

# Enhanced fracture toughness of boron carbide from microalloying and nanotwinning



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## ABSTRACT

Fracture toughness is one of the most important mechanical properties of structural materials. Particularly, enhancing the fracture toughness of super-hard materials is essential for their applications. Here, we applied density functional theory to examine how the microalloying and nanotwinning affect the fracture toughness of superhard boron carbide ( $B_4C$ ). We find that replacing C-B-C chains with two-atom chains especially weakly coupled O atoms can significantly improve the fracture toughness of  $B_4C$ . In addition, inserting nanotwins can significantly enhance the fracture toughness of  $B_4C$  and boron phases. Our results provide useful information to design boron based superhard materials with enhanced fracture toughness.

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Fracture toughness ( $K_c$ ), describing as the materials capability of resisting cracks propagation, is one of the most important mechanical properties [1,2]. In particular, it is essential to measure and predict the fracture toughness of brittle materials where cracks rather than dislocations and twins are most involved in the mechanical failure and hardness measurement [2]. For decades, many experimental techniques, such as double-torsion [3], fractography [4] and indentation [5–7], have been developed to determine this physical quantity. The choice for measuring fracture toughness depends on resources, availability of time, and level of precision required [7]. Among these techniques, the Vickers indentation fracture (VIF) test, imposing the load from hardness test machine, has been a popular one for brittle materials because of its expediency [5,6]. At a VIF test, the sample generates cracks and the length of cracks, along with elastic modulus, applied load and indent half-diagonal, which are used to determine the fracture toughness of the sample [7].

Besides the above experimental methods, many theoretical studies have predicted the fracture toughness of materials based on various fracture modes [2,8–11]. It is well known that the applied forces on materials could enable a crack to propagate in three ways [11,12]: (1) Mode I, known as opening mode, occurs when the applied tensile stress is normal to the plane of crack; (2) Mode II, known as sliding mode, dominates when the applied shear stress is normal to the leading

edge of crack but in the plane of crack; (3) Mode III, known as tearing mode, happens when the applied shear stress is parallel to the leading edge of crack but out of the crack plane. For the materials with Mode I crack, the strength is governed by the Griffith theory in which the theoretical fracture toughness is determined by the relation:  $K_{Ic} = 2\sqrt{\gamma_s G/(1-\nu)}$  where  $\nu$  is Poisson's ratio,  $G$  is shear modulus and  $\gamma_s$  is surface energy [13]. However, a fracture toughness  $K_c$  measured from experiments is normally considerably different from the theoretical  $K_{Ic}$  because defects such as dislocations under shear stress always affect the  $K_{Ic}$ . In addition, the dislocations emission is generally a major factor in determining the intrinsic ductile or brittle behaviors of materials [14]. Thus, in order to estimate the resistance of a material to dislocation involved fracture, Rice et al. [9] applied unstable stacking fault energy ( $\gamma_{us}$ ) to derive the fracture toughness for Mode II and Mode III:  $K_{IIC} = 2\sqrt{\gamma_{us} G/(1-\nu)}$  and  $K_{IIIC} = \sqrt{2\gamma_{us} G}$ . Although these methods have been established for a decade, their applications to superhard materials are very limited and the accuracy is not well validated.

To significantly accelerate the discovery and optimization of novel superhard materials with enhanced fracture toughness, it is urgent to employ quantum-mechanics (QM) based rational design (in silico optimization) so that the most promising candidates can be identified before experiments. This computational design requires accurate prediction of fracture toughness from QM simulations. To validate the fracture toughness modes, we use boron carbide ( $B_4C$ ) as a prototype material because of such excellent properties as super strong, low density, high melting point and high abrasion resistance [15–17]. Moreover,

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$B_4C$  has suffered from the brittle failure under impact, characterized by its low fracture toughness, which restrict its extended engineering applications [18–20]. Thus, it is essential to predict the fracture toughness of  $B_4C$  and provide rational design to improve it.

The brittle failure of  $B_4C$  has been investigated by many theoretical efforts [18,20–27]. In particular, recent studies indicate that the failure of  $B_4C$  arises from icosahedra ( $B_{11}C$ ) cracking [18], which leads to the formation of high density amorphous shear bands [21]. Recent theoretical studies suggested that the mechanical behaviors of  $B_4C$  can be tuned by nanotwinning or modifying the chain and icosahedral structures (microalloying) [21–27]. However, it is not well established how the fracture toughness was affected by these modifications.

Here, in order to examine how microalloying and nanotwinning affect the fracture toughness of  $B_4C$ , we first predicted the theoretical fracture toughness of  $B_4C$  and its microalloyed structures of  $B_{13}C_2$ ,  $B_{14}C$ ,  $B_6O$ ,  $B_{12}P_2$ ,  $o\text{-}B_{10}Si_2Si_2$ . Then, the nanotwinned structures including nanotwinned  $B_4C$  ( $\tau\text{-}B_4C$ ),  $B_6O$  ( $\tau\text{-}B_6O$ ) were investigated and compared with their crystalline structures. Finally, we examined the pure boron phases including  $\alpha\text{-}B_{12}$ ,  $\gamma\text{-}B_{28}$ ,  $\beta\text{-}B_{106}$  and  $\tau\text{-}B_{106}$  to illustrate how the various boron structures affect the fracture toughness. Based on our results, the design principles of enhanced toughness of  $B_4C$  are established.

It is well established that shear has a weaker resistance against external deformation compared to tension or compression for the selected super-hard materials because their shear modulus is much lower than bulk modulus [24,25,28–32,34]. Thus, we focus on the fracture toughness of Mode II and Mode III in this study. According to previous work by Rice et al. [9], the fracture toughness of Mode II and Mode III fracture could be derived as [9,10]:

$$K_{IIc}^2 = \frac{2\gamma_{us}G}{1-\nu},$$

$$K_{IIIc}^2 = 2\gamma_{us}G,$$

where  $\nu$  is Poisson's ratio and can be calculated from bulk modulus (B) and shear modulus (G) through equation:  $\nu = \frac{3B-2G}{2(3B+2G)}$ . The unstable stacking energy ( $\gamma_{us}$ ) can be calculated by integrating the engineering shear-stress-displacement curves which could be derived from the true shear-stress-shear-strain relationships. The detailed estimation method could be found in previous literature [25]. The details of DFT simulations are provided in the supporting information (SI).

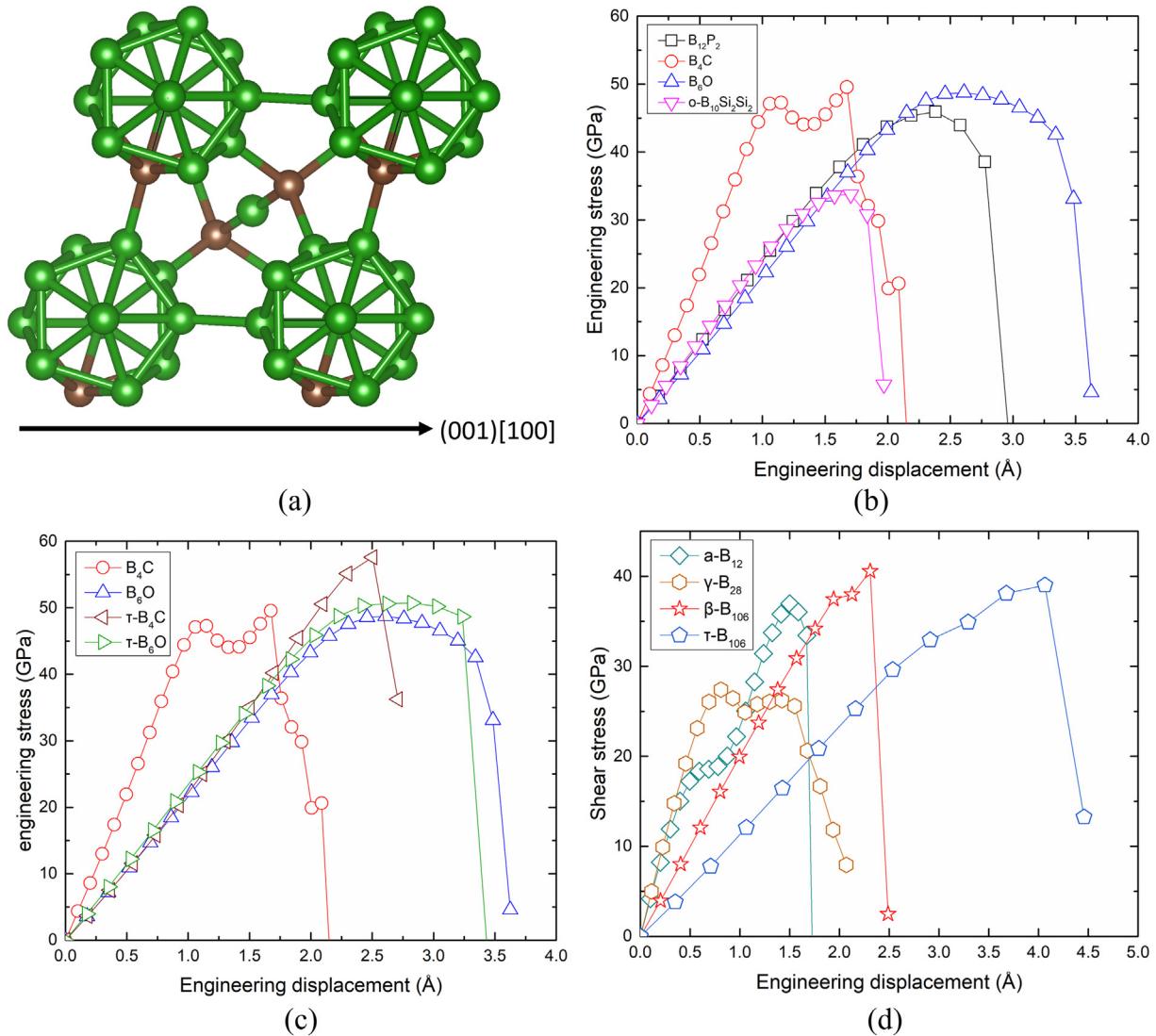
To compute the fracture toughness, we first derive the engineering shear-stress-displacement curves from the true shear-stress-shear-strain relationships from the previous shear deformation on selected materials (Fig. S1(a) and Fig. S2 of SI).  $B_4C$  and its related compounds  $\tau\text{-}B_4C$ ,  $B_{13}C_2$ ,  $B_{14}C$ ,  $B_{12}P_2$ ,  $\tau\text{-}B_6O$ , and  $o\text{-}B_{10}Si_2Si_2$  have the same plausible slip system of (001)[100] [18,22,23,25,26,28–30] (Fig. 1a) while the most plausible slip system for  $B_6O$  is changed to (011)[211] [24]. For the boron phases of  $\alpha\text{-}B_{12}$ ,  $\beta\text{-}B_{106}$ ,  $\gamma\text{-}B_{28}$  and  $\tau\text{-}B_{106}$ , the most plausible slip system is (001)[100] [31–33]. The converted shear-stress-displacement curves for all selected materials are summarized in Fig. 1 (b–d). Then we used the predicted bulk modulus (B) and shear modulus (G) of all selected structures from previous literature, as listed in Table 1 [24,25,28–32,34]. The computed fracture toughness is summarized in Fig. 2 and Table 1.

The low fracture toughness of  $B_4C$  arises from its unique crystal structure in which the ground state configuration is  $(B_{11}C)^{-1}\text{-}(C\text{-}B^+\text{-}C)$  where  $B_{11}C$  is the icosahedral cluster and the  $C\text{-}B^+\text{-}C$  represents a 3-atom chain, as shown in Fig. 1(a) [16,18,35,36]. Our predicted fracture toughness of  $B_4C$  are  $K_{IIc} = 1.62 \text{ MPa m}^{1/2}$  and  $K_{IIIc} = 1.47 \text{ MPa m}^{1/2}$ , which agree well with experimental measurements although the loading conditions are different in experiments and defects exist in experimental samples [33–35]. It is worth to note that the measured fracture toughness in experiments depends on the samples synthesized from

different approaches. The measured fracture toughness of fully dense  $B_4C$  is  $4.7 \text{ MPa m}^{1/2}$  based on radial-median cracks [37]. This sample prepared by pressure assisted sintering has a high density and avoids undesirable grain growth, leading to better properties than usual samples. In another experiment, the static mechanical behaviors of spark plasma sintering (SPS)  $B_4C$  samples were measured as a function of porosity [38]. The obtained fracture toughness ( $K_c$ ) is between  $3.9$  and  $4.9 \text{ MPa m}^{1/2}$  which were higher than the values reported for samples produced by pressureless sintering and Plasma Pressure Compaction technique ( $2.8\text{--}3.6 \text{ MPa m}^{1/2}$ ) [39,40]. In addition, these experimental values were measured from Vickers indentation experiments in which some uncertainties exist in measuring crack lengths and plastic behaviors beyond critical strain [41]. These uncertainties account for the discrepancy between our theoretical prediction and experimental measurements.

To understand how microalloying affects the fracture toughness of  $B_4C$ , we examined the fracture toughness of various  $B_4C$  based compounds:  $B_6O$ ,  $B_{12}P_2$ ,  $B_{13}C_2$ ,  $B_{14}C$  and  $o\text{-}B_{10}Si_2Si_2$ . These compounds can be considered as the modification of  $B_4C$ . Replacing C atoms in  $B_{11}C$  icosahedra leads to the configuration  $B_{13}C_2$  and replacing C-B-C chain with C-B-B chain in  $B_{13}C_2$  leads to the formation of two  $B_{14}C$  configurations: linear chain structure and kink chain structure. The kink chain structure is considered here since it is more stable than linear chain structure. Furthermore, replacing C-B-C chains with oxygen chains (OO) and phosphorus chains (P—P) in  $B_{13}C_2$  lead to  $B_6O$  and  $B_{12}P_2$ , respectively. It is worth noticing that two chain oxygen atoms in  $B_6O$  are not bonded to each other while P atoms in P—P are bonded to each other [25,30]. Si doping is considered to be a practical approach to improve the ductility of  $B_4C$  [23,42]. Here we consider the  $o\text{-}(B_{10}Si_2)Si_2$  compounds in which all C atoms in  $B_4C$  are replaced by Si atoms and an additional Si is replacing B in icosahedra to satisfying Wade's rule [29]. Among all these compounds, the fracture toughness of  $B_6O$  ( $K_{IIc} = 2.13 \text{ MPa m}^{1/2}$  and  $K_{IIIc} = 1.96 \text{ MPa m}^{1/2}$ ) and  $B_{12}P_2$  ( $K_{IIc} = 1.77 \text{ MPa m}^{1/2}$  and  $K_{IIIc} = 1.64 \text{ MPa m}^{1/2}$ ) are larger than those of other compounds. Particularly,  $B_6O$  shows the highest fracture toughness among all selected compounds. This is because  $B_6O$  and  $B_{12}P_2$  display larger displacements ( $\Delta d = 3.49 \text{ \AA}$  for  $B_6O$  and  $\Delta d = 2.78 \text{ \AA}$  for  $B_{12}P_2$ ) before fracturing compared to other compounds. The large fracture displacements suggest that  $B_6O$  and  $B_{12}P_2$  are more ductile than other compounds, which is in agreement with our previous studies showing that replacing 3-atoms chain with 2-atoms chain can improve the ductility [25]. In addition to the large displacement,  $B_6O$  also has the largest shear modulus ( $G = 210.9 \text{ GPa}$ ) among all selected compounds, which also accounts for its highest fracture toughness. It is worth noticing that no icosahedron is broken in  $B_{12}P_2$  and  $B_6O$  during large shear deformation, making them attractive superhard compounds [25]. For other modified structures, such as  $B_{13}C_2$ ,  $B_{14}C$ , and  $o\text{-}B_{10}Si_2Si_2$ , their fracture toughness are lower than that of  $B_4C$  due to the lower shear modulus or critical shear stress. Their fracture modes are the disintegration of icosahedra, which is similar to  $B_4C$  [21,26,28,29]. Particularly,  $o\text{-}(B_{10}Si_2)Si_2$  displays much lower fracture toughness ( $K_{IIc} = 1.05 \text{ MPa m}^{1/2}$  and  $K_{IIIc} = 0.96 \text{ MPa m}^{1/2}$ ) than those of  $B_4C$  ( $K_{IIc} = 1.62 \text{ MPa m}^{1/2}$  and  $K_{IIIc} = 1.47 \text{ MPa m}^{1/2}$ ),  $B_{14}C$  ( $K_{IIc} = 1.37 \text{ MPa m}^{1/2}$  and  $K_{IIIc} = 1.26 \text{ MPa m}^{1/2}$ ) and  $B_{13}C_2$  ( $K_{IIc} = 1.34 \text{ MPa m}^{1/2}$  and  $K_{IIIc} = 1.20 \text{ MPa m}^{1/2}$ ), as shown in Fig. 2(a). This arises from the very low critical shear stress of  $o\text{-}(B_{10}Si_2)Si_2$ , as shown in Fig. 1(b) and Fig. S1(a). These results suggested that it is essential to possess both the large displacement and critical shear stress to achieve high fracture toughness.

Nanotwins has been widely observed in  $B_4C$  and related materials [43,44]. In order to understand how nanotwins affect the fracture toughness of  $B_4C$ , we examined the fracture toughness of nanotwinned  $B_4C$  ( $\tau\text{-}B_4C$ ) and  $B_6O$  ( $\tau\text{-}B_6O$ ), as shown in Fig. 2(b). Comparing the fracture toughness of  $\tau\text{-}B_4C$  with  $B_4C$ , we found that even though  $\tau\text{-}B_4C$  shows similar elastic modulus to  $B_4C$  (Table 1), the fracture toughness ( $K_{IIc} = 1.82 \text{ MPa m}^{1/2}$  and  $K_{IIIc} = 1.65 \text{ MPa m}^{1/2}$ ) of  $\tau\text{-}B_4C$  is higher than those of  $B_4C$  ( $K_{IIc} = 1.62 \text{ MPa m}^{1/2}$  and  $K_{IIIc} = 1.47 \text{ MPa m}^{1/2}$ ).



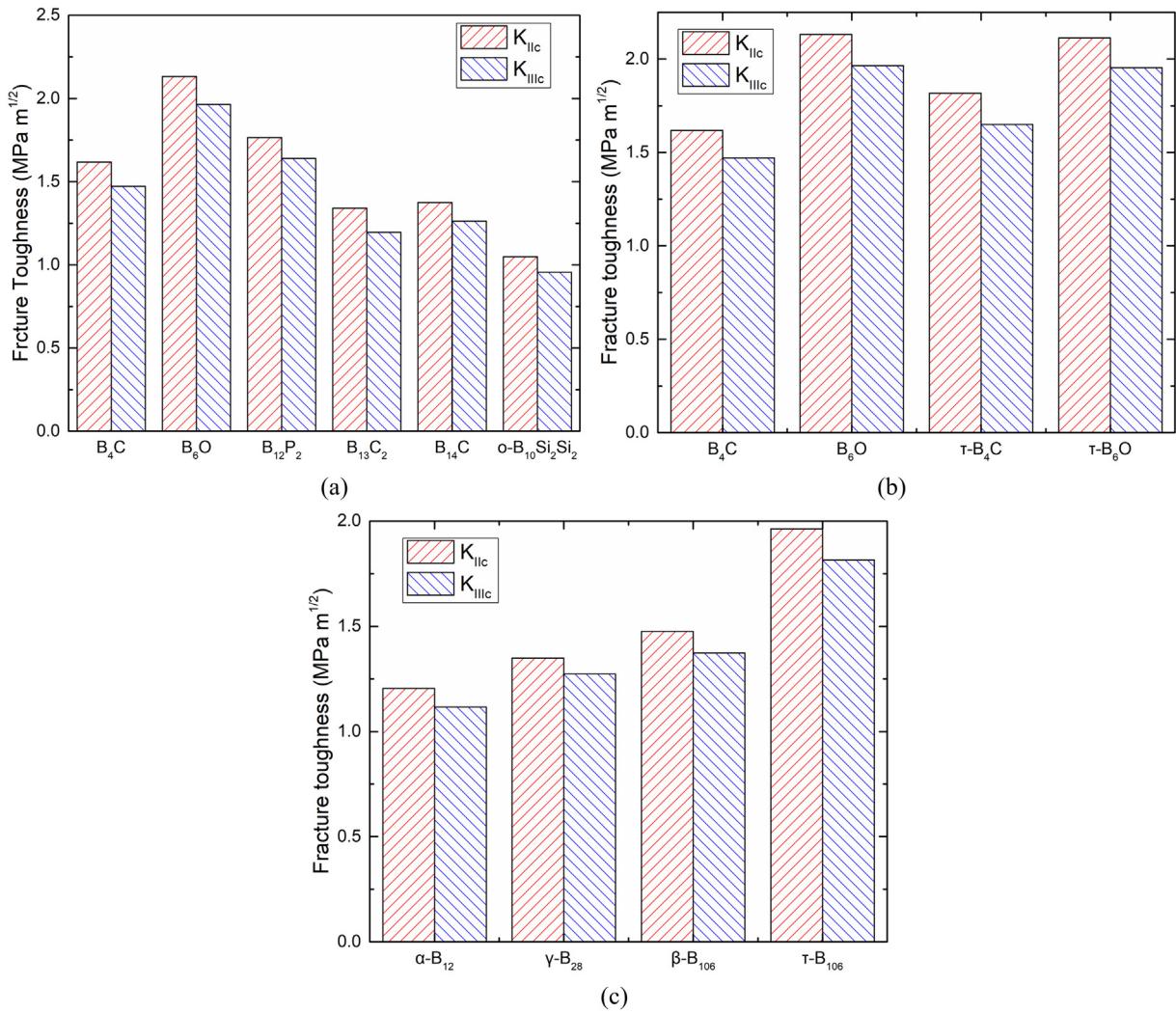
**Fig. 1.** The structure of B<sub>4</sub>C with most plausible slip system of (001)[100] and the ideal engineering shear-stress-displacement relationships for selected superhard materials. (a) The structure of B<sub>4</sub>C; (b-d) shear-stress-displacement relationships of (b) the B<sub>4</sub>C based compounds including B<sub>4</sub>C, B<sub>6</sub>O, B<sub>12</sub>P<sub>2</sub> and o-B<sub>10</sub>Si<sub>2</sub>Si<sub>2</sub>; (c) nanotwinned B<sub>4</sub>C (τ-B<sub>4</sub>C), B<sub>6</sub>O (τ-B<sub>6</sub>O); (d) elemental boron phases including α-B<sub>12</sub>, γ-B<sub>28</sub>, β-B<sub>106</sub> and nanotwinned τ-B<sub>106</sub>.

This is due to the higher critical engineering shear stress of τ-B<sub>4</sub>C (57.60 GPa) compared to B<sub>4</sub>C (47.26 GPa), as shown in Fig. 1(c). For B<sub>6</sub>O, the fracture toughness of nanotwinned B<sub>6</sub>O (τ-B<sub>6</sub>O) ( $K_{Ic} = 2.11 \text{ MPa m}^{1/2}$  and  $K_{IIIc} = 1.95 \text{ MPa m}^{1/2}$ ) is very similar to rhombohedral

B<sub>6</sub>O ( $K_{Ic} = 2.13 \text{ MPa m}^{1/2}$  and  $K_{IIIc} = 1.96 \text{ MPa m}^{1/2}$ ), as shown in Fig. 2 (b). This suggests that introducing nanotwins in B<sub>6</sub>O does not affect the fracture toughness because the τ-B<sub>6</sub>O has a slightly shorter displacement ( $\Delta d = 3.23 \text{ \AA}$ ) than rhombohedral B<sub>6</sub>O ( $\Delta d = 3.48 \text{ \AA}$ ) before fracture, as

**Table 1**  
Our estimated bulk modulus (B), shear modulus (G) and fracture toughness of Mode I ( $K_{Ic}$ ), Mode II ( $K_{IIc}$ ) and Mode III ( $K_{IIIc}$ ) for all examined structures. All values of B and G are given in GPa while values of  $K_{Ic}$ ,  $K_{IIc}$  and  $K_{IIIc}$  are given in MPa m<sup>1/2</sup>. The  $V_0$  is the equilibrium volume per atom (unit in Å<sup>3</sup>). A comparison of B and G with previous experimental results is also provided.

Structure	B	G	Experimental B	Experimental G	$V_0$ (Å <sup>3</sup> )	$K_{Ic}$	$K_{IIc}$	$K_{IIIc}$
B <sub>4</sub> C	238.0	199.0	240[51]	196[52]	7.268	3.029	1.618	1.471
B <sub>6</sub> O	232	210.9	222[53]	204[54]	7.386	3.087	2.132	1.964
B <sub>12</sub> P <sub>2</sub>	199.1	190.9	192[55]	207[56]	8.782	2.800	1.765	1.640
B <sub>13</sub> C <sub>2</sub>	216.3	154.2			7.469	2.553	1.340	1.196
B <sub>14</sub> C	212.3	188.7			7.309	2.788	1.375	1.262
o-(B <sub>10</sub> Si <sub>2</sub> )Si <sub>2</sub>	160.1	136.6			10.620	2.192	1.045	0.956
α-B <sub>12</sub>	211.7	200.8			7.249	2.868	1.204	1.117
γ-B <sub>28</sub>	223.0	236.0			6.995	3.173	1.349	1.273
β-B <sub>106</sub>	204.2	196.7			7.712	2.817	1.476	1.373
τ-B <sub>106</sub>	202.5	189.1			7.715	2.751	1.962	1.816
τ-B <sub>4</sub> C	238.2	197.6			7.270	3.020	1.817	1.650
τ-B <sub>6</sub> O	225.9	209.2			7.380	3.033	2.114	1.953



**Fig. 2.** The estimated fracture toughness of (a) the  $B_4C$  based compounds including  $B_4C$ ,  $B_6O$ ,  $B_{12}P_2$ ,  $B_{13}C_2$ ,  $B_{14}C$  and  $o\text{-}B_{10}Si_2Si_2$ ; (b) nanotwinned  $B_4C$  ( $\tau\text{-}B_4C$ ),  $B_6O$  ( $\tau\text{-}B_6O$ ); (c) elemental boron phases including  $\alpha\text{-}B_{12}$ ,  $\gamma\text{-}B_{28}$ ,  $\beta\text{-}B_{106}$  and nanotwinned  $\tau\text{-}B_{106}$ .

shown in Fig. 1(c). Thus, it is likely that introducing nanotwins in  $B_4C$  improves its fracture toughness, but it does not have much effects in  $B_6O$ .

The  $B_4C$  crystal structure is the modification of  $\alpha\text{-}B_{12}$  phase. To understand how the fracture toughness is affected by other possible boron structures, we also examined the fracture toughness of various boron phases, as shown in Fig. 2(c). We consider  $\alpha\text{-}B_{12}$ ,  $\beta\text{-}B_{106}$  and  $\gamma\text{-}B_{28}$  phases which have been identified as the most stable pure phases [45–47]. Comparing to  $\alpha\text{-}B_{12}$  ( $B_{12}$  icosahedron), the  $\gamma\text{-}B_{28}$  has a crystal structure containing two  $B_{12}$  icosahedra and two  $B_2$  pairs in the unit cell [32]. For  $\beta\text{-}B_{106}$ , the structure is very complex with 106 atoms and partially occupied sites (POS) in the unit cell, in which the main units are  $B_{12}$  icosahedra and icosahedral fused  $B_{28}$  clusters [48,49]. In addition, our recent study combined high resolution transmission electron microscopy (HRTEM) and QM simulation identified a  $\beta\text{-}B_{106}$  transformed twinlike structure, named  $\tau\text{-}B_{106}$  [33]. Here, we examined the fracture toughness of these four boron phases. Comparing the fracture toughness of these structures, the sequence from high to low is  $\tau\text{-}B_{106} > \beta\text{-}B_{106} > \gamma\text{-}B_{28} > \alpha\text{-}B_{12}$ , as shown in Fig. 2(c). The  $\tau\text{-}B_{106}$  with twinlike structure exhibits larger fracture toughness than those of other three boron phases because it undergoes larger elastic deformation before fracture, as shown in Fig. 1(d). The engineering shear stress increases to its maximum at the displacement  $\Delta d = 4.065 \text{ \AA}$ , which is much larger than those of other three boron phases. This indicates that nanotwins in elemental boron can improve its fracture toughness. In addition, the  $\beta\text{-}B_{106}$  shows a much higher fracture toughness ( $K_{Ic} =$

$1.48 \text{ MPa m}^{1/2}$  and  $K_{IIc} = 1.37 \text{ MPa m}^{1/2}$ ) compared to other two crystalline phases due to its larger critical shear strength (40.56 GPa) and critical displacement ( $\Delta d = 2.308 \text{ \AA}$ ) compared to  $\gamma\text{-}B_{28}$  (26.06 GPa,  $\Delta d = 0.809 \text{ \AA}$ ) and  $\alpha\text{-}B_{12}$  (36.97 GPa,  $\Delta d = 1.671 \text{ \AA}$ ) before fracturing, as shown in Fig. 2(c). Even though  $\gamma\text{-}B_{28}$  has the largest shear modulus (236.0 GPa) among these four boron phases, its lowest ideal shear strength lowers the fracture toughness, as shown in Fig. 1(d). However, its fracture toughness is still higher than that of  $\alpha\text{-}B_{12}$ .

Our results provide useful information of improving the ductility of  $B_4C$ . To achieve the enhanced ductility of  $B_4C$  under shearing loading, it is essential to (1) replacing the 3-atoms chains with 2-atoms chains; and (2) introducing the nanotwins in  $B_4C$ .

It is interesting to predict the theoretical fracture toughness of Mode I fracture ( $K_{Ic}$ ) for these superhard materials and compare with  $K_{Ic}$  and  $K_{IIc}$ . Here we computed the  $K_{Ic}$  of  $B_4C$  based materials using the recent developed method by Niu et al. [50] as below equation:

$$K_{Ic} = V_0^{1/6} \times G \times \left( \frac{B}{G} \right)^{1/2}$$

where  $V_0$  is the equilibrium volume per atom,  $B$  is the bulk modulus and  $G$  is the shear modulus.

The obtained  $K_{Ic}$  for all selected materials are listed in Table 1. For the microalloying effects on  $B_4C$ , the  $K_{Ic}$  of  $B_6O$  (3.09 MPa m<sup>1/2</sup>) is higher than that of  $B_4C$  (3.03 MPa m<sup>1/2</sup>) because of its larger shear modulus

( $G = 210.9$  GPa) than that of  $B_4C$  (199.0 GPa). The  $K_{Ic}$  of  $o\text{-}B_{10}Si_2Si_2$  displays the lowest  $K_{Ic}$  (2.19 MPa m $^{1/2}$ ) among all compounds due to the lowest shear modulus ( $G = 136.6$  GPa). These results agree well with our theoretical predictions on the fracture toughness from Mode II and III fracture.

For nanotwinned structures, the  $K_{Ic}$  of  $B_4C$  and  $B_6O$  are similar to those of perfect crystal, indicating that inserting nanotwins into these two materials does not have much effect on  $K_{Ic}$ . This is consistent with  $K_{IIC}$  and  $K_{IIIc}$  for  $B_4C$ . While for  $B_6O$ , the  $K_{IIC}$  and  $K_{IIIc}$  of  $\tau\text{-}B_6O$  are higher than those of  $B_6O$ .

For the pure boron phases, the sequence of  $K_{Ic}$  from high to low is  $\gamma\text{-}B_{28} > \alpha\text{-}B_{12} > \beta\text{-}B_{106} > \tau\text{-}B_{106}$ , which is in accordance with the sequence of shear modulus. While for  $K_{IIC}$  and  $K_{IIIc}$ , the sequence from high to low is  $\tau\text{-}B_{106} > \beta\text{-}B_{106} > \gamma\text{-}B_{28} > \alpha\text{-}B_{12}$ .

The comparison suggests that the fracture toughness of Mode I fracture mainly depends on shear modulus. It is worth to notice that this  $K_{Ic}$  prediction only depends on the elastic modulus and equilibrium volume. Application of this approach on superhard materials needs to be validated by comparing experiments and theoretical predictions.

In summary, we predict the fracture toughness of  $B_4C$ , its microalloyed structures, the nanotwinned structures  $\tau\text{-}B_4C$ ,  $\tau\text{-}B_6O$ , and four pure boron phases using DFT simulations. Under shear loading, among all crystalline structures,  $B_6O$  and  $B_{12}P_2$  exhibit higher fracture toughness than other compounds due to large displacement before fracture. For nanotwinned structures,  $\tau\text{-}B_4C$  displays higher fracture toughness than its crystal structures because of its higher critical engineering shear stress, while the fracture toughness of  $\tau\text{-}B_6O$  is similar to rhombohedral  $B_6O$  because  $\tau\text{-}B_6O$  has a shorter range of displacement before fracture. For pure boron phases,  $\tau\text{-}B_{106}$  exhibits the highest fracture toughness because of large elastic deformation. Our results suggest that (1) replacing C-B-C chains with two-atom chains especially weakly coupled O atoms can significantly enhance the fracture toughness of  $B_4C$ ; (2) nanotwins can be applied to improve the fracture toughness of  $B_4C$  and elemental boron.

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## Appendix A. Supplementary data

Supplementary material includes additional information on computational methodology, engineering shear-stress-displacement curves for all crystalline structures and shear-stress-shear-strain curves for all selected materials. Supplementary data associated with this article can be found in the online version, at doi: <https://doi.org/10.1016/j.scriptamat.2018.11.035>.

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