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Network-based Intervention Strategies to Reduce Violence among Homeless

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Abstract Violence is a phenomenon that severely impacts homeless youth who are at an increased risk of experiencing it as a result of many contributing factors such as traumatic childhood experiences, involvement in delinquent activities and exposure to perpetrators due to street-tenure. Reducing violence in this population is necessary to ensure that the individuals can safely and successfully exit homelessness and lead a long productive life. Interventions to reduce violence in this population are difficult to implement due to the complex nature of violence. However, a peer-based intervention approach would likely be a worthy approach as previous research has shown that individuals who interact with more violent individuals are more likely to be violent, suggesting a contagious nature of violence. We propose Uncertain Voter Model to represent the complex process of diffusion of violence over a social network, that captures uncertainties in links and time over which the diffusion of violence takes place. Assuming this model, we define Violence Minimization problem where the task is to select a predefined number of individuals for intervention so that the expected number of violent individuals in the network is minimized over a given time-frame. We also extend the problem to a probabilistic setting, where the success probability of converting an individual into non-violent is a function of the number of "units" of intervention performed on them. We provide algorithms for finding the optimal intervention strategies for both scenarios. We demonstrate that our algorithms perform significantly better than interventions based on popular centrality measures in terms of reducing violence. Finally, we use our optimal algorithm for probabilistic intervention to recruit peers in a homeless youth shelter as a pilot study. Our surveys before and after intervention show significant reduction in violence.

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1 Introduction

There are an estimated 1.6–2.8 million youths experiencing homelessness in the United States (Terry et al. 2010). Although violence in the United States has steadily decreased during the past decade, homeless youth remain disproportionately susceptible to violent victimization and perpetration (of Justice. 2013). These youths experience all types of violence at higher rates than their housed counterparts (Petering et al. 2014; Heerde et al. 2014; Eaton et al. 2012a).

Violence perpetuates violence and diffuses through a network like a contagious disease (Fagan et al. 2007). Cure Violence program[1] is based on a similar idea of treating violence as a contagious disease, and has shown significant reduction in violence. Motivated by the contagious nature, a diffusion model is ideal for modeling spread of violence. Doing so can lead to optimal intervention strategies under certain assumptions.

While many diffusion models exist that are variations of Independent Cascade Models, Linear Threshold Model, and Susceptible-infected, they are "progressive" models, i.e., they assume that once activated (or infected), the individuals remain activated. However, in the context of violence, it would mean that a violent person can never become non-violent, which is not applicable. Although some non-progressive extensions do exist (Kempe et al. 2003), accurate analytical solutions of those models are hard to obtain. While these models reached popularity in the era of online social networks,

[1] See `http://cureviolence.org/`

a popular model of non-progressive diffusion of competing behaviors on real life networks, that has existed for a longer time is voter model (Holley et al. 1975; Even-Dar and Shapira. 2007). In voter model individuals are influenced by a randomly selected neighbor[2]. But application of voter model in real-life scenarios such as diffusion of violence has the following drawbacks. (a) There is some uncertainty in the network structure, in the sense that, individuals may forget to mention someone as their peer, and yet be influenced by them (Rice et al. 2014). (b) The number of discrete time steps over which the diffusion process unfolds (a parameter required by Voter Model) is often unknown in practice. To deal with these uncertainties, we proposed Uncertain Voter Model (UVM) as an extension of Voter Model (Srivastava et al. 2018). UVM allows for some uncertainty in the knowledge of the neighborhood that may arise from an individual being influenced by someone they did not explicitly state as their "friend" during the survey to create the network. Our model also incorporates uncertainty in number of time-steps of the diffusion process. Under UVM, we find the optimal intervention strategies to minimize violence. The task is to perform interventions on individuals with constrained "resources" so that they change their state from "violent" to "non-violent" resulting in others adopting "non-violent" state, eventually minimizing violence. We consider two types of interventions: (i) deterministic, where selecting an individual turns them into non-violent, with the constraint being the number of individuals to select; (ii) probabilistic, where an individual's probability of becoming non-violent varies with the individual and is increased based on number of "units" (hours, sessions, etc.) of intervention, with the constraint being the total number of units available. We also conducted a pilot study to test the real life effectiveness of our probabilistic intervention based on UVM. The pilot study consisted of two rounds of data collection - one before the intervention and a longitudinal follow-up. Due to the time consuming nature of the data collection and fast changing network of homeless youth, it is difficult to conduct more rounds of data collection and verify the modeling itself (see Section 8). We will explore this issue in a future work.

Specifically, our contributions are as follows:

- We propose Uncertain Voter Model (UVM) for violence that can capture its non-progressive nature and takes into account the uncertainty in neighborhood as well as uncertainty in the time period over which the diffusion of violence unfolds. Under UVM, we define Violence Minimization problem where the task is to perform intervention with finite resources, i.e., changing the state of some violent individuals so that the total expected number of violent individuals is minimized.
- We show that Uncertain Voter Model can be reduced to the classic Voter Model, and thus a greedy algorithm forms the optimal solution to Violence Minimization.
- We extend our solution to "probabilistic" intervention, where the intervention reduces the probability of violence of selected individuals as a concave non-decreasing function.
- We perform experiments on synthetic networks and a real-life network of homeless youths and find the nodes to be selected for intervention and demonstrate that baselines that do not take the diffusion model into account perform significantly worse.
- We present results from a pilot study of intervention based on our algorithms, that shows significant reduction in violence.
- We make the real-life homeless network publicly available for further research by the community.

[2] We use the terms "neighbor" and "neighborhood" to refer to the links of a given individual in the network and not their physical neighborhood

2 Prior Work

In our prior work (Srivastava et al. 2018), we proposed Uncertain Voter Model to model the spread of violence as a non-progressive diffusion process. The model takes into account the uncertainty in the knowledge of the network, and based on available network data, models interactions through an existing edge or a non-existent edge that may be created in the future. Based on the voter model we defined Violence Minimization problem and showed that greedy solutions are optimal for both deterministic and probabilistic version of intervention. In this work, we provide further analysis of interventions based on UVM, suggesting that better results are obtained considering uncertainty in edges through Katz-centrality based edge-prediction compared to ignoring the uncertainty. Further, while the prior work was based on surveyed data, the intervention results were obtained through simulations on the real-life network. This paper presents results from a pilot study where actual intervention was performed. Owing to the fast changing nature of homeless youth network, new data was collected along with a followup to measure the effect of intervention. We discuss the transition to practice where further assumptions were made to model the probability of response to intervention. The followup data suggests that there was a significant decrease in the number of individuals involved in violence. Also, there was a significant increase in the practice of "mindfulness" which constituted the intervention.

To the best of our knowledge, intervention strategy to reduce violence using diffusion models has received very little attention in the literature (Myers. 2000; Myers and Oliver. 2008). Violence is modeled based on susceptibility and infectiousness in (Myers. 2000). In (Myers and Oliver. 2008) the idea of opposing forces, "provocation" and "repression", is used to model violence as two diffusing processes. This is more accurate as it captures the non-progressive nature of violence, where an individual may switch between the state of "violence" and "non-violence". However, it is a macroscopic approach, which disregards the network structure that can be crucial in identifying best intervention strategies (Valente. 2012). Finally, gang violence has been modeled as a diffusion process in (Shakarian et al. 2014) in order to deter violence by convincing members to dis-enroll, and exit the gang network. Our setting is different as we aim to only change the violent behavior of the individuals while they remain in the same network and continue to interact with their "friends".

3 Initial Data Collection

A sample of 481 homeless youth from ages of 18 to 25 years accessing services from two day-service drop-in centers for homeless youth in Hollywood and Santa Monica, CA, were approached for study inclusion in October 2011 and February 2012. The research team approached all youths who entered the service agencies during the data collection period and invited them to participate in the study. The final sample consisted of 366 individuals who agreed to participate.

The study consisted of a social network interview, where each participant was asked to name anyone they interacted with in person, on the phone, or through the internet in the previous month prompted by interviewers stating, "These might be friends; family; people you hang out with/chill with/kick it with/ have conversations with; people you party with–use drugs or alcohol; boyfriend/girlfriend; people you are having sex with; baby mama/baby daddy; case worker; people from school; people from work; old friends from home; people you talk to (on the phone, by email); people from where you are staying (squatting with); people you see at this agency; other people you know from the street." The social network obtained through the survey has been made publicly available[3].

The variable of interest is violent behavior. Violent behavior was assessed by recent participation in a physical fight. Participants were asked: "During the past 12 months, how many times were you in a physical fight?" Eight ordinal responses ranged from "zero times" to "over 12 times." The responses were dichotomized similar to previous literature on youth violence (Duong and Bradshaw. 2014; Eaton et al. 2012b) to distinguish between participants who had been in no physical fights and participants who had been in at least one physical fight during the previous year (In real-life implementation (see Section 7), we utilized actual values intead of binary 0/1). This question was adopted from the Youth Risk Behavior Survey, Centers for Disease Control and Prevention (Kann et al. 2014) and did not distinguish between victims and perpetrators of violence.

[3] `www-scf.usc.edu/~ajiteshs/datasets/HoSM.txt`

4 Model

To model the spread of violence we model the network of homeless youth as a graph $G(V,E)$ where every individual is a node which can exist in one of two states: 'violent' or 'non-violent'. We chose to model violence as a non-progressive diffusion process, i.e, a node may switch its state unlike the progressive diffusion where once a node is violent it cannot become non-violent again. Next, we provide a background on Voter Model (Even-Dar and Shapira. 2007) on which our model is based.

4.1 Voter Model

In the Voter Model (Even-Dar and Shapira. 2007), at every time step a node u picks an incoming neighbor v at random with a probability $p_{v,u}$. The incoming probabilities are normalized such that $\sum_v p_{v,u} = 1$. Let $x_{u,t}$ represent the probability of node u being violent at time t. According to the model, $x_{u,t} = \sum_v p_{v,u} x_{v,t-1}$. Let $\mathbf{x_t}$ represent the state of all the nodes at time t, with ith element representing the probability that $v_i \in V$ is violent at time t. Suppose matrix M represents the transpose of the adjacency matrix of the weighted network, i.e., $M_{u,v} = p_{v,u}$. Then $\mathbf{x_t} = M\mathbf{x_{t-1}}$. It follows that $\mathbf{x_t} = M^t \mathbf{x_0}$. Here $\mathbf{x_0}$ is the initial state of nodes, which is assumed to be known. Now we wish to select k nodes out of those who are violent at $t = 0$ and turn them into non-violent so that the expected number of nodes that are violent at time t is minimized. Define I_X for $X \subseteq V$ as the vector in which the i-th element is 1 if $v_i \in X$. Then the expected number of violent nodes at time t is

$$\sum_i P(v_i \text{ is violent at time}) = \sum_i x_{i,t} = I_V^T \mathbf{x_t} \tag{1}$$

4.2 Uncertain Voter Model

A network formed through a survey may have missing edges due to the uncertainty in a person's ability to recall all "friends" they might be influenced by (Rice et al. 2014). To capture this aspect, we propose the Uncertain Voter Model, where we assume that a node which is not directly connected to the node of interest may also influence it. In this model, two mutually exclusive events happen: (i) with probability θ a node randomly selects one incoming neighbor and adopts its state, (ii) with probability $(1-\theta)$ it selects a node that is not its neighbor in the network and adopts its state. We propose two ways of selecting the node form outside the neighborhood: (i) random and (ii) Katz-based.

4.2.1 Random

In this case every node which is not a neighbor is equally likely to be selected. Mathematically,

$$x_{u,t} = \theta \sum_{\{v|p_{v,u}>0\}} p_{v,u} x_{v,t-1} + (1-\theta) \sum_{\{v|p_{v,u}=0\}} \frac{1}{|\{v|p_{v,u}=0\}|} x_{v,t-1} \tag{2}$$

If n is the total number of nodes and d_u is the number of incoming neighbors of u, then $|\{v|p_{v,u}=0\}| = n - d_u$. Suppose we define,

$$q_\theta(v,u) = \begin{cases} \theta p_{v,u} & \text{if } p_{v,u} > 0 \\ \frac{1-\theta}{n-d_u} & \text{if } p_{v,u} = 0\,. \end{cases} \tag{3}$$

4.2.2 Katz-bazed

We treat the influence from outside the neighborhood as the problem of finding missing edges (Lü and Zhou. 2011). A popular method for missing edge detection is using Katz similarity (Liben-Nowell and Kleinberg. 2007), which is based on exponentially weighted number of paths between two nodes:

$$K(u,v) = \sum_i \alpha^i |\text{path of length } i \text{ to } u \text{ from } v|\,. \tag{4}$$

We chose Katz-based method for link prediction as it has consistently performed well (Liben-Nowell and Kleinberg. 2007) and can also predict links between nodes that may be more than 2 hops away, unlike common neighbor-based methods. Since, we are only interested in nodes that are not directly in the neighborhood we take the above summation for $i \geq 2$. The entire similarity matrix is given by:

$$K = \sum_{i \geq 2} \alpha^i M^i = \alpha^2 M^2 (I - \alpha M)\,, \tag{5}$$

We choose a small value of $\alpha = 0.005$ (Liben-Nowell and Kleinberg. 2007). We normalize the scores for each node u over all nodes v which are not in its neighborhood, so that that the probability of selecting node v is proportional to $K(u,v)$, i.e.,

$$K'(u,v) = K(u,v) / \sum_w K(u,w)\,. \tag{6}$$

Now, the Katz-based Uncertain Voter Model is given by

$$x_{u,t} = \theta \sum_{\{v|p_{v,u}>0\}} p_{v,u} x_{v,t-1} + (1-\theta) \sum_{\{v|p_{v,u}=0\}} K'(u,v) x_{v,t-1}\,. \tag{7}$$

Again, we can define

$$q_\theta(v,u) = \begin{cases} \theta p_{v,u} & \text{if } p_{v,u} > 0 \\ (1-\theta) K'(u,v) & \text{if } p_{v,u} = 0\,. \end{cases} \tag{8}$$

From Equations 3 and 8, both random and Katz-based Uncertain Voter Model lead to reduction of Equations 2 and Equations 7 to

$$x_{u,t} = \sum_v q(v,u) x_{v,t-1} \text{ or } \mathbf{x_t} = Q_\theta \mathbf{x_{t-1}} \tag{9}$$

where $[Q_\theta]_{u,v} = q_\theta(u,v)$ which reduces to Voter Model (Section 4.1) of a graph of which the transpose of the adjacency matrix is Q_θ. Now, we define the problem of Violence Minimization as follows.

Problem Definition 1 (Violence Minimization) *Given a weighted graph $G(V,E)$, an initial set of violent nodes S, a time frame t, and an integer k, find $T \subseteq S$ such that $|T| = k$, turning the nodes in T into non-violent minimizes the expected number of violent nodes after time t, i.e., $I_V^T \mathbf{x_t}$ under Uncertain Voter Model.*

5 Greedy Minimization

Let $\mathbf{x'_0}$ be the vector formed by turning some k nodes into non-violent initially. Suppose this results in the vector of probabilities $\mathbf{x'_t}$ at time t. Now, minimizing $I_V^T \mathbf{x'_t}$ is equivalent to maximizing $I_V^T(\mathbf{x_t} - \mathbf{x'_t}) = I_V^T Q_\theta^t (\mathbf{x_0} - \mathbf{x'_0})$, i.e., the problem reduces to maximizing

$$I_V^T \Delta\mathbf{x_t} = I_V^T Q_\theta^t \Delta\mathbf{x_0} = \sum_{\{u|\Delta\mathbf{x_0(u)}=1\}} I_V^T Q_\theta^t I_u \tag{10}$$

which can be optimized using greedy strategy (Even-Dar and Shapira. 2007) as presented in Algorithm 1.

Algorithm 1 Greedy algorithm to minimize violence

```
function MinViolence(G, S, θ, k, t)
    Compute Q_θ^t for G
    ∀u ∈ S compute σ(u) = I_V Q_θ^t I_u
    Sort {σ(u)} in descending order and return top k.
end function
```

The most expensive step of the algorithm is the computation of Q_θ^t which can be computed in $O(|V|^{2.4} \log t)$.

5.1 Uncertainty in Time

Uncertain Voter Model requires t as a parameter which is unknown in real life. While we may have a certain time period (days or weeks) over which we want the intervention to work, finding a relation between that time period and the

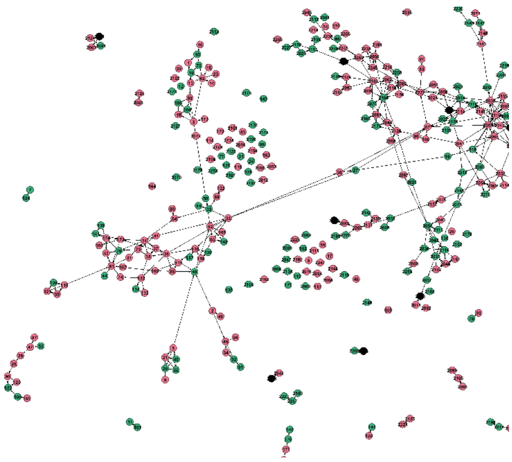


Fig. 1 Visualization of the homeless youth network. The red nodes represent the violent nodes and the green ones represent non-violent ones. The black nodes have unknown state.

parameter t is non-trivial as it depends on how often the individuals interact. To capture this uncertainty, we assume that time t takes a value τ with probability $P(t=\tau)$. Now, we wish to minimize $\mathbb{E}(I_V \mathbf{x_t})$ where the expectation is taken over t. Therefore,

$$\mathbb{E}(I_V^T \mathbf{x'_t}) = \sum_\tau P(t=\tau) I_V^T Q_\theta^\tau \mathbf{x'_0} = I_V^T \left(\sum_\tau P(t=\tau) Q_\theta^\tau \right) \mathbf{x'_0}. \tag{11}$$

Notice from Equation (11) that a greedy solution like Algorithm 1 still applies.

5.2 Probabilistic Intervention

In the previous section, we assumed that performing intervention on a "violent" node turns it into "non-violent", i.e., an intervention is always successful. However, in real life this may not be true, and some nodes may require more "units" (hours, sessions, etc.) of intervention than others. Let $s_u(z_u)$ be the probability of success after applying z_u units of intervention to node u. These functions can be different for different nodes, as different individuals may respond differently to interventions. We assume that these functions $\{s_i\}$ are non-decreasing, i.e, adding more units of intervention cannot decrease the probability of success. We also assume that theses functions are concave, i.e., the marginal increase in probability reduces with increasing number of interventions. Such assumptions are similar to those made in immunization literature (Prakash et al. 2013). Mathematically, if $z' \geq z$, $s_i(z') \geq s_i(z)$, and $s_i(z'+1) - s_i(z') \leq s_i(z+1) - s_i(z)$, $\forall i$. Rewriting Equation 10 for probabilistic intervention, the utility (reduction in violence) obtained by an allocation of $\{z_1, z_2, \ldots, z_n\}, z_i \in \mathbb{N} \cup \{0\}$ is

$$I_V^T Q_\theta^t \Delta \mathbf{x_t} = \sum_u I_V^T Q_\theta^t I_u s_u(z_u) \tag{12}$$

Let $f_u(z_u) = I_V^T Q_\theta^t I_u s_u(z_u)$. This leads to the probabilistic intervention version of Violence Minimization problem, which is equivalent to maximizing $\sum_u f_u(z_u)$, such that $\sum_u z_u = k$. Note that, $I_V^T Q_\theta^t I_u$ is a non-negative constant and $s_u(z_u)$ is non-decreasing concave function, and so, $f_u(z_u)$ is also non-decreasing and concave. Formally, we define this as follows.

Problem Definition 2 (Units Assignment Problem) *Given $k \in \mathbb{Z}$ resources and n concave non-decreasing utility functions $f_i : \mathbb{Z} \to \mathbb{R}$, where $f_i(z_i)$ represents the utility of assigning z_i units to function f_i, maximize the total utility $F = \sum_i f_i(z_i)$ subject to $\sum_i z_i = k$.*

Algorithm 2 Greedy Maximization using Marginal Returns

function GREEDYMAX$((f_1, f_2, \ldots, f_n), k)$
  for $i \leftarrow 1 : n$ **do**
    $z_i \leftarrow 0$
  end for
  for $j \leftarrow 1 : k$ **do**
    $idx \leftarrow \arg\max_i (f(z_i + 1) - f(z_i))$
    $z_{idx} \leftarrow z_{idx} + 1$
  end for
  return $(z_1, z_2, \ldots, z_n)$
end function

Lemma 1 *For a non-decreasing concave function $f : \mathbb{Z} \to \mathbb{R}$, and $h \geq 1$,*

$$f(x+h) - f(x) \leq h(f(x) - f(x-1)) \tag{13}$$

$$f(x) - f(x-h) \geq h(f(x) - f(x-1)) \tag{14}$$

We prove the following.

Theorem 1 *Algorithm 2 produces the optimal assignment for Units Assignment Problem.*

Proof Suppose the greedy assignment results in an assignment of z_i to the function f_i. Without loss of generality, we assume that the functions are ordered as following: if $i < j$ then $f_i(z_i) - f_i(z_i - 1) \geq f_j(z_j) - f_j(z_j - 1), \forall i, j$ such that $z_i, z_j \geq 1$.

Assume that the optimal assignment $(z_1^*, z_2^*, \ldots, z_n^*)$ is different from greedy assignment and produces a greater $F = \sum_i f_i(z_i^*)$.

Choose the smallest index p such that $f_p(z_p) \neq f_p(z_p^*)$.

Case $f_p(z_p) > f_p(z_p^*)$ Since, $\sum_i z_i = \sum z_i^* = k, \exists p < i_1 < i_2 < \cdots < i_M$, for some $M > 0$ such that $z_{i_r}^* > z_{i_r}$ and $\sum_r (z_{i_r}^* - z_{i_r}) \geq z_p - z_p^*$. Therefore, it is possible to pick $h_{i_r} \leq (z_{i_r}^* - z_{i_r})$ such that $\sum_r h_{i_r} = z_p - z_p^*$. Suppose we take h_{i_r} out of the optimal assignment $z_{i_r}^*, \forall r$ and assign them to the function f_p, then we should not expect any gain ($\Delta F \leq 0$) as the assignment we started with was optimal. We note that

$$\sum_r (f_{i_r}(z^*_{i_r}) - f_{i_r}(z^*_{i_r} - h_{i_r}))$$
$$\leq \sum_r h_{i_r}(f_{i_r}(z^*_{i_r} - h_{i_r}) - f_{i_r}(z^*_{i_r} - h_{i_r} - 1)) \text{ [Using Lemma 1]}$$
$$\leq \sum_r h_{i_r}(f_{i_r}(z_{i_r}) - f_{i_r}(z_{i_r} - 1)) \text{ [Due to concavity]}$$
$$\leq \sum_r h_{i_r}(f_p(z_p) - f_p(z_p - 1)) \text{ [Due to ordering of functions]}$$
$$\leq (z_p - z^*_p)(f_p(z_p) - f_p(z_p - 1)) \text{ [Since } \textstyle\sum_r h_{i_r} = z_p - z^*_p]$$
$$\leq (f_p(z_p) - f_p(z^*_p)) \text{ [Using Lemma 1]}$$

Therefore, the gain obtained in this case is $\Delta F = (f_p(z_p) - f_p(z^*_p)) + \sum_r (f_{i_r}(z^*_{i_r} - h_{i_r}) - f_{i_r}(z^*_{i_r})) \geq 0$.

Case $f_p(z_p) < f_p(z^*_p)$ Since, $\sum_i z_i = \sum z^*_i = k, \exists p < i_1 < i_2 < \cdots < i_M$, for some $M > 0$ such that $z_{i_r} - z^*_{i_r}$ and $\sum_r (z_{i_r} - z^*_{i_r}) \geq z_p - z^*_p$. Therefore, it is possible to pick $h_{i_r} \leq (z_{i_r} - z^*_{i_r})$ such that $\sum_r h_{i_r} = z^*_p - z_p$. Now, we take $z^*_p - z_p$ resources out of the optimal assignment on f_p and distribute them such that f_{i_r} gets $z_{i_r} + h_{i_r}$. Since we have obtained the sequence $z_1, z_2, \ldots, z_n$ using greedy assignment, $f_i(z_i + 1) - f_i(z_i) \leq f_j(z_j) - f(z_j - 1), \forall i \neq j$, otherwise the last unit that went to f_j would have gone to f_i instead. We have $f_p(z^*_p) - f_p(z_p)$

$$\leq (z^*_p - z_p)(f_p(z_p + 1) - f_p(z_p)) \text{ [Using Lemma 1]}$$
$$\leq \sum_{i_r} h_{i_r}(f_p(z_p + 1) - f_p(z_p)) \text{ [Since } \textstyle\sum_r h_{i_r} = z_p - z^*_p]$$
$$\leq \sum_{i_r} h_{i_r}(f_{i_r}(z_{i_r}) - f_{i_r}(z_{i_r} - 1)) \text{ [From of Algorithm 2]}$$
$$\leq \sum_{i_r} h_{i_r}(f_{i_r}(z^*_{i_r} + h_{i_r}) - f_{i_r}(z^*_{i_r} + h_{i_r} - 1))$$
$$\leq \sum_{i_r} (f_{i_r}(z^*_{i_r} + h_{i,r}) - f_{i_r}(z^*_{i_r})) \text{ [Using Lemma 1]}$$
$$\leq \sum_{i_r} (f_{i_r}(z_{i_r}) - f_{i_r}(z^*_{i_r})) \text{ [} f_{i_r} \text{ is non-decreasing].}$$

Therefore, $\Delta F = \sum_{i_r} h_{i_r}(f_{i_r}(z_{i_r}) - f_p(z^*_{i_r})) + (f_p(z_p) - f_p(z^*_p)) \geq 0$.

But $\Delta F \leq 0$., and so ΔF must be zero, i.e., for any optimal assignment that differs from the greedy assignment first at index p, we can perform a reassignment that retains optimality so that they no longer differs at index p. Proceeding thus we get $z^*_i = z_i, \forall i$. Hence, the greedy assignment is optimal.

Suppose the exact response to intervention for individuals is hard to predict, and instead we have some estimation of the response. In other words, if the exact functions f_i are not known but we have an approximation g_i of f_i, the following can be shown.

Theorem 2 *If concave non-decreasing functions $\{g_i\}$ estimate $\{f_i\}$, such that $(1-\varepsilon)f_i(z) \leq g_i(z) \leq (1+\varepsilon)f_i, \forall i, z$, for some $\varepsilon \geq 0$, then Algorithm 2 applied on the functions $\{g_i\}$ produces a $(1-\varepsilon)$-approximation for Units Assignment Problem.*

Proof Let the vector $\mathbf{z}^* = [z^*_1, z^*_2, \ldots, z^*_n]$ represent the optimal solution for Units Assignment Problem. Let $F(\mathbf{x}^*) = \sum_i f_i(z^*_i)$. Suppose, Algorithm 2 produces the vector $\mathbf{z}' = [z'_1, z'_2, \ldots, z'_n]$ based on the functions $\{g_i\}$, and $G(\mathbf{z}') = \sum_i g_i(z'_i)$. Since, $(1-\varepsilon)f_i(z) \leq g_i(z) \leq (1+\varepsilon)f_i(x), \forall i, z$, taking the sum over all i, we get $(1-\varepsilon)F(\mathbf{z}) \leq G(\mathbf{z}) \leq (1+\varepsilon)F(\mathbf{z}), \forall \mathbf{z}$. Then,

$$G(\mathbf{z}') \geq G(\mathbf{z}^*) \geq (1-\varepsilon)F(\mathbf{z}^*). \tag{15}$$

6 Experiments

We have shown that the greedy algorithms described in Algorithms 1 and 2 are optimal under Uncertain Voter Model for deterministic and probabilistic interventions, respectively. However, to study how prominent the difference is from other choices of intervention strategies, we compare it against the following baselines:

- Degree: We define the degree of a node based on the weighted graph as $d_v = \sum_u p_{v,u}$. Then we select top k nodes.
- Betweenness Centrality: Top k nodes are selected based on the betweenness centrality in the graph.

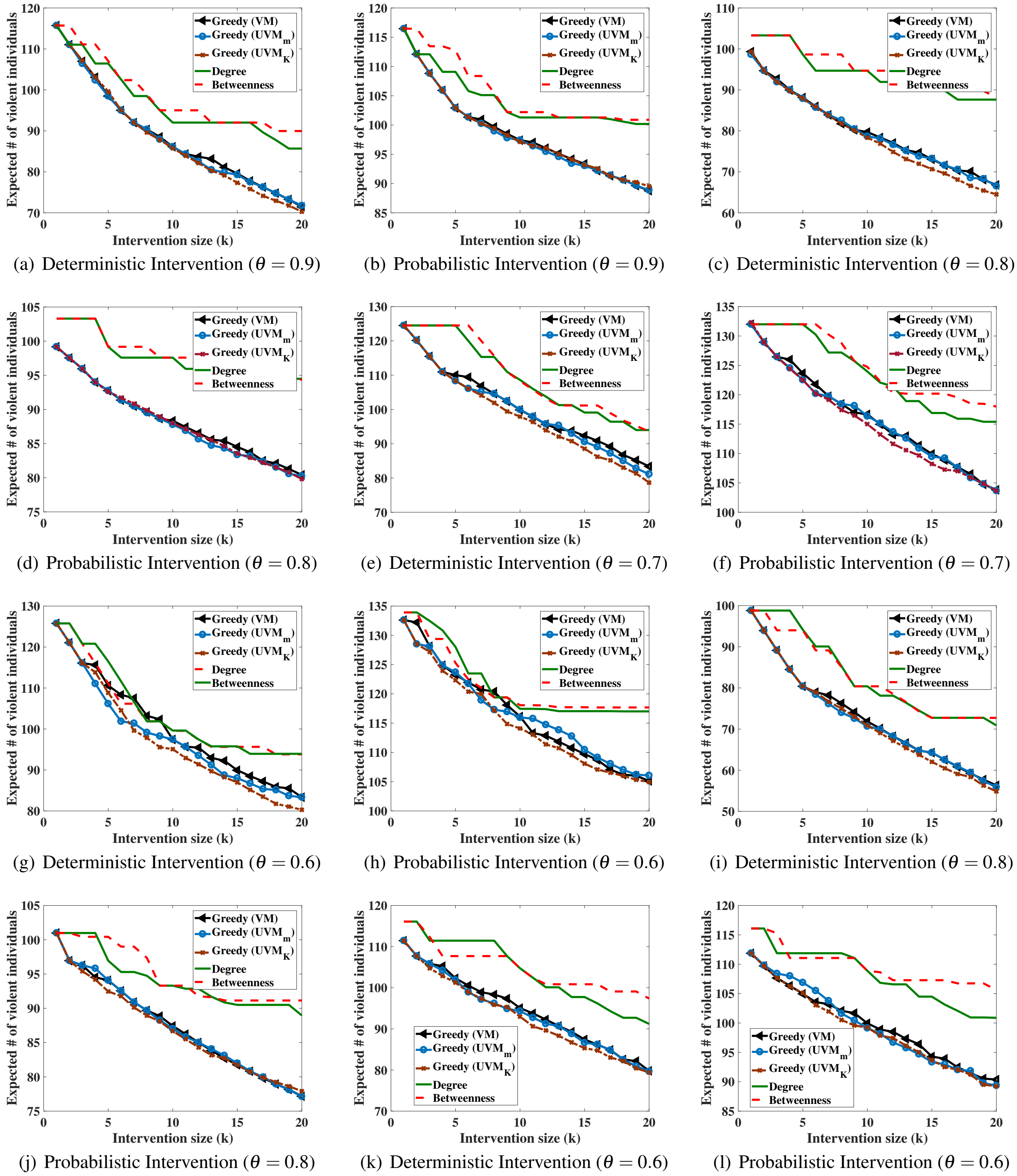
We have performed two sets of experiments:

Synthetic Kronecker graphs We generated random Kronecker graphs (Leskovec et al. 2010) with roughly same number of nodes and edges as the real Homeless Youth network, described next.

Real-world Homeless Youth Network We constructed the network obtained by the surveyed data, which consists of 366 nodes and 558 directed edges. Due to the lack of the knowledge of edge-weights, we assume that all incoming links for a node are equally weighted.

6.1 Synthetic Networks

To simulate the fact that individuals often forget to mention some individuals they might be influenced by (Rice et al. 2014), for every Kronecker graph G, we randomly removed a certain fraction ϕ of edges to form graph G'. We applied, our greedy algorithm to obtain optimal set of nodes for intervention under UVM for both random and Katz-based variation, assuming $\theta = 1 - \phi$ on G'. We chose ϕ from $[0, 0.5]$.



(a) Deterministic Intervention ($\theta = 0.9$) (b) Probabilistic Intervention ($\theta = 0.9$) (c) Deterministic Intervention ($\theta = 0.8$)

(d) Probabilistic Intervention ($\theta = 0.8$) (e) Deterministic Intervention ($\theta = 0.7$) (f) Probabilistic Intervention ($\theta = 0.7$)

(g) Deterministic Intervention ($\theta = 0.6$) (h) Probabilistic Intervention ($\theta = 0.6$) (i) Deterministic Intervention ($\theta = 0.8$)

(j) Probabilistic Intervention ($\theta = 0.8$) (k) Deterministic Intervention ($\theta = 0.6$) (l) Probabilistic Intervention ($\theta = 0.6$)

Fig. 2 Comparison of the baseline against the greedy algorithm for varying intervention sizes under the Uncertain Voter Model on random Kronecker graphs.

We consider this to be a sensible range for ϕ as $\phi > 0.5$ (i.e., $\theta < 0.5$) would represent very low confidence in the collected data, i.e, it would mean that a node is more likely to be influenced by one of the nodes it is not connected to. The parameter t was assumed to be uniformly distributed between 1 and 5. We also applied the greedy algorithm assuming $\theta = 1$, which would be the optimal for Voter Model. Intervention was performed by selecting these sets, but on original graph G. Figure 2 shows the number of violent nodes that result from different intervention strategies, while varying the intervention size k. Figures 2(a), 2(c), 2(e), 2(g), 2(i) and 2(k) are for deterministic intervention scenario, where selected nodes become non-violent. Figures 2(b), 2(d), 2(f), 2(h), 2(j) and 2(l) are for probabilistic intervention scenario, where a selected node u becomes non-violent with a probability $1 - {r_u}^{z_u}$, where z_u is the number of units of intervention applied on u, and $r_u \in [0,1]$ is randomly selected to simulate how well u responds to the intervention. We obtained the plots for many synthetic Kronecker graphs, but only report a few as they all had the same trends.

UVM_m and UVM_K represent UVM with random and Katz-based selection of out of neighborhood nodes, respectively. Greedy algorithm on Katz-based UVM significantly outperforms the baselines. Greedy algorithm on Voter Model ($\theta = 1$) performs worse and UVM based on random selection while comparable, is still slightly worse. This suggests that taking the uncertainty of edges into account by predicting links produces better intervention strategy. Most of these graphs were generated with approximately 50% initial violent nodes to match the real-world network. As expected, for high values of θ VM and UVM have similar performances.

6.2 Homeless Youth Network

For the real Homeless Youth Network, we performed selection and simulated intervention on the same graph, as the network that includes the "forgotten" links is not available. Out of the 366 nodes, 55.01% were "violent" ($x_{u,0} = 1$) and 42.55% are "non-violent" ($x_{u,0} = 0$). Data on the rest of 2.44% are missing and are assumed to be equally likely to be of either state ($x_{u,0} = 0.5$). Based on this "initial state" we run Greedy Minimization for Uncertain Voter Model.

Figure 3 show the deterministic intervention comparison for expected number of nodes that are violent after $t = 5$ and $t = 10$. Figure 4 shows the comparison for probabilistic intervention. The value of θ was set to 0.75 to generate these plots. Other values for parameters t and θ show similar trends and hence, have been omitted. We set $s_u(z_u) = r_u \forall z_u \geq 1$, where r_u is chosen randomly in $[0,1)$. We compare the results obtained using Algorithm 2 labeled *Greedy (PI)* against a number of baselines: (1) Algorithm 1 labeled *Greedy (DI)*, deterministic intervention ignoring s_u; (2) Degree; (3) Betweenness; (4) PageRank; (5) IMM, state-of-the-art influence maximization seed set selection algorithm under Independent/Weighted Cascade Model (Tang et al. 2015). We observe that the greedy algorithm maximizing marginal returns for probabilistic intervention significantly outperforms all baselines.

The upper limit of number of time steps (t) was chosen to be a small number in our experiments, keeping in mind that homeless youth networks are dynamic, and so in practice, the intervention should be performed in short-term.

Choosing individuals in practice So far we have presented the comparison of our greedy method against the baseline centrality measures in terms of reduction in violence. Now, we proceed to examine individuals chosen for intervention based on our method. We experimented with different values for parameter $\theta = 1, 0.9, 0.8, 0.7, 0.6$ and 0.5, i.e., increasing edge uncertainty. Table 1 presents the top 10 nodes (in terms of PID assigned in the survey) chosen for intervention (deterministic). Note that there are many nodes such as PIDs 47, 4, 2086, 2156, and 51, that consistently appear in the top 10, suggesting that the set of chosen individuals is not highly sensitive to the choice of parameters within a sensible range. However, the significant deviation from betweenness and degree centralities (Figure 3) suggests that finding this set is non-trivial. We also varied the value of $t = 2, 4, 6, 8, 10$, and 12. The lists of seeds obtained for different values of t have not been presented for brevity, as they had the same PIDs frequently occurring in the lists. These individuals were selected based on deterministic intervention, which should be applied when the knowledge of personal traits is not available. However, with the availability of personal traits sufficient to model how an individual may respond to intervention ($s_u(z_u)$), probabilistic intervention should be used. Next, we describe how we utilized our approach to perform real-life intervention to decrease violence.

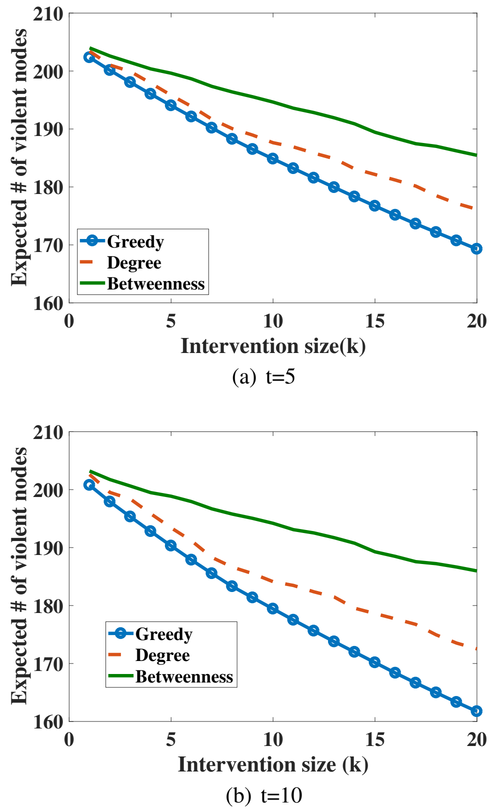


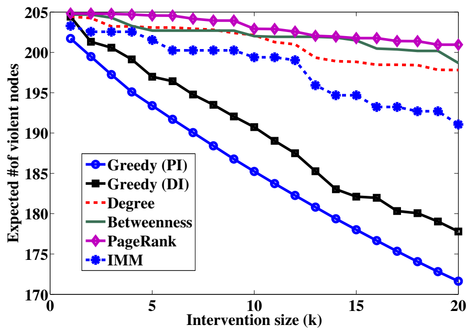
Fig. 3 Comparison of the baseline against the greedy algorithm for varying intervention sizes under the UVM for deterministic intervention.

Table 1 Top 10 seeds for various values of θ output by Greedy Minimization

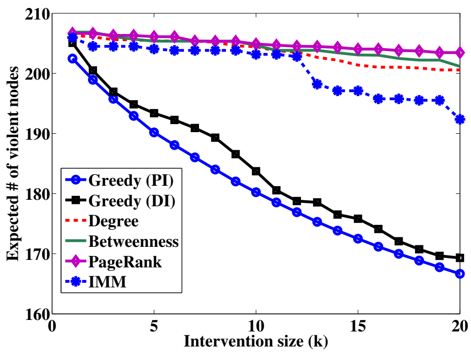
θ	Selected Seeds										$\mathbb{E}(I_V^T \mathbf{x}'_\mathbf{t})$
1	47	4	2156	51	13	2086	169	2115	2099	2056	179.43
0.9	47	4	2156	2086	51	13	169	2115	2056	2099	183.327
0.8	47	4	2086	2156	51	13	169	2115	2056	89	185.86
0.7	47	4	2086	2156	51	2115	13	169	2056	2125	187.54
0.6	47	4	2086	2115	2156	51	169	13	2056	2125	188.66
0.5	47	4	2086	2115	2156	51	169	13	2056	2125	189.43

Table 2 Top 10 seeds for various values of t output by Greedy Minimization

t	Selected Seeds										$\mathbb{E}(I_V^T \mathbf{x}'_\mathbf{t})$
2	47	2086	4	2115	51	2156	169	13	2056	2125	189.92
4	47	4	2086	2115	51	2156	169	13	2056	2125	188.66
6	47	4	2086	51	2156	2115	169	13	2056	2125	187.81
8	47	4	2086	51	2156	2115	13	169	2056	2125	187.22
10	47	4	2086	2156	51	13	2115	169	2056	2125	186.79
12	47	4	2086	2156	51	13	2115	169	2056	2125	186.45



(a) t=5



(b) t=10

Fig. 4 Comparison of the baseline against the greedy algorithm for varying intervention sizes under the UVM for probabilistic intervention.

7 Pilot Study

While the data collection discussed in Section 3 was used only to perform simulated experiments, we also performed a pilot study to deploy actual intervention based on UVM on a different site. A group of homeless youth (HY) were recruited in the summer of 2018 from a homeless support services drop-in agency for in Los Angeles that serves more than 1000 unique HYA every year. Baseline data collection was conducted for approximately six days. The survey consisted of questions as described in Section 3 with added emphasis on mindfulness related questions (Baer et al. 2008). During the agency's open hours, study staff approached all HY utilizing services to verbally describe the study. HY who endorsed interest in participating signed a voluntary consent form which assured youth of the confidentiality of their information. All HY that were approached were interested in participating, however, some HY were not enrolled due to limits on staffing capacity, time constraints and other commitments that prevented enrollment during the study recruitment period. We also collected detailed contact information for each participant to ensure longitudinal follow-up, which was performed after a month and more follow-ups are planned. Research staff members sat with them while youth completed the baseline survey on an iPad. During this process, network data was also collected by eliciting participants information about their social connections with other participants in the study. This network data is also made publicly available[4]

The intervention consisted of Yoga and Mindfulness sessions informed by Brofenbrenner's ecological theory (Bronfenbrenner. 1977) and Bandura's theory of social learning (Bandura. 1986). The intervention also derives from Positive Youth Development model (Catalano et al. 2004) that suggests an integrated approach for prevention that incorporates strategies that promote youth development such as social skills, communication, self-awareness and community commitment.

7.1 Peer Selection

The selection of "peers" (seeds/individuals to invite for intervention), we used Algorithm 2 with some heuristics to represent the probability of violence $x_{u,t}$ and how well an individual responds to intervention s_u. For computing $x_{u,0}$, we

[4] www-scf.usc.edu/~ajiteshs/datasets/SPY.txt

computed a score of violence X_u as the number of times an individual was involved in violent incidents, and then took the sigmoid of the standardized X_u, i.e.,

$$x_{u,0} = sigmoid\left(\frac{X_u - \mu(X_u)}{std(X_u)}\right)$$

. Response to the intervention was calculated as a function of an aggregate score of mindfulness measure S_u based on the 15 item version of the Five Factor Mindfulness Questionnaire (Baer et al. 2008). Again, the probability of responding to intervention was assumed to be sigmoid function of standardized S_u, i.e.,

$$s_u = sigmoid\left(\frac{S_u - \mu(S_u)}{std(S_u)}\right).$$

Using these values of $x_{u,0}$ and s_u, Algorithm 2, 11 top candidates where invited who attended the intervention sessions. The parameter θ was set to 0.75 and t was assumed to be uniformly distributed in $\{1,\ldots,5\}$.

7.2 Intervention Results

Table 3 summarizes the effects of intervention by providing a comparison of mindfulness and violence related counts before and after intervention. Note that approximately 90 HY where present in the baseline data (t1), only 58 of them where present in the second round of data collection (t2). The reason is difficult to assess formally, but it may be due to the fact that HY often move and are difficult to locate or contact. We observe that a significant increases in the number of individuals who reported daily mindfulness practice (from 13 to 24). Also, there is a significant decrease in the number of individuals who had a physical fight (from 20 to 12).

Table 3 Summary of results from Pilot Study of proposed intervention

Variables	t1	t2	p-value
Practice mindfulness daily	13	24	< 0.05
Practice Yoga at least once in the previous month	22	25	< 0.02
Had a physical fight	20	12	< 0.05
Had a verbal fight that you felt might escalate to a physical fight	37	32	< 0.05

8 Discussion

UVM with Individual-specific Uncertainty We have assumed that the uncertainty parameter θ is same for all individuals. However, it is possible to incorporate different levels of uncertainty for different individuals θ_u. In our collected dataset, there is a variable that measures how attentive the individual was, which can be used as a proxy to individual-specific uncertainty. We can rewrite Equation 8

$$q_\theta(v,u) = \begin{cases} \theta_u p_{v,u} & \text{if } p_{v,u} > 0 \\ (1-\theta_u)K'(u,v) & \text{if } p_{v,u} = 0\,. \end{cases} \tag{16}$$

Therefore, we can still use Equation 9 to represent the dynamics.

UVM with Personal Traits. We have taken certain personal traits into account only to model response to intervention along with the network structure and history of violence of the individuals. A more complex diffusion model can be learned that accurately models the dynamics of violence by accounting for personal traits in the diffusion as well. Here, we discuss one such extension of our model.

In our collected dataset, one feature of particular interest is Difficulty in Emotion Regulation (DERS). Intuitively, an individual with high DERS is likely to have a higher propensity for violence. Other factors such as gender may affect the propensity of violence as well. Suppose $\alpha(u)$ is the propensity of a node u for being violent. Mathematically,

$$\begin{aligned} x_{u,t} &= \\ &\frac{\alpha(u)\sum_j q_\theta(v,u)x_{v,t-1}}{\alpha(u)\sum_v q_\theta(v,u)x_{v,t-1} + (1-\alpha(u))\sum_v q_\theta(v,u)(1-x_{v,t-1})} \\ &= \frac{\alpha(u)\sum_j q_\theta(v,u)x_{v,t-1}}{\alpha(u)\sum_v q_\theta(v,u)x_{v,t-1} + (1-\alpha(u))(1-\sum_v q_\theta(v,u)x_{v,t-1})}\,. \end{aligned} \tag{17}$$

Note that this model can be represented as

$$\mathbf{x_t} = \mathscr{C}(Q_\theta \mathbf{x_{t-1}}) = \mathscr{C}(Q_\theta \mathscr{C}(\ldots\mathscr{C}(Q_\theta \mathbf{x_0}))), \tag{18}$$

where $\mathscr{C}(Q\mathbf{x}_\tau)$ is a vector of functions with u^{th} element given by $\mathscr{C}_u(\sum_v q_\theta(v,u)x_{u,t})$. Each $\mathscr{C}_u$ is a non-decreasing concave function, and since a linear combination of concave functions and composition of concave functions is also concave, RHS of Equation 18 is also concave. Let that function be $\mathscr{C}^t(\mathbf{x_0})$. Therefore, effect of intervention is given by

$$I_V^T \Delta\mathbf{x_t} = I_V^T \mathbf{x_0} - I_V^T \mathscr{C}^t([x_{1,0}(1-s_1(z_1))\ldots x_{n,0}(1-s_n(z_n))]) \tag{19}$$

The utility of intervention can be represented as a function over multisets $U(T) = I_V^T \Delta\mathbf{x_t}$, where $T = \{(u,z_u)|z_u$ units assigned to node $u\}$. The following can be shown.

Theorem 3 *$U(T)$ is submodular and non-decreasing.*

Due to Theorem 3 the greedy algorithm maximizing marginal returns admits a $(1-1/e)$-approximation (Soma and Yoshida. 2016).

Data Limitations. To verify the modeling, multiple snapshots of the network as well as data associated with the individuals are required. Multiple snapshots would also enable the analysis of how quickly the diffusion process takes place. This would aid in understanding a mapping between time parameter of the model vs time elapsed in real life. Another advantage of multiple snapshots would be to understand and take into account the evolution of the network itself, i.e., rate of addition and removal of nodes and links. However, surveying individuals is a time consuming process, and the homeless youth networks change rapidly. Thus, collecting multiple snapshots of data within a short period of time is not feasible, and alternate methods of data collection need to be explored. For the current scenario, we suggest that the intervention should be performed iteratively, over short periods of time and the effects should be observed. This new set of data would help in retraining the model and finding more accurate nodes for intervention in the next iteration.

9 Conclusions

We have proposed Uncertain Voter Model (UVM) to capture the non-progressive diffusion of violence. Under UVM, a node selects one of its neighbors with probability θ or one of the remaining nodes with probability $1-\theta$, and adopts its state. The parameter θ captures the certainty of being influenced by the neighbors. The model also captures uncertainty in time over which the diffusion of violence takes place. We have shown that a greedy algorithm is the optimal intervention strategy to minimize violence under this model. We have also extended the deterministic intervention by considering a scenario where the intervention succeeds only with a certain probability as a function of number of resources (units of interventions) allocated to the individual. We have also shown that the greedy algorithm maximizing marginal returns forms the optimal intervention strategy. Experiments on synthetic Kronecker graphs suggest that UVM is a better choice than the classic Voter Model, where edges may have been omitted during data collection. Experiments on real-world Homeless Youth network have demonstrated that our intervention strategy significantly outperforms interventions based on popular centrality based measures. We show in our experiments that for sensible choices of parameters the top individuals selected for intervention roughly remain the same. We also presented the results from a pilot study that demonstrates that probabilistic intervention based on proposed model led to significant reduction in violence.

As future work, we plan to incorporate personal traits (such as Difficulty in Emotion Regulation Score) to more accurately model the diffusion process and the response to the intervention. We also plan to apply our modeling and intervention scheme to other behaviors that are contagious and non-progressive in nature, such as such as drug-abuse.

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