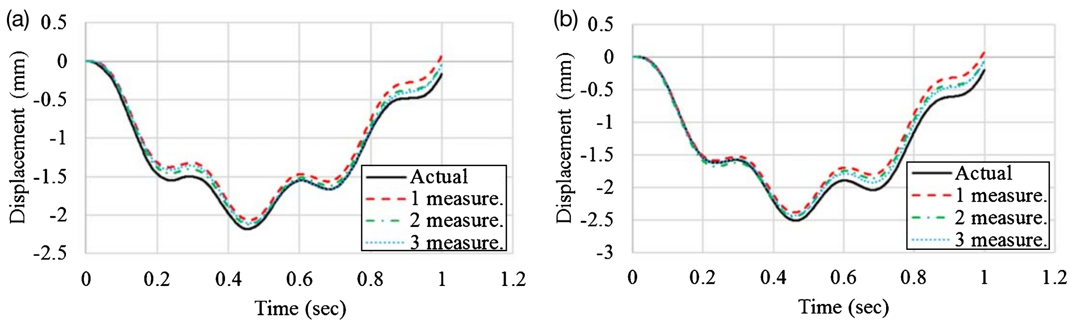

Fig. 7. Estimated displacement at locations a) A1, (b) A2, using different measurement number (1 measure. = One measurement, 2 measure. = two measurement . . . etc.).

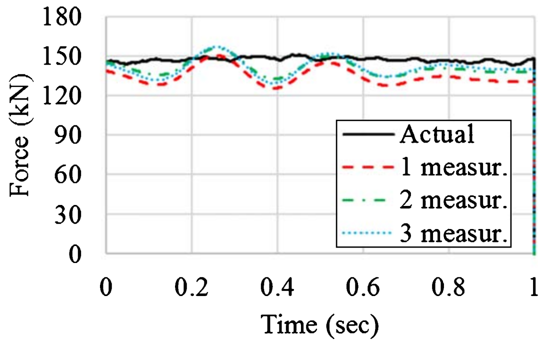

Fig. 8. Force history according to the number of measurements used compaing with the actual force.

Table 2
Percentage error in the predicted force for different speeds

Speed (m/s)	10	15	20	25	30
GVW (Kn)	145.1	144.3	143	142.5	143
Error %	–2.1	–2.8	–4.2	–4.7	–4.2

and the first mode of vibration are considered. The velocity is varied from 10 m/s to 30 m/s in increments of 5. Table 2 illustrate the error in detecting the force history for five different speed, and as expected, the error increases while the speed increase.

6. Conclusion

In conclusion, this paper propose a B-WIM system based on acceleration data instead of the strain data. The system allow to use limited acceleration measurements to estimate the weight of moving vehicles. The new approach was presented based on the Kalman filter and MFI algorithm. Kalman filter has been used to estimate the state vector, and then MFI algorithm is applied to estimate the force history. The effectiveness and performance of the proposed method were investigated based on the numerical simulations of quarter car model on simply supported bridge. The effect of noise, speed, measurement number, and modes number has been addressed in this paper. The error in force history using the acceleration ranged from 2% to 7.5% based on the speed, number of modes used, number of measurements and the noise level. The highest error found in case of high noise level and when using one sensor only. All other cases have error less than 5.0 % which is acceptable.

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