Corrigendum

Corrigendum to "Controllability of multi-agent systems with directed and weighted signed networks" [Systems Control Lett. 116 (2018) 47–55]

Yongqiang Guan [a,*], Mengran Xue [b], Sandip Roy [b]

[a] *School of Mechano-Electronic Engineering, Xidian University, Xi'an, 710071, China*
[b] *School of Electrical Engineering and Computer Science, Washington State University, Pullman, WA, 99164, United States*

Recently, as a fundamental and important research topic in the cooperative control of multi-agent systems, controllability has attracted the attention of researchers in distinct disciplines, e.g. [1–5]. The article [1] develops a number of conditions on the controllability of multi-agent systems whose networks are signed, in addition to being directed and weighted. Specially, the controllability of generic linear multi-agent systems has been studied, where the system question in [1] is $\dot{X} = \tilde{A}X + \tilde{B}U$ (system (10)), where $\tilde{A} := I_n \otimes A - L \otimes HK$, $\tilde{B} := B \otimes HK$, $A \in \mathbb{R}^{N \times N}$ and $H \in \mathbb{R}^{N \times M}$ are the system matrix and the input matrix, respectively. $K \in \mathbb{R}^{M \times N}$ is a feedback gain matrix to be designed. $L \in \mathbb{R}^{n \times n}$ is the Laplacian of interaction network and $B = [e_1, e_2, \ldots, e_m] \in \mathbb{R}^{n \times m}$, e_i is the ith canonical vector in R^n. Theorem 3 in [1] shows that controllability of system (10) is congruously determined by the interaction network and the agent dynamics. Specifically, it claims that the system (10) is controllable if and only if the pairs (L, B) and (A, HK) are simultaneously controllable.

The conditions given in Theorem 3 and Corollary 1 of [1] are necessary but not sufficient for controllability of system (10). A counterexample is shown as follows:

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}, H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$L = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 4 & -2 \\ 0 & -2 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}.$$

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* Corresponding author.
E-mail addresses: guan-jq@163.com (Y. Guan), morashu@wsu.edu (M. Xue), sandip@wsu.edu (S. Roy).

It is easy to check that the pairs (L, B) and (A, HK) are simultaneously controllable. However, the system (10) (or the pair $(\tilde{A}, \tilde{B})$) is not controllable. In fact the controllability matrix of system (10) has rank 7 rather that 9.

The counterexample connects to several recent studies showing that input–output properties of diffusively-coupled networks (e.g., controllability, observability, invariant zeros, fixed modes) may depend in complicated ways on subsystem-level and network-level properties [2–7]. Because of this complicated dependence, input–output properties often cannot be deduced solely from the network's graph topology, as indeed is seen in the counterexample. Of particular note, the presented counterexample here is closely connected to the counterexample developed for a slightly different model in [4]. It also relates closely to Example 1 in the article [5], which defines the notion of a *network invariant mode* and studies its implications on controllability.

The proof of sufficiency in Theorem 3 relies on the Jordan canonical form of the Laplacian of interaction network. Through a similarity transform the controllability of $(\tilde{A}, \tilde{B})$ is equivalent to that of the pairs $(I_{n_i} \otimes A - J_{n_i} \otimes HK, \bar{B}_i^T \otimes HK)$, $i = 1, 2, \ldots, t$, see [1] for details. The gap in the proof arises from the fact that controllability of the pair (A, HK) is not sufficient for controllability of $(I_{n_i} \otimes A - J_{n_i} \otimes HK, \bar{B}_i^T \otimes HK)$, $i = 1, 2, \ldots, t$. Insufficiency may arise, specifically, because repeated eigenvalues across the diagonal blocks $A - \lambda_i HK$ of $\tilde{A}$ may have multi-dimensional eigenspaces across them. In this case, $\tilde{A}$ may have a left eigenvector with nonzero entries across multiple blocks which is in the null space of $\tilde{B}$, even though none of the left eigenvectors of each diagonal block $A - \lambda_i HK$ are in the null space of $\tilde{B}$. Therefore, it is possible that (L, B) and (A, HK) are simultaneously controllable, yet $(\tilde{A}, \tilde{B})$ and hence system (10) are uncontrollable. Indeed, for the counterexample presented above, the matrix $\tilde{A}$ has an eigenvalue at 1, which is

each repeated across three diagonal blocks, i.e., $1 \in \sigma(A)$, $1 \in \sigma(A - 2HK)$, $1 \in \sigma(A - 6HK)$, where $0, 2, 6 \in \sigma(L)$.

Although the example presented above is a counterexample to Theorem 3, it is in some sense a rare case. In fact, if the feedback gain matrix K is chosen properly, the possibility for repeated eigenvalues across the blocks $A - \lambda_i HK$ can be ruled out, and then uncontrollability of system (10) can be avoided. For example, the system (10) becomes controllable when the gain matrix K is chosen as

$$K = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 2 & 4 \end{bmatrix}.$$

This means that the conditions in Theorem 3 and Corollary 1 would still be appropriate, provided that the gain matrix K is properly designed.

The following results are now correct.

Theorem 3. *The system (10) is controllable only if the pairs (L, B) and (A, HK) are simultaneously controllable.*

Corollary 1. *The system (10) is controllable only if the pair (A, HK) is controllable and there does not exist some λ such that any of the statements (i), (ii) and (iii) of Proposition 1 is satisfied.*

In addition, it is important to note that the proof of necessity in Theorem 3 (Corollary 1) is valid, and the result of Theorem 3 (Corollary 1) in this paper is independent of and unrelated to the results presented in other parts.

References

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