Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 52-3 (2019) 25–30

Structural Controllability of Linear Dynamical Networks with Homogeneous Subsystems ⋆

Mengran Xue and Sandip Roy *

* *Washington State University, Pullman, WA 99164 USA (e-mail: {morashu,sandip}@wsu.edu).*

Abstract: Structural controllability of a linear dynamical network made up of interconnected homogeneous subsystems is examined. Subsystem- and network- level structural controllability are shown to be necessary but not sufficient for structural controllability of the full model. It is shown that the presence of certain high-multiplicity structural modes, which we call structural network-invariant modes, are barriers to structural controllability. An equivalence between structural network-invariant modes and the classical notion of a structural decentralized fixed mode is obtained, which allows testing and characterization of structural controllability.

Keywords: Dynamical networks, structural controllability, structural fixed modes

1. INTRODUCTION

Dynamical network models with interconnected homogeneous subsystems have been widely studied (Wu (1995a,b); Watts (1998); Li (2010); Yu (2011); Wang (2011); Xue (2014, 2017)). Such models were originally developed to represent synchronization phenomena in coupled-oscillator circuits and emergent processes in nature (Wu (1995a,b); Watts (1998)). They have since been used for myriad large scale systems ranging from the power grid to vehicle teams (Li (2010); Yu (2011); Wang (2011); Xue (2014, 2017)).

A primary focus of the research on linear network models with homogeneous subsystems has been to establish conditions for stability and stabilization, in terms of the network's graph topology. The foundation for these stability analyses is a decomposition of the model's spectrum, in terms of global (network-level) and subsystem-level constructs (Wu (1995a,b); Watts (1998); Li (2010); Yu (2011); Wang (2011)). Recently, researchers have begun to study input-output properties of the models, including controllability, observability, centralized and decentralized fixed modes, and transfer-function zeros (Xue (2014, 2017); Abad (2015); Hao (2018); Wang (2016, 2017); Xue (2018); Zhang (2014); Xue (2018)). These input-output analyses have also sought to decompose model properties in terms of global and subsystem- level constructs.

There has been a particular interest in the controllability of linear coupled-subsystem network models (Wang (2016, 2017); Xue (2018); Zhang (2014); Xue (2018); Trumpf (2018)). Several studies posited that the condition for controllability could be decomposed into network-level and subsystem- level conditions (e.g. Zhang (2014)). However, a sequence of recent studies have demonstrated that such a decomposition is not possible, in the case where the subsystems are multi-input multi-output devices (Xue (2017); Wang (2016, 2017); Xue (2018); Zhang (2014); Xue (2018); Trumpf (2018)). In particular, the network model may be uncontrollable even when the network-level dynamics is controllable, and the subsystems are observable and controllable. The complexity in the controllability analysis arises due to subtleties in the eigenvector analysis of the coupled-subsystem model in the repeated-eigenvalue case. Very recently, we have developed some characterizations of repeated eigenspaces in the coupled-subsystem model, which give insight into the intertwining of network-level and subsystem-level conditions for controllability (Xue (2018)). Specifically, these characterizations demonstrate that the model may have repeated eigenvalues with very high multiplicity, which we term network-invariant modes, that are essential barriers to controllability.

In many large-scale systems, notions of *structural controllability* – which are concerned with controllability across state-space models with a specified zero pattern, rather than for a particular set of model parameters – are of significant interest. Structural notions are often important because they indicate robustness to parametric uncertainties, which are ubiquitous in large-scale system applications. For this reason, structural controllability of canonical network models has been widely studied (Pequito (2016); Zamani (2009); Rahimian (2013); Chapman (2013); Carvalho (2017); Guan (2017)), following on an older literature on structural controllability of linear systems (e.g. Lin (1974)). These efforts have also distinguished between strong structural controllability notions (where every network of the given structure must be controllable) and weak notions (where only one network of the given structure need be controllable). Particularly relevant to this work, structural controllability has been studied for linear network models that comprise interconnections of heterogeneous linear subsystems, however the focus was on a numerical technique for evaluating (weak) structural controllability (Carvalho (2017)). Also of significant relevance, structural controllability of a homogeneous-

⋆ The authors acknowledge support from United States National Science Foundation grants 1545104 and 1635184.

2405-8963 © 2019, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.
Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2019.06.005

subsystem model has also been considered in Guan (2017), however this study considers design of the subsystem-level dynamics to achieve controllability rather than analyzing already-built networks.

In this work, we examine the structural controllability of a network model comprising couplings of arbitrary homogeneous linear subsystems. We demonstrate via an example that the coupled-subsystem model may be structurally uncontrollable even though the global and subsystem- level models are both strongly structurally controllable. We then argue that structural controllability is lost due to certain structural modes of high multiplicity, which we call*structural network-invariant modes.* A characterization of these modes in terms of the subsystem's structural fixed modes is developed, and their implications on controllability are also examined.

2. MODELING AND PROBLEM FORMULATION

A network model made up of interconnected homogeneous linear subsystems, which can also be actuated by external inputs, is considered. Formally, a model with n nodes labeled $1, \ldots, n$ and m inputs labeled $1, \ldots, m$, is defined. The state $\mathbf{x}_i \in R^N$ of each node i is governed by:

$$\dot{\mathbf{x}}_i = A\mathbf{x}_i + B(\sum_{j=1}^{n} g_{ij} C\mathbf{x}_j + \sum_{q=1}^{m} s_{iq}\mathbf{u}_q), \quad (1)$$

where $A \in R^{N\times N}$, $B \in R^{N\times M}$, $C \in R^{M\times N}$, g_{ij} and s_{iq} are scalar weights, and $\mathbf{u}_q \in R^M$ is the qth external input.

We refer to the model given by Equation 1 as the **coupled-subsystem network model** (or simply the full model). We note that the triple (C, A, B) describes the identical (square) subsystem at each node, hence we refer to the triple as the **subsystem model**. The local input to each subsystem is a weighted linear combination of outputs from other subsystems as specified by the $n \times n$ network matrix $G = [g_{ij}]$. and external inputs as specified by the $n \times m$ matrix $S = [s_{iq}]$. Thus, the pair (G, S) specifies the topology of interactions among the subsystems, and of the external inputs' impacts on the subsystems; for this reason, we refer to the pair as the **global model** or network-level model. We also refer to the matrix G as the **network matrix** of the model. The global model (G, S) and the subsystem model (C, A, B) together entirely specify the coupled-subsystem network model.

The coupled-subsystem network model can be expressed in vector form using the Kronecker product notation:

$$\dot{\mathbf{x}} = (I_n \otimes A + G \otimes BC)\mathbf{x} + (S \otimes B)\mathbf{u}, \quad (2)$$

where I_n is an $n \times n$ identity matrix, $\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_n \end{bmatrix}$, and $\mathbf{u} = \begin{bmatrix} \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_m \end{bmatrix}$. This state-equation form makes explicit the dependence of the coupled-subsystem network model on the subsystem and global models.

The focus of this study is on the structural controllability of the coupled-subsystem network model, and its relationship to the global and network-level models. The structure of the model is encoded in the zero-nonzero pattern of its defining matrices (C, A, B, G, and S). A number of notations can be used to indicate the zero-nonzero patterns of matrices. We follow the notation used in e.g. Tsitsiklis (1984), by indicating the zero entries in the matrices as '0', and the non-zero entries as '*'. The non-zero entries are assumed to be real.

Following the recent literature, we consider both strong and weak notions of structural controllability for the coupled-subsystem network model (Chapman (2013)). These notions are defined as follows:

1) The coupled-subsystem network model is said to be strongly structurally controllable if, for all matrices (C, A, B, G, S) with the specified structure, the model (Equation 1) is controllable.

2) The coupled-subsystem network model is said to be weakly structurally controllable if there exists matrices (C, A, B, G, S) with the specified structure for which the model (Equation 1) is controllable.

3) The coupled-subsystem network model is said to be structurally uncontrollable if it is not weakly structurally controllable, i.e. no choice of parameter matrices permits controllability.

Our analysis also requires parallel definitions for structural controllability and observability for the subsystem and global models, as presented next:

- The global model is said to be strongly structurally controllable if the pair (G, S) is controllable for all matrices G and S of the specified structure.
- The global model is said to the weakly structurally controllable if there exist matrices G and S of the specified structure, such that (G, S) is controllable.
- The global model is said to be structurally uncontrollable if it is not weakly structurally controllable.
- The subsystem model is said to be strongly structurally controllable if the pair (A, B) is controllable for all matrices A and B of the specified structure.
- The subsystem model is said to the weakly structurally controllable if there exist matrices A and B of the specified structure, such that (A, B) is controllable.
- The subsystem model is said to be structurally uncontrollable if it is not weakly structurally controllable.
- The subsystem model is said to be strongly structurally observable if the pair (C, A) is observable for all matrices C and A of the specified structure.
- The subsystem model is said to the weakly structurally observable if there exist matrices C and A of the specified structure, such that (C, A) is observable.
- The subsystem model is said to be structurally unobservable if it is not weakly structurally observable.

In many domains, the network matrix may be further restricted. For instance, network matrices for diffusion, synchronization processes, and some linearized infrastructure-dynamics models are typically Metzler (i.e. having nonnegative off-diagonal entries). It is also common that the network matrix G is restricted to be symmetric or diagonally symmetrizable. We also consider structural controllability subject to these restrictions on the network matrix.

Our goal in this study is to examine whether strong and weak structural controllability of the coupled-subsystem model can be decomposed into network-level and subsystem-level conditions, and in turn to give characterizations for structural controllability through a modal approach. Decompositions of this sort are appealing because they can allow application of graph-theoretic results on structural controllability for scalar network synchronization/consensus processes, and also enable simplified numerical analysis of structural controllability.

3. CONDITIONS FOR STRUCTURAL CONTROLLABILITY: NEGATIVE RESULT

We show that, while structural controllability of the subsystem and global models are necessary for structural controllability of the full network model, they are not sufficient. First, the following two lemmas affirm that subsystem- and global- conditions are necessary for structural controllability of the full model. We omit the proofs of these results, because they closely follow standard (non-structural) controllability analyses of the full model (Xue (2018); Zhang (2014)).

Lemma 1. The coupled-subsystem network model is strongly structurally controllable only if the global and subsystem models are both strongly structurally controllable.

Lemma 2. The coupled-subsystem network model is weakly structurally controllable only if the global and subsystem models are both weakly structurally controllable.

Remark: Similar necessary conditions can be obtained when the network matrix is restricted to be Metzler, symmetric, or diagonally symmetrizable.

Remark: Lack of structural observability of the subsystem model also restricts the structural controllability of the full model, because it prevents indirect regulation of subsystems via remote inputs. In general, subsystem observability is required for controllability of the full model, unless the external inputs are sufficient in number and richness to directly move the state of all network components at will. We will revisit this dependence later.

Previous work on exact controllability has shown that the full model may be uncontrollable even though the subsystem and global models are controllable, because of the presence of high multiplicity modes (Xue (2018)). One might hope that considering structural matrices may eliminate such repeated modes and hence allow decomposition of the controllability condition. However, the following example shows that the full coupled-subsystem network model may be only weakly structurally controllable or even structurally uncontrollable, even though the subsystem model is strongly structurally controllable and observable, and the global model is strongly structurally controllable.

The example is defined by the following structured matrices: $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & * \\ * & 0 & * \end{bmatrix}$, $B = \begin{bmatrix} * & 0 \\ 0 & * \\ 0 & 0 \end{bmatrix}$, $C = \begin{bmatrix} * & 0 & 0 \\ 0 & * & 0 \end{bmatrix}$, $G = \begin{bmatrix} * & * & 0 \\ * & * & * \\ 0 & * & * \end{bmatrix}$, and $S = \begin{bmatrix} * \\ 0 \\ 0 \end{bmatrix}$. Let us first evaluate the structural controllability of the subsystem model for the example. To do so, consider the controllability matrix for the subsystem model, given by $\mathcal{C}_s = \begin{bmatrix} B & AB & A^2B \end{bmatrix}$. The first three columns of $\mathcal{C}_s$ take the form $\begin{bmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{bmatrix}$, hence these columns are necessarily linearly independent. The subsystem model is therefore seen to be strongly structurally controllable. By an analogous argument, the subsystem model can also be verified to be strongly structurally observable.

Next, let us consider the structural controllability of the global model. The controllability matrix for the global model is given by $\mathcal{C}_g = \begin{bmatrix} S & GS & G^2S \end{bmatrix}$. The controllability matrix has the structural form $\mathcal{C}_g = \begin{bmatrix} * & * & ? \\ 0 & * & ? \\ 0 & 0 & * \end{bmatrix}$, where '?' indicates an entry that may be either zero or nonzero. The three columns of $\mathcal{C}_g$ are necessarily linearly independent, and hence the global model is strongly structural controllable.

Finally, let us consider structural controllability of the full coupled-subsystem network model for the example. Our main conclusion is that the full model cannot be controllable, except perhaps for some very special choices of the network matrix G. To understand why, we recall that the eigenvalues of the state matrix $I \otimes A + G \otimes BC$ can be found as the union of the eigenvalues of $A + \lambda_i BC$, where λ_i are the eigenvalues of the nextwork matrix G. However, notice that the matrix $Q = A + \lambda BC$ has the structure $Q = \begin{bmatrix} * & 0 & 0 \\ 0 & * & * \\ * & 0 & * \end{bmatrix}$, where only the upper-left and middle diagonal entries depend on λ (and may be complex numbers). From this structure, it is immediately clear that the lower right entry, say z, is an eigenvalue of $Q = A + \lambda BC$ for any λ. It thus follows that z (which depends on the subsystem model parameters but not the global model) is an eigenvalue of each of the blocks $A + \lambda_i BC$. In consequence, the eigenvalue z has algebraic multiplicity of at least 3, no matter what the the coupled-subsystem model's parameters are. Provided that the matrix G is diagonalizable, it also follows that the geometric multiplicity of the eigenvalue is also at least 3. However, notice that the full model has only two input channels (the input matrix is defined by $S \otimes B$, which has dimension 9×2). Thus, it follows that the full coupled-subsystem network model is uncontrollable, except perhaps in the special case that the network matrix G is not diagonalizable. The above discussion demonstrates that the coupled-subsystem network model for the example is not strongly structurally controllable, and indeed generic choices for the model parameters will lead to uncontrollability.

Next, it is instructive to study whether the presented example is weakly structurally controllable, or in fact structurally uncontrollable. Toward such a characterization, we note that the model can be controllable only if the network matrix G is not diagonalizable. However, for this particular example, G is necessarily diagonalizable if it is restricted to be Metzler. This is because G is an irreducible Metzler matrix whose digraph is a tree (i.e., the graph has no cycles of length greater than 2). Thus, the matrix G in this example can always be symmetrized using a diagonal similarity transformation regardless of the parameter values and hence is diagonalizable. Thus, the full model is structurally uncontrollable if the network matrix is restricted to be Metzler. When there is no restriction on G, the network matrix G may not be diagonalizable, which leaves open the possibility for controllability. Indeed, it is possible to construct network matrices of the specified structure which are not diagonalizable. However, even for these examples, we have found that the coupled-subsystem network model is not controllable. We thus conjecture that the model is in fact structurally uncontrollable.

Conceptually, the example was chosen so that the network model's modes are constrained to be modes of a perturbation of the subsystem state matrix A, where only two of the diagonal entries can be perturbed. In consequence, the full model has a mode equal to the bottom-right entry of A with high multiplicity, and thus controllability is lost unless a large number of control inputs can be used. The result is structural since the high multiplicity of the mode is independent of the particular entries of A and the diagonal perturbation. We note that the constraint on the open-loop modes is an essential consequence of the network's interconnection structure: the subsystem in isolation would not be constrained in this way.

Remark: Our negative result contrasts with the structural analyses of heterogeneous networks in (Guan (2017)).

4. STRUCTURAL NETWORK-INVARIANT MODES AND THEIR IMPLICATIONS

In the example in Section III, strong structural controllability is lost because the coupled-subsystem network model has a structural eigenvalue with high multiplicity, which serves as a barrier to controllability. In this section, we identify structures for which the coupled-subsystem network model necessarily has modes (eigenvalues) of high multiplicity, and study the implications of such modes on the controllability of the network model.

The coupled-subsystem network model necessarily has an eigenvalue of high multiplicity, if $A + \lambda BC$ alway has an eigenvalue μ that is invariant with respect to the complex scalar λ. In this case, each of the matrices $A + \lambda_i BC$ has μ as an eigenvalue, and hence the algebraic multiplicity of μ for the full model is at least equal to the number of nodes n. Thus, it is of interest to identify conditions under which the $A + \lambda BC$ has an eigenvalue that is invariant with respect to λ. This motivates the following definition for a structural network-invariant mode:

Definition 1. Consider the coupled-subsystem network model with structured parameter matrices (C, A, B, G, S). The model is said to have a structural network-invariant mode if, for every A, B, and C of the specified structure, the matrix $A + \lambda BC$ has a common eigenvalue μ for all values of the complex scalar λ. We note that the eigenvalue μ may depend on the triple (C, A, B), but there must be an invariant eigenvalue with respect to λ for each triple.

The example coupled-subsystem network model introduced in Section III has a structural network-invariant mode, with the invariant eigenvalue μ equal to the lower-right entry of the matrix A (called q in Section III).

Two types of analyses are pursued with regard to the structural network-invariant modes. First, we develop tests for whether a coupled-subsystem network model has a structural network-invariant mode. Second, we explore the implications of structural network-invariant modes on the structural controllability of the full model.

Tests for and characterizations of structural network-invariant modes can be developed by recognizing an equivalence with *structural decentralized fixed modes* for the subsystem model. To develop the equivalence, let us first review the concept of a structural decentralized fixed mode (see Tsitsiklis (1984); Sezer (1981)) , in the context of the subsystem model:

Definition 2. Consider the subsystem model in the case where the parameter matrices A, B, and C are structured matrices. The subsystem model is said to have a structural decentralized fixed mode if, for every choice of A, B, and C of the specified structure, the subsystem model has a decentralized fixed mode in the sense that $A + BKC$ has a common eigenvalue for all real diagonal matrices K.

The following theorem formalizes the equivalence between structural network-invariant modes and structural decentralized fixed modes:

Theorem 1. The coupled-subsystem network model has a structural network-invariant mode if and only if the subsystem model has a structural decentralized fixed mode.

Proof: First assume that the subsystem model has a structural decentralized fixed mode. Consider a particular subsystem model (C, A, B). Then, by the definition of a structural decentralized fixed mode, the matrix $A + BKC$ has a common eigenvalue μ for all real diagonal matrices K. Choosing $K = \lambda I$, where λ is a real scalar, we immediately find that μ is an eigenvalue of $A + \lambda BC$ for all real λ. To check whether μ is also an eigenvalue $A + \lambda BC$ for complex λ, let us consider solving for λ such that $A+\lambda BC$ has eigenvalue μ. For these λ, $det(A+\lambda BC-\mu I) = 0$. Notice that the determinant is a polynomial in λ of degree at most n (it may be either identically 0 or a nondegenerate polynomial). Hence, the equation either has a finite set of solutions λ, or holds for all complex λ. Since the equation has already been shown to hold for all real λ, it thus holds for all λ in the complex plane. Since this argument holds for any triple (C, A, B), it follows that the full model has a structural network-invariant mode.

Conversely, assume that the coupled-subsystem network model has a structural network invariant mode. Now consider a particular triple (C, A, B). It follows that $A + \lambda BC$ has a set of modes, say $\mathcal{M}$, which are common for all real λ (there is at least one, and may be more such invariant modes). Notice that the modes in $\mathcal{M}$ are a subset of the eigenvalues of A. Notice further that, for all but perhaps a finite set of λ, the remaining eigenvalues of $A + \lambda BC$ are distinct from those of A. Now consider the eigenvalues of $R = A + \lambda B(I + \epsilon\widehat{K})C$, where $\widehat{K}$ is an arbitrary diagonal matrix. For all sufficiently small ϵ, each eigenvalue of R is arbitrarily close to one of the eigenvalues of $A + \lambda BC$. Next, notice that R could alternately be written as $R = A + \lambda B\tilde{C}$, where $\tilde{C} = (I + \epsilon\widehat{K})C$ has the same structure as the matrix C. However, we recall that the coupled-subsystem network model is assumed to have a structural network invariant mode, which means that $R = A + \lambda B\tilde{C}$ must have an eigenvalue in common with A. Thus, one of the eigenvalues of R must be in the set $\mathcal{M}$. Since this argument holds for any $\widehat{K}$, we have thus shown that the matrices $A+BKC$ have a fixed eigenvalue in the set $\mathcal{M}$, for all diagonal K of the form $K = \lambda(I+\epsilon\widehat{K})$, which forms a cone in the space defined by the diagonal entries of K. Iterating the argument using small perturbations at the boundary of the cone, it eventually follows that matrices $A + BKC$ have a fixed eigenvalue in the set $\mathcal{M}$ for all diagonal real matrices K. If the argument is initiated using the diagonal K which has the lowest-cardinality set of fixed eigenvalues, it then follows that matrices $A+BKC$ have a common set of eigenvalues for all real diagonal K. It thus follows that the triple (C, A, B) has a decentralized fixed mode. Since the argument holds for all triples (C, A, B), the model has a structural decentralized fixed mode. ■

Remark: The converse argument in the proof crucially depends on the fact that the network-invariant mode is structural. An immediate further consequence is that network-invariant modes do not depend on the entries of B and C, only possibly on A.

The equivalence given in Theorem 1 enables algorithmic testing for structural network-invariant modes, using a standard algorithm for finding structural decentralized fixed modes (Tsitsiklis (1984)). The equivalence also allows characterizations of structural network-invariant modes based on known characterizations of decentralized-fixed modes. For instance, if the subsystem is single-input single-output (SISO), it is immediate that the subsystem model has structural decentralized fixed modes if and only if the subsystem model is either structurally uncontrollable or structurally unobservable. Thus, for the SISO subsystem case, the full model has structural network-invariant modes if and only if the subsystem model is either structurally uncontrollable or structurally unobservable. In contrast, for MIMO subsystems, structural network-invariant modes may be present even if the subsystem is strongly structurally controllable and observable, as the example shows. In summary, the equivalence allows analysis of structural network-invariant modes using machinery for testing and characterization of structural decentralized fixed modes. The equivalence also clarifies that the structural network-invariant modes include subsystem uncontrollable and unobservable modes.

Remark: The above theorem exposes an interesting difference between structural network-invariant modes as compared to the network-invariant modes concept introduced in Xue (2018). In general, network-invariant modes are a superset of the decentralized fixed modes of the subsystem model (Xue (2018)), however structural network-invariant modes are precisely identical to the structural decentralized fixed modes of the subsystem model.

Finally, we discuss implications of structural network-invariant modes on the structural controllability of the coupled-subsystem network model. One main insight is that, if the model has a structural network-invariant mode, the full model necessarily has an eigenvalue with algebraic multiplicity of at n. In these circumstances, the full model can only be structurally controllable if the the network has a sufficient number of inputs, except perhaps if G cannot be diagonalizable. This notion is formalized next:

Theorem 2. Consider a coupled-subsystem network model which has a structural network-invariant mode. Assume that the number of external input channels m satisfies $m < \frac{n}{M}$, where M is the number of input variables per channel (the number of columns of B). Also assume that the network matrix G does not have a structural defective mode at the origin. Then the coupled-subsystem network model cannot be strongly structurally controllable.

Proof: The eigenvalues of the full coupled-subsystem network model are the union of the eigenvalues of $A + \lambda_i BC$, where λ_i are the n eigenvalues of G. If the model has a structural network-invariant mode, it is then immediate that, for the coupled-subsystem network model, this mode has algebraic multiplicity of at least n. Under the assumption on G, it immediately follows that G is diagonalizable for at least one choice of the global model (G, S) of the given structure. If G is diagonalizable, the structural network invariant mode also necessarily has geometric multiplicity of at least n, since the eigenvalues of the full coupled-subsystem network model are those of a block diagonal matrix with diagonal blocks equal to $A + \lambda_i BC$ in this case. For this choice of G, since one mode has geometric multiplicity of at least n, it follows from basic linear systems concepts that at least n input variables are needed for controllability. Thus, if the total number of input variables mM is less than n, the model is not controllable for this choice. Thus, strong structural controllability is lost. ■

Several remarks about this result are needed:

1) The mild condition on G that it does not have a structural defective mode is sufficient to guarantee diagonalizabilty for at least one choice of G. This eliminates structures such as $G = \begin{bmatrix} 0 & * \\ 0 & 0 \end{bmatrix}$ are always defective.

2) An immediate consequence is that, if the subsystem model is not strongly structurally observable, then a significant number of input channels is needed for controllability of the full model ($m \geq \frac{n}{M}$). This is because unobservability of the subsystem model implies that the coupled-subsystem network model has a structurally network invariant mode, and the result above applies.

3) If the full model has a structural invariant mode and has insufficient input channels ($m < \frac{n}{M}$), in many cases the model will not only lose strong structural controllability, but become structurally uncontrollable. This is always the case when the network matrix is restricted to be symmetric, diagonally symmetrizable, or Metzler with an associated acyclic graph. In these cases, the matrix G can be diagonalized, which means controllability of the full model is lost no matter how the subsystem model is chosen. Even if G is defective, fewer inputs generally do not suffice.

4) The theorem can be refined to show that any partition of the network must have a sufficient number of input channels for strong structural controllability, see Xue (2018) for a parallel result concerned with exact controllability.

The reader will notice that the analyses of controllability have been phrased in terms of the presence/absence of structural-invariant modes, which are a property of the subsystem model. The global model influences controllability, as follows. If the subsystem model does not have structural network invariant modes, then structural controllability resolves to structural controllability of the global model. We leave a full treatment to future work.

5. CONCLUSIONS

We have demonstrated that structural controllability for networks comprising homogeneous subsystems may not decompose into subsystem-level and network-level (global) conditions, due to the presence of structural network invariant modes. We have also obtained an equivalence between structural network-invariant modes and the subsystem structural decentralized fixed modes, and discussed the limitations on controllability imposed by invariant modes. An important next step is to study when the subsystem structure makes control difficult (e.g. requiring much energy), even if it does not prevent control entirely.

REFERENCES

Wu, C. W., Chua, L. O. (1995). “Application of Kronecker products to the analysis of systems with uniform linear coupling”. *IEEE Transactions on Circuits and Systems I: Fundamental theory and applications*, 42(10), 775-778.

Wu, C. W., Chua, L. O. (1995). “Synchronization in an array of linearly coupled dynamical systems”. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, 42(8), 430-447.

Watts, D. J., Strogatz, S. H. (1998). “Collective dynamics of small-worldnetworks”. *Nature*, 393(6684), 440.

Li, Z., Duan, Z., Chen, G., Huang, L. (2010). “Consensus of multiagent systems and synchronization of complex networks: A unified viewpoint”. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 57(1), 213-224.

Yu, W., Chen, G., Ren, W., Kurths, J., Zheng, W. X. (2011). “Distributed higher order consensus protocols in multiagent dynamical systems”. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 58(8), 1924-1932.

Wang, X., Roy, S., Wan, Y. (2011, February). “Using deliberate-delay decentralized controllers to stop spread dynamics in canonical network models”. In Computational Advances in Bio and Medical Sciences (ICCABS), 2011 IEEE 1st International Conference on (pp. 178-183). IEEE.

Xue, M., Wang, W., Roy, S. (2014). “Security concepts for the dynamics of autonomous vehicle networks”. *Automatica*, 50(3), 852-857.

Xue, M., Roy, S. (2017, December). “Input-output properties of linearly-coupled dynamical systems: Interplay between local dynamics and network interactions”. In Decision and Control (CDC), 2017 IEEE 56th Annual Conference on (pp. 487-492). IEEE.

Abad Torres, J., Roy, S. (2015, December). “A two-layer transformation for characterizing the zeros of a network input-output dynamics”. In Decision and Control (CDC), 2015 IEEE 54th Annual Conference on (pp. 902-907). IEEE.

Hao, Yuqing, Zhisheng Duan, and Guanrong Chen. “Decentralised fixed modes of networked MIMO systems.” *International Journal of Control* 91, no. 4 (2018): 859-873.

Wang, Lin, Guanrong Chen, Xiaofan Wang, and Wallace KS Tang. “Controllability of networked MIMO systems.” *Automatica* 69 (2016): 405-409.

Wang, L., Wang, X., Chen, G. (2017). “Controllability of networked higher-dimensional systems with one-dimensional communication”. *Phil. Trans. R. Soc. A*, 375(2088), 20160215.

Xue, M., Roy, S. (2018). “Modal Barriers to Controllability in Networks with Linearly-Coupled Homogeneous Subsystems. In 2018 Network Controls and Systems Symposium (NecSys 2018). Extended version submitted to *IEEE Transactions on Automatic Control*.

Zhang, S., Cao, M., Camlibel, M. K. (2014). “Upper and lower bounds for controllable subspaces of networks of diffusively coupled agents”. *IEEE Transactions on Automatic Control*, 59(3), 745-750.

Xue, M., Roy, S. (2018). “Comments on ‘Upper and Lower Bounds for Controllable Subspaces of Networks of Diffusively Coupled Agents’ ”. *IEEE Transactions on Automatic Control*, 63(7), 2306-2306.

Trumpf, Jochen, and Harry L. Trentelman. ”Controllability and stabilizability of networks of linear systems.” IEEE Transactions on Automatic Control (2018).

Dhal, R., Roy, S. (2016). “Vulnerability of Network Synchronization Processes: A Minimum Energy Perspective”. *IEEE Trans. Automat. Contr.*, 61(9), 2525-2530.

Pequito, Sergio Daniel, Soummya Kar, and A. Pedro Aguiar. “A Framework for Structural Input/Output and Control Configuration Selection in Large-Scale Systems.” *IEEE Trans. Automat. Contr.* 61, no. 2 (2016): 303-318.

Zamani, M., Lin, H. (2009, June). “Structural controllability of multi-agent systems”. In American Control Conference, 2009. ACC’09. (pp. 5743-5748). IEEE.

Rahimian, M. A., Aghdam, A. G. (2013). “Structural controllability of multi-agent networks: Robustness against simultaneous failures”. *Automatica*, 49(11), 3149-3157.

Chapman, A., Mesbahi, M. (2013, June). On strong structural controllability of networked systems: A constrained matching approach. In American Control Conference (pp. 6126-6131).

Carvalho, J. F., Pequito, S., Aguiar, A. P., Kar, S., Johansson, K. H. (2017). “Composability and controllability of structural linear time-invariant systems: Distributed verification”. *Automatica*, 78, 123-134.

Guan, Y. and Wang, L., 2017. “Structural controllability of multi-agent systems with absolute protocol under fixed and switching topologies”. Science China Information Sciences, 60(9), p.092203.

Lin, C. T. (1974). “Structural controllability”. *IEEE Transactions on Automatic Control*, 19(3), 201-208.

Papadimitriou, C. H., Tsitsiklis, J. (1984). “A simple criterion for structurally fixed modes”. *Systems and Control Letters*, 4(6), 333-337.

Sezer, M. E., iljak, D. D. (1981). “Structurally fixed modes”. *Systems and Control Letters*, 1(1), 60-64.