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# Impacts of High and Low Gain Controllers on Remote Channels in Dynamical Networks

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*Abstract*— **The impacts of high and low gain controllers on remote channels in a linear diffusive network model are studied, from a topological perspective. Specifically, we study how a high or low gain controller deployed at one point in the network influences the finite and infinite zero structure of a second channel. The analysis shows how the network's graph topology, the positions of the control channels relative to the topology, and the specifics of the built controller influence the zero structure. The analysis yield conditions under which a deployed controller can induce nonminimum phase behaviors, and also conditions where minimum-phase dynamics are preserved.**

## I. Introduction

Engineered networks such as power and transportation systems are regulated by multiple control systems, which act on different network components and at varied spatial and temporal scales. Today, the paradigm for controlling such engineered networks is changing, as new sensing and actuation technologies are deployed, and control systems are cyber-enabled and networked. At their essence, these changes are enabling the deployment of many new wide-area control loops (loops that use geographically-remote sensing and actuation assets) with sophisticated regulatory functions. They are also often leading to the democratization of control, in the sense that different control loops are operated by different stakeholders, and indeed sometimes are modulated by the network's users. The control of large-scale engineered networks has been extensively studied, and many methods are available for both centralized and decentralized control [1]–[3]. However, these studies primarily approach controller design from a holistic perspective, i.e. the design is achieved by solving a single multi-input multi-output (centralized or decentralized) control problem. Theoretically, approaching controller design from a holistic perspective is appropriate, since MIMO control can achieve improved performance and obviate fragilities that are present when SISO controls are used [4]. However, practical considerations in the control of engineered networks often dictate that control systems are deployed piecemeal and operated independently, particularly when varied stakeholders are involved. Managers of engineered networks have long been concerned that these piecemeal-deployed and operated control systems may have complex interdependencies and adverse impacts on each other. These impacts may be exacerbated by the growing use of wide-area controls, and the involvement of new stakeholders in the control process. Experimental analysis of such control-channel interactions after deployment is sometimes possible, but may be difficult and costly [5]. Thus, network managers would benefit from systematic analyses of control-channel interdependencies at design time, and particularly simple structural or topological rubrics for identifying possible adverse impacts.

With this motivation in mind, we pursued a preliminary study of interdependencies or interactions among control channels in a representative dynamical-network model, with the aim of gaining graph-theoretic insights into the problem [6]. While the initial study was concerned with interdependencies among two SISO control channels, here we also consider the impact of multi-terminal controllers on a remote channel. Noting the wide use of low- and/ or high- gain controller design methodologies in multi-time-scale network control, we also characterize interdependencies caused by controllers with low or high gain properties. In short, our main focus here is to understand the impacts of low- or high- gain controls deployed in a dynamical network on the transfer function seen across a second control channel, from a topological or graph-theoretic perspective.

Low- and/or high- gain controller designs have been proposed and deployed in various large-scale network applications, including in managing the fast dynamics of the power transmission network and in queueing/communication networks. In many of these networks, multiple controllers are used, which are designed and deployed in a piecemeal fashion. Traditionally, the low and high gain control designs have been concerned with shaping the model's internal dynamics and the particular control channel's response. Hence, these designs do not indicate how a designed controller at one network location may affect other input-output channels and consequent behaviors of other control systems. Our focus here is to understand how low- and high-gain controls in a network may impact other remote channels [7]–[11], in the context of a standard linear diffusive network model defined on a digraph. Specifically, we focus on characterizing the infinite and finite zero structure of the second channel, which dictates its input-output behavior, and limits and modulates control performance [12], [13]. The analysis shows that the positions of the two control channels relative to the network graph, as well as the form of the deployed controller, affects the zero structure.

The remainder of the article is organized as follows. The model and problem formulation are given in Section II.

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The results from previous work [6] are briefly reviewed, and then the main results on low- and high- gain control-channel interactions are presented, in Section III. *Due to space constraints, proofs and some details are excluded, see [14].*

## II. Modeling and Problem Formulation

A standard linear diffusive network model defined on a digraph is enhanced to explicitly represent a built control, and a second single-input single-output channel of interest. The formulation generalizes that given in [6].

A network with $n$ components or nodes, labeled as $1, 2, ..., n$, is considered. Each node $j$ has associated with it a scalar state $x_j(t)$. The nodes' states are nominally governed by a linear dynamical model with diffusive state matrix $A$, and are further modulated by an in-built linear feedback control. Further, a channel of interest (an input-output pair) is defined. The full model of the system is given by:

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + \mathbf{P} + \mathbf{e}_i u \\ \mathbf{P} &= \mathbf{e}_q H_c(s)\mathbf{z}^T\mathbf{x} \\ y &= \mathbf{e}_n^T\mathbf{x} \end{aligned} \quad (1)$$

Here, $\mathbf{x} = \begin{bmatrix} x_1 \ldots x_n \end{bmatrix}^T$ is the **full state** of the network, and $\mathbf{e}_i$ is 0–1 indicator vector with $i$th entry equal to 1. The state matrix $A$ is assumed to have nonnegative off-diagonal entries, while the diagonal entries are negative and satisfy $A_{l,l} \leq -\sum_{j=1, j\neq l}^{n} A_{l,j}$. This diffusive model form encompasses many of the canonical models for synchronization/consensus, diffusion, and spread in dynamical networks (e.g., [15], [16]). The in-built feedback control is modeled as providing an additive input $\mathbf{P}$ to the state dynamics. Specifically, the control vector $\mathbf{P}$ is assumed to be determined from the state vector $\mathbf{x}$ according to a Laplace-domain relationship of the following form: $\mathbf{P} = \mathbf{e}_q H_c(s)\mathbf{z}^T\mathbf{x}$, where the vector $\mathbf{z}$ indicates a combination of state variables that is being used in feedback, $\mathbf{e}_q$ is 0–1 indicator vector which identifies a single node $q$ which is being actuated by the in-built controller, and $H_c(s)$ is the transfer function of the in-built controller. We alternatively also consider the common circumstance that the built controller incorporates local proportional controllers at multiple nodes. In this case, the control vector takes the form $\mathbf{P} = -\sum_{j\in V_b} \mathbf{e}_j k_j x_j$, where $V_b$ is a subset of the nodes in the network. For both cases, our focus here is primarily on low or high gain controllers: that is, the controller transfer function (or proportional gain) is assumed to take the form $H_c(s) = kH_u(s)$, where $k$ is either sufficiently large or small. Finally, a single-input single-output channel is considered, which is defined by an additive input $u$ at a single network node $i$, and an output $y$ which measures the state at a single node (labeled $n$ without loss of generality).

The goal of this work is to study how the built low- or high- gain controller affects the input-output characteristics of the channel of interest, specifically its finite- and infinite-zero structure. We particularly seek for topological results, which give an indication of how the network's connectivity structure dictates the impact of the built controller. To develop topological analyses, it is convenient to associate a graph with the network dynamics. Specifically, a weighted digraph $G$ with $n$ vertices is defined, where each vertex $l = 1, 2, \ldots, n$ in the graph corresponds to the network node $l$. Formally, an arc (directed edge) is drawn from vertex $l$ to vertex $j$ in the graph ($l$,$j$ distinct) if and only if $A_{j,l} \neq 0$, and is assigned a weight of $A_{j,l}$. The vertices corresponding to the input and output network nodes are referred to as the input and output vertices. The state matrix $A$ can be viewed as a grounded Laplacian matrix associated with the directed graph. In this paper, the notation $d_{ab}$ is used the directed distance from vertex $a$ to vertex $b$ in the digraph $G$.

The introduced control problem is relevant to a number of network applications where multiple stakeholders are involved in regulation, or controllers are integrated in a piecemeal fashion. For instance, problems of this type may arise in the context of distributed-decision-making algorithms, coordination of groups of mobile autonomous vehicles, mobile sensor networking, infrastructure health monitoring, or price consensus for commodity markets [17], [18].

## III. Results

The results are organized as follows. First, a characterization of the zeros of a channel of interest when high-gain proportional controllers are applied at remote nodes is reviewed (III.A). The main focus of this work is to understand when high or low gain controllers (whether static or dynamic) can make the channel of interest nonminimum phase (III.B), and when they do not influence the phase properties of the channel (III.C). The results developed in this section assume that the network graph is strongly connected (i.e., there is a directed path between each pair of vertices); we focus on this case to avoid trivial cases where the built controller does not have any influence on the channel of interest. To improve readability, the results are presented in a concise for here, and the proofs are relegated to an appendix.

### *A. Control Schemes that Promote Minimum-Phase Dynamics: Review*

The following result from our previous work [6] characterizes the zeros of the input-output channel of interest, when proportional controllers of sufficiently large gain are applied remotely to the channel. The result is included because it serves as a basis for further analyses developed here, and because it relates to the main focus on low- and high- gain control.

This result, as well as further analyses presented here, require some further terminology regarding the network input-output model. The term *special input-output path* is used to refer to a path of minimum length (least number of edges) between the input and output in graph $G$. As defined before, the notation $d_{in}$ is used for the length of the special input-output path, i.e. for the distance between the input vertex $i$ and output vertex $n$. Additionally, we define a modified system based on a subgraph of $G$. Specifically, we consider the uncontrolled input-output model, with a subset of vertices deleted. Formally, let us consider a subset of vertices $V_b \subset \{1, \ldots, n\}$, which does not include the input and output vertices ($i$ and $n$). Let us also define the vectors $\mathbf{e}_r^{(V_b)}$ as a modified version of the vector $\mathbf{e}_r$, where the entries $i \in V_b$ are omitted. Similarly, $A^{(V_b)}$ is defined as a submatrix of $A$ obtained by deleting the rows and columns specified in $V_b$. Then, the **deletion subsystem** is defined as:

$$\dot{\mathbf{x}}^{(V_b)} = A^{(V_b)}\mathbf{x}^{(V_b)} + \mathbf{e}_i^{(V_b)}\widehat{u} \qquad (2)$$
$$\widehat{y} = {\mathbf{e}^{(V_b)}}_n^T \mathbf{x}^{(V_b)},$$

where $\mathbf{x}^{(V_b)}$, $\widehat{u}$, and $\widehat{y}$ are the state, input, and output, respectively. The deletion system (2) is associated with a weighted directed **deletion graph** $G^{(V_b)} = G - V_b$ . Also, we define $d_{in}^{(V_b)}$ as the distance between the input and output vertices (i.e. from vertex $i$ to $n$) in graph $G^{(V_b)}$.

Here is the result on zeros, see [6] for the proof.

*Theorem 1:* Consider the network input-output model (1). Assume that local proportional controllers are applied at network nodes in the set $V_b$, i.e. $\mathbf{P} = -\sum_{j\in V_b} \mathbf{e}_j k_j(x_j)$ where $k_j > 0$. Also, suppose that the input-output channel is remote from the local controllers ($i, n \notin V_b$), and that $d_{in}^{(V_b)} = d_{in}$. When the gains $k_j$ ($j \in V_b$) are scaled up, a subset of the zeros of (1) approach the zeros of the deletion system (2), while all other zeros are in open left half plane (OLHP).

The theorem immediately permits us to define local proportional control schemes that make the input-output model minimum phase. For example, one way to make a system minimum phase is to put high gain local controllers at all vertices adjacent to special input-output path.

### *B. Controls that Cause Nonminimum-Phase Dynamics*

In many large-scale networks, there is a significant concern that the actions of a control authority may make other regulation and control tasks difficult, or alter properties of remote input-output channels in undesirable ways. For instance, operators of the electric power grid have recognized that newly-integrated fast controls may alter performance of other controls, or cause unexpected disturbance responses [11]. These concerns suggest that, while controllers are typically designed to achieve desirable internal properties, they may incidentally alter the input-output characteristics of remote channels in undesirable ways (e.g., cause the channel to become nonminimum phase, or increase susceptibility to disturbances). Here, we identify conditions under which the built control causes the network's input-output channel to become non-minimum-phase.

First, we consider the possibility that proportional controllers applied at one or more nodes may result in nonminimum-phase dynamics for a channel of interest:

*Theorem 2:* Consider the input-output model (1). Assume that local proportional controllers are applied at the nodes in the set $V_b$, i.e. $\mathbf{P} = -\sum_{j\in V_b} \mathbf{e}_j k_j(x_j)$ where $k_j > 0$. Consider an input-output channel that is remote from the local controllers (i.e. $i, n \notin V_b$). The input-output model (1) is non-minimum phase for sufficiently scaled-up gains $k_j$ ($j \in V_b$), if $d_{in}^{(V_b)} \geq d_{in} + 2$.

The theorem shows that high-gain proportional control schemes necessarily incur nonminimum-phase dynamics, if they disrupt the special input-output path of the channel of interest. Precisely, nonminimum-phase dynamics result if the shortest input-output path with the controlled nodes removed is longer by at least two edges as compared to the original special input-output path. This result shows that proportional negative feedback controls, which always improve the internal stability of the diffusive network model, nevertheless can cause other channels to become nonminimum phase.

Next, the possibility that a linear dynamic controller applied at a single node can cause remote channels to become nonminimum-phase is explored. In the following two theorems, it is shown that a negative feedback controller with sufficiently large gain can cause non-minimum phase behavior if its relative degree is larger than a threshold, depending on its location in the network. On the other hand, a controller with sufficiently small gain can cause non-minimum phase behavior if its relative degree is less than a threshold, again depending on the controller's position in the network. These thresholds depend on the network structure (i.e. graph $G$) as well as the locations of the input node, output node, and control channel in the network. The two theorems address controllers applied at a single node and across a link, respectively.

*Theorem 3:* Consider the input-output model (1). Assume that the controller $\mathbf{P} = \mathbf{e}_q H_c(s)x_j$ is applied. Consider any input-output channel that is either remote from the built controller (i.e. $j \neq i, n$ and $q \neq i, n$), or across the built controller (i.e. $j = i$ and $q = n$). Assume that the controller $H_c(s) = kH_u(s)$ has relative degree $\boldsymbol{n_c}$. The input-output model (1) is non-minimum phase for:

- all sufficiently large negative feedback gain $k$ (i.e., for all $k \leq \widehat{k}$ for some $\widehat{k} < 0$) if $(d_{ij} + d_{qn} + 1 = d_{in} + d_{qj})$ and $(\boldsymbol{n_c} = 1 - d_{qj})$.
- all sufficiently small negative feedback gain $k$ (i.e., for all $0 > k \geq \widehat{k}$ for some $\widehat{k} < 0$) if $(d_{ij} + d_{qn} < d_{in} + d_{qj})$, $(\boldsymbol{n_c} \leq d_{in} - d_{ij} - d_{qn} - 2)$, and $(\boldsymbol{n_c} \geq -d_{qj} - 2)$

*Theorem 4:* Consider the input-output model (1). Assume that the controller $\mathbf{P} = \mathbf{e}_q H_c(s)(x_j - x_k)$ is applied. Consider any input-output channel. Also, assume that the controller $H_c(s) = kH_u(s)$ has relative degree $\boldsymbol{n_c}$. The input-output model (1) is non-minimum phase for:

- all sufficiently large negative feedback gain $k$ (i.e., for all $k \leq \widehat{k}$ for some $\widehat{k} < 0$) if: $(d_{ij} < d_{ik})$, $(d_{ij} + d_{qn} + 1 = d_{in} + d_{qj})$, $(\boldsymbol{n_c} = 1 - d_{qj})$, and $(d_{qj} < d_{qk})$.
- all sufficiently small negative feedback gain $k$ (i.e., for all $0 > k \geq \widehat{k}$ for some $\widehat{k} < 0$) if $(d_{ij} \neq d_{ik})$, $(d_{qj} \neq d_{qk})$, $(\min\{d_{ij}, d_{ik}\} + d_{qn} < d_{in} + \min\{d_{qj}, d_{qk}\})$, $(\boldsymbol{n_c} \leq d_{in} - \min\{d_{ij}, d_{ik}\} - d_{qn} - 4)$, and $(\boldsymbol{n_c} \geq -1 - \min\{d_{qj}, d_{qk}\})$

In addition to the cases that are discussed in Theorems 3 and 4, there are many other cases where high gain or low gain may cause nonminimum-phase dynamics, however many of these circumstances are concerned with positive feedback controls and hence may be of less interest.

### C. Control Schemes that Do Not Alter Channel Phase Characteristics

Often, it is important to ascertain whether a control scheme can alter a channel's phase characteristics, and specifically whether the control will preserve minimum phase dynamics on the channel. The following theorems identify control schemes that are guaranteed to maintain minimum-phase dynamics, i.e. not to change a minimum-phase channel to a nonminimum-phase channel. For this development, we say that the *phase property of the network input-output model is maintained*, to indicate that the model remains strictly minimum phase (respectively strictly nonminimum phase) upon inclusion of the controller $\mathbf{P}$ when the uncontrolled model is strictly minimum phase (respectively strictly nonminimum phase).

In the previous work [6], it was shown that low-gain proportional controllers necessarily only move system eigenvalues, and hence poles of any defined channel, by a small amount. In contrast, they can introduce/remove zeros or cause zeros to jump in general. The graph-theoretic condition in the following Theorem 5 from previous work [6] is sufficient to guarantee that this does not happen, and hence that the phase property is maintained.

*Theorem 5:* Consider the network input-output model (1), and assume that a controller $\mathbf{P} = k\mathbf{e}_q\,(x_j - x_q)$ is applied. Also, consider the network graph $G$ as well as a modified graph $\tilde{G}$ where the directed edge $j \to q$ is added to $G$. Consider any input-output channel. If the distance between the input and output vertices in the original and modified graphs is identical, then the phase property of the network input-output model is maintained for any sufficiently small gain $k$ ( i.e. for all $k < f$ for some threshold $f < \infty$).

The result shows that small-gain controllers acting across network edges maintain the phase property of the uncontrolled system, if they do not introduce any new dependencies that alter the distance between the input-output distance in the network graph for the channel of interest.

*Remark:* The conditions for maintaining the phase property in Theorem 5 is necessarily satisfied, if the proportional controller is local ($q = j$), or acts across a link that is already present in the network. Likewise, it is necessarily satisfied if the channel of interest is local ($i = n$), or is placed across an existing network link.

The following two theorems give more general conditions under which a proportional controller of the form $\mathbf{P} = k\mathbf{e}_q(x_j - x_q)$, or a local proportional controller $\mathbf{P} = -k\mathbf{e}_j(x_j)$, does not change the phase property of the network input-output model. The theorems require some further graph-theoretic notation for the network input-output model (1). In particular, let us consider a set of three vertices $V_d = \{i, j, n\}$ in the network graph. The vertex $r$ is said to be a *disjointing vertex* of the set $V_d$ if following paths pass through or reach the vertex $r$: 1) all paths from vertex $i$ to vertex $j$, 2) all paths from vertex $i$ to vertex $n$, 3) all paths from vertex $j$ to vertex $n$. We note that the disjointing vertex may be one of the vertices in the set $V_d$. The concept of a disjointing vertex is illustrated in Fig. 1. In this example, vertex 1 is a disjointing vertex for the set $V_d = \{1, 3, 15\}$, vertex 2 is a disjointing vertex for both sets $V_{d_1} = \{1, 2, 15\}$ and $V_{d_2} = \{1, 5, 15\}$.

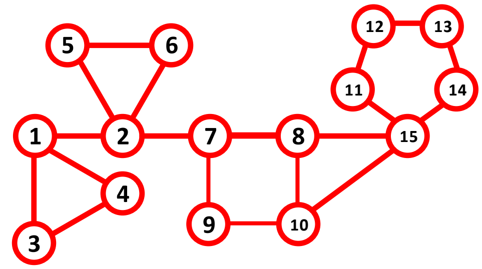


Fig. 1: Graph $G$ with several disjointing vertices.

*Theorem 6:* Consider the network input-output model (1). Assume that the local proportional controller $\mathbf{P} = -k_j\mathbf{e}_j(x_j)$ is applied, and consider any input-output channel where the input and output are not collocated (i.e. $i \neq n$). The phase property of the model is maintained if the network graph $G$ has at least one disjointing vertex for the set $V_d = \{i, j, n\}$.

Theorem 6 is illustrated in an example. In particular, consider an input-output model (1) with network graph $G$ shown in Fig. 1, with input at vertex $i = 1$ and output at vertex $n = 15$. The local proportional controller $\mathbf{P} = -k_j\mathbf{e}_j(x_j)$ does not change the phase property of the input-output model (1) if $j \in \{1, 2, 3, 4, 5, 6, 7, 11, 12, 13, 14, 15\}$, because a disjointing vertex can be found for the set $V_d = \{i, j, n\}$ in this case.

*Theorem 7:* Consider the input-output model (1). Assume that the proportional controller $\mathbf{P} = k\mathbf{e}_q(x_j - x_q)$ is applied, and consider an input-output channel where the input and output are not collocated (i.e. $i \neq n$). The phase property of the model is maintained if there is a common disjointing vertex in the network graph $G$ for the sets $V_{d_1} = \{i, j, n\}$ and $V_{d_2} = \{i, q, n\}$.

Next, conditions under which dynamic controllers maintain the phase property of a system are discussed. First, a special case is identified where the transfer function $H_c(s)$ does not influence the phase property of the input-output channel, provided that the controller is stable.

*Theorem 8:* Consider the input-output model (1). Suppose that the local controller $\mathbf{P} = \mathbf{e}_j H_c(s) x_j$ is applied where the controller transfer function $H_c(s)$ is stable. Consider any input-output channel that is adjacent to the built control (i.e. $j = i$ or $j = n$). The phase property of the input-output model (1) is maintained.

The following Theorem discusses the case where low gain controller does not change the phase behavior for the interested channel.

*Theorem 9:* Consider the input-output model (1). Assume that the controller $\mathbf{P} = \mathbf{e}_q H_c(s) x_j$ is applied. Consider any input-output channel that is either remote from the built controller (i.e. $j \neq i, n$ and $q \neq i, n$), or across the built controller (i.e. $j = i$ and $q = n$). Assume that the controller transfer function $H_c(s) = kH_u(s)$ is stable and has relative degree $\boldsymbol{n_c}$. The phase property of the model is maintained for any sufficiently small gain $k$ ( i.e. for all $k < f$ for some threshold $f < \infty$) if one of the following condition satisfies:

- $(d_{ij} + d_{qn} > d_{in} + d_{qj})$ and $(\boldsymbol{n_c} \geq -1 - d_{qj})$.
- $(d_{ij} + d_{qn} < d_{in} + d_{qj})$, $(\boldsymbol{n_c} \geq d_{in} - d_{ij} - d_{qn} - 1)$, and $(\boldsymbol{n_c} \geq -d_{qj} - 2)$

## IV. Examples

The graph-theoretic results on control channel interactions are illustrated and enhanced using three examples. For all of the examples, we consider an input-output model (1) with 8 nodes, with input and output at nodes 1 and 3, respectively. A common network graph $G$ is considered for the examples, as shown in Fig. 2, however we modify the state matrix and the controller among the examples.

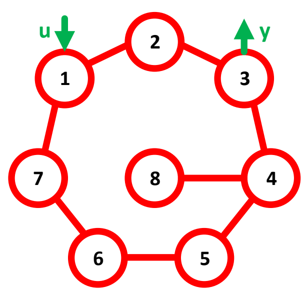


Fig. 2: Graph $G$ associated with a network with 8 nodes.

In the examples, we consider the case where local proportional controllers are applied to a subset of network nodes: $\mathbf{P} = -\sum_{j \in V_b} \mathbf{e}_j k_j(x_j)$ where the $k_j$ are positive (i.e. a negative feedback control is used). Controls of this form improve stability or damping of the network, in the sense that the real parts of poles become more negative as the gains are increased. The analyses developed in the paper have suggested that such controllers may or may not move zeros in desirable ways, in the sense that they may preserve or promote minimum-phase dynamics or may lead to nonminimum-phase dynamics. In previous work [6], we showed that for a similar example which was non-minimum phase, based on Theorem 1, a local proportional controller at node 7 ($\mathbf{P} = -\mathbf{e}_7 k_7(x_7)$) with a high-gain controller promoted minimum-phase dynamics. Here, We consider two cases where, per the presented theorems, the dynamics may transition from nonminimum phase to minimum phase or vice versa.

### A. Example 1

For the first example, the state matrix $A$ is the following:

$$A = \begin{bmatrix} -1.2 & 0.1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0.1 & -0.3 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & -1.2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3.1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2.1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2.1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & -2.1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1.1 \end{bmatrix}$$

In this case, the uncontrolled network input-output model is minimum phase. Now consider a local proportional controller at node 2 ($\mathbf{P} = -\mathbf{e}_2 k_2(x_2)$). We note here that vertex 2 is on the special input-output path in the network graph. Per Theorem 2, increasing the controller gain should yield a nonminimum-phase dynamics. In Fig. 3, the real part of the dominant zero (i.e. the zero with maximum real part) is seen to move from negative values to the positive values as the gain is increased, and so the input-output model become non-minimum phase.

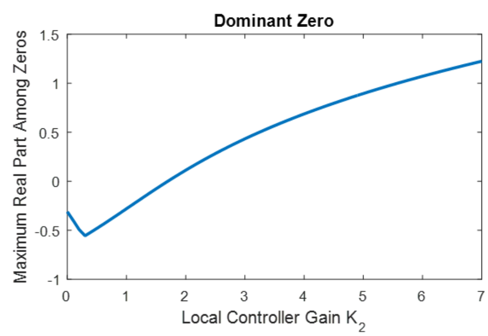


Fig. 3: The dependences of the dominant zero location (the largest real part among the zeros) on local controller gain.

### B. Example 2

The state matrix $A$ for the second example is the following:

$$A = \begin{bmatrix} -1.64 & 0.54 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0.54 & -0.65 & 0.01 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & -1.2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3.1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2.1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2.1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & -2.1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1.1 \end{bmatrix}$$

In this case, the uncontrolled input-output model is minimum phase. Now consider a local proportional controller applied at node 8 ($\mathbf{P} = -\mathbf{e}_8 k_8(x_8)$), We note that vertex 8 is not on any path from the input vertex to output vertex. In this case, as the gain of this controller is increased (see Fig. 4), first the input-output channel becomes non-minimum phase, and then with further increase it again becomes minimum phase.

In conclusion, we see that local proportional controllers might cause minimum phase behavior or non-minimum phase behavior in other channels.

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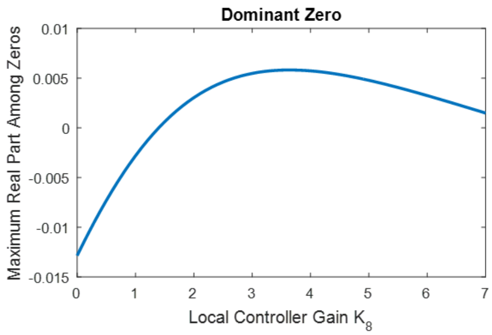


Fig. 4: The dependences of the dominant zero location (the largest real part among the zeros) on local controller gain.

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