Hidden Disruptions in the Air Traffic System: Modeling, Identification, and Monitoring

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1 Motivation and Objective

The United States' air transportation system is subject to myriad disruptions. Traditionally, the the management of the air transportation system has primarily been focused on mitigating the impact of disruptions caused by severe weather, including convection, winter weather, and high winds [1, 2, 17, 18]. Recently, however, there has been growing concern about assorted other disruptions to the air transportation system, including cyber- failures and attacks, space vehicle operations, and vulnerabilities introduced by the system's human operators and clients (ranging from operator error due to fatigue to kinetic threats caused by bad actors) [3–6, 19, 20]. Indeed, several high-profile disruptions to the air transportation system have originated from non-weather causes in recent years. These disruptions, while localized at their source, have incurred costly propagative impacts across the air traffic system.

Motivated by the recent disruption events, several several studies have sought to develop models and tools for threat assessment and mitigation of heterogeneous disruptions to the air transportation system. The approach taken by these studies has been to introduce multi-layered models that capture traffic flows, weather, and human/cyber components in the air traffic management system [3–6]. In turn, these models are used to evaluate disruption impacts, and to design traffic flow management (TFM) strategies to mitigate impact. Additionally, the models have been used for counterfactual analyses [6,21], which identify vulnerable locations in the traffic system (i.e. locations where disruptions have the widest or most severe propagative impact).

While the recently-developed techniques for assessment of disruptions to the air traffic system are promising, a key barrier to their practical implementation is that disruption sources are often *hidden* to system planners and operators. This is the case because disruptions arise from extremely complex spatiotemporal processes and operations, which are usually incompletely known to system managers, and poorly monitored in real-time operations. For instance, the cyber networks used in air traffic management facilities (e.g. Air Route Traffic Control Centers) are specialized, complex, and legacy systems; to the best of our knowledge, no global blueprint of thes cyber networks are available, and certainly almost no real-time monitoring of the cyber systems are undertaken. Similarly, weather forecasting and monitoring products are often at a course resolution, which limits observability of weather impacts on air traffic dynamics. For instance, ensemble forecasts typically forecast convective-weather and wind-speed probabilities in grid squares ranging in dimension from $10mi \times 10mi$ to $40mi \times 40mi$. However, terminal-area operations (e.g., runway closure events, capacity reductions) often depend on much finer resolution weather characteristics, which are not forecasted or even persistently monitored. Many behavioral characteristics of human operators (e.g., fatigue of air traffic controllers, or even staffing profiles at facilities) are similarly hidden.

The hidden nature of the processes and operations where air traffic system disruptions originate greatly complicates the modeling and monitoring of disruptions. In particular, the hidden structure makes it difficult to build and learn models at the level of detail and accuracy needed for statistical prediction of disruptions. Additionally, the hidden structure complicates real-time monitoring of disruptions, in particular making it difficult to rapidly give alarms when disruptions are initiated. Indeed, whenever non-weather disruptions have occurred, there has been considerable initial uncertainty about their cause and source. Even for weather disruptions, there is often a lack of clarity in modeling and monitoring the disruptions' impacts on traffic-system resources, which complicates mitigation of their impacts.

The specific objective of this study is to introduce a framework for modeling and monitoring heterogeneous hidden disruptions to the air transportation system. The eventual aim in developing the framework is to facilitate rapid alarming of emerging disruptions, so as to enable mitigation of disruption impacts. The focus in this initial work is to motivate and present the tractable framework, and conduct some initial studies regarding its usability.

The remainder of the article is organized as follows. In Section 2, we overview the technical approach. In Section 3, we formally introduce the modeling framework and define associated identification and monitoring problems. Prior works which are relevant to the monitoring and identification problems are briefly described in Section 4. Finally, in Section 5, some preliminary analyses using the framework are given, with a focus on exploring whether the approach can indeed allow identification/monitoring of hidden process.

2 Overview of the Approach

Broadly, we pursue development of an abstract stochastic modeling framework for temporal evolution of disruptions in the air traffic system, which is generic enough to capture multi-faceted disruptions (e.g. weather and cyber disruptions), but structured enough to allow efficient model learning and disruption monitoring. While different types of disruptions have widely varying dynamics, they have in common that they are highly-stochastic discrete-valued processes with complex spatiotemporal evolutions. For instance, weather impacts on air traffic resources are correlated discrete-valued signals (e.g., resources are constrained or not), which are highly unpredictable. Likewise, cyber- and human-operator- originated disruptions are complex discrete-valued processes (e.g., each cyber- device may transition to failure states), which are highly uncertain. Thus, abstractly, we are motivated to use a *discrete stochastic network model* to generically capture disruption dynamics. Unfortunately, such stochastic network dynamics are generally very difficult to learn from data and to analyze/monitor, because their complexity generally grows exponentially with the model dimension.

In this article, we introduce a framework for modeling hidden disruptions in the air traffic system, which is based on a special discrete stochastic network model known as the *influence model*. The influence model, which describes a class of networked discrete-state Markov chains with quasi-linear interactions [8, 9], has proved useful for representing social processes in human groups [7], environmental phenomena (e.g. convective weather propagation) [10, 11], and decision-making algorithms [12], among other stochastic network dynamics. The model is appealing for myriad applications because of: 1) the tractability enabled by its quasi-linear structure, and 2) the model's ability to represent high-dimensional stochastic network processes with a terse parameterization. In particular, the terseness of the model description suggests that model learning may be possible using limited data. The influence model has already been used to capture convective-weather dynamics in the air traffic system [10, 11], as well as propagation of cyber- failures among air traffic system resources [3, 4]. Given the tractabilities of the influence model, and its effective use in modeling air traffic disruptions, we believe it to be an appealing tool for generically representing disruption processes in the air traffic system. To enable study of hidden disruption dynamics, we consider an enhanced influence modeling framework which represents partial observation of the state dynamics (e.g., observation of a subset of network locations, or of projections of the nodes' statuses); we note that this enhanced model with an explicit observation model follows on the partially-observed influence modeling concept introduced in [13, 29].

Toward developing a framework for modeling and monitoring hidden disruptions, this study defines three questions related to the partially-observed influence model. First, the identification of the model from the partial-observation sequence is considered. That is, we seek to understand whether, and how, the influence model can be inferred from the partial observation sequence, to enable construction of models for hidden disruption processes using data from a limited set of manifest variables. Second, real-time monitoring of disruption events from the measurement sequence is pursued. Third, statistical analysis of the disruption processes, either *a priori* or based on measurements, is considered.

3 Mathematical Formulation

An abstract representation of disruption processes in the air transportation system is developed, wherein discrete disruption statuses of system components are modeled as evolving stochastically in a correlated way. A pool or set of n system components, labeled $1, \ldots, n$, is considered. Each component i is modeled as having a disruption status $s_i[k]$ which evolves along a discrete or sampled time axis ($k \in Z^+$). The status at each time k is taken from a finite set $\{1, \ldots, m_i\}$, i.e. each component has an evolving discrete disruption status. Components and their disruption statuses may include, for instance: 1) discrete capacity levels of airspace regions (e.g., nominal vs. degraded), 2) airport runway configurations, 3) convective-weather status near an airport (e.g., presence vs. absence of convection), 4) attack or failure statuses of Center cyber systems, and 5) human-operator fatigue or compromise statuses.

The components' statuses are modeled as evolving in a Markov fashion, but with interactions among the components allowing for correlation. The representation can capture, for instance, that airport or airspace-region capacities are correlated with each other or with convective-weather or cyber-failure statuses. Similarly, the representation can capture dependencies among cyber- components, or between human-operator disruptions and physical-world responses.

Specifically, the disruption statuses are modeled as evolving according to an influence model [8, 9], which has a special stochastic update rule that permits quasi-linear analyses of status probabilities and provides a terse description of the process. The influence model update rule at each time step can be described as follows:

1) Each component $i \in 1, \ldots, n$ is probabilistically influenced or determined by a single component $j \in 1, \ldots, n$ (possibly including itself) with probability $d_{ij}[k]$, where $d_{ij}[k] \geq 0$ and $\sum_j d_{ij}[k] = 1$.

2) The current status of the determining component j probabilistically specifies the next status of the component i. Specifically, if the determining component has status $s_j[k] = q$, then the next status $s_i[k+1]$ of component i is generated according to the probability distribution of the qth column of the $m_i \times m_j$ matrix $A_{ji}[k]$. We note that the matrices $A_{ji}[k]$ are nonnegative, with column sums equal to 1.

We note here that, in general, time-variation in the parameters is allowed, although the variation is expected to be slow compared to the disruption dynamics in many cases. We will distinguish cases without time variation (or with sufficiently slow time variation) as the time-invariant model. We also stress that the model is heterogeneous, in the sense that each component may have different numbers of statuses and varying probabilistic update rules.

The heterogeneous influence model, as defined above, has been proposed as as a representation of diverse network processes such as cascading outages in infrastructures and inter-personal interactions in human groups. The model is appealing for threat modeling for the air transportation system because is abstractly captures essential correlations among discrete disruption statuses, while stripping away details associated with different components. Although a gross over-simplification of reality, we believe that the model can permit rough forecasting of disruption events in ways that can support traffic management. The model is particularly appealing for two reasons. First, the model permits low-computation statistical analyses of disruptions, because it enjoys a moment-closure property. Second, the model gives a terse description of the disruption evolution. Specifically, the system has $\prod_i m_i$ disruption configurations, and a Markov description would entail an $\prod_i m_i \times \prod_i m_i$ dimensional transition matrix. Instead, the influence model captures the disruption process using $(\sum_i m_i) \times (\sum_i m_i) + n \times n$ parameters. This sparse description is especially appealing for model identification purposes.

A main premise of our modeling framework is that the disruption process, as represented by the influence model, is partially hidden to operators. Thus, model identification and state monitoring using partial data is a prerequisite for disruption modeling. Partial visibility of disruption processes in the air transportation system may arise for many reasons. For instance:

1) Many system components, such as many cyber- subsystems, near-airport above-surface weather conditions, or weather hazards such as clear-air turbulence, may not be monitored persistently. Some of these

components may be occasionally or partially modeled: for instance, a few servers may be monitored for cyber- attack activity, or pilot reports may give some real-time observations of turbulence.

2) Weather forecasts and measurements may not capture the airspace system at a sufficient spatial or temporal resolution for disruption management.

3) Only limited data on airport or en-route airspace conditions may be recorded and stored. For instance, while runway configurations are typically archived, weather events causing configuration changes or departure/arrival bank information may not be maintained in the record.

Given the limited visibility of disruption processes, we are motivated to extend the influence-modeling framework to explicitly represent partial measurements. Three types of partial measurements can be envisioned:

1) *Measurement at a subset of components.* In this case, the measurement $\mathbf{y}[k]$ at each time k consists of the states of all components in a subset $\mathcal{Z}$ of the full component set $\{1, \ldots, n\}$.

2) *Time-snapshot measurements.* The measurement $\mathbf{y}[k]$ contains all components' statuses, but only at some time points. At other times, the measurement is nil.

3) *State-projection measurements.* The measurements $\mathbf{y}[k]$ are projections of network-wide statuses, for instance a count of the number of components in a warning status.

Combinations of these measurement paradigms, for instance time-snapshot measurements at a subset of components, may also be considered.

We refer to the model as a whole as the *partially-observed influence model for air traffic system disruptions.* We are interested in three key questions regarding the model:

1) Learning or identification of the model from archived data. Mathematically, the problem of interest is to recover the influence model parameters $d_{ij}[k]$ and $A_{ji}[k]$ from measurements $\mathbf{y}[k]$ over a period of time. Often, identification will be done with the assumption that the model is time-invariant, and possibly also homogeneous (all A_{ji} are identical matrices).

2) Disruption monitoring from streaming data, once the model has been learned. The problem of interest is to recover the all components' statuses ($s_i[k]$, $i = 1, \ldots, n$) at a current, past, or future time k, using a sequence of measurements $\mathbf{y}[k]$.

3) Statistical analysis of disruption events using learned model, either *a priori* of any disruption events or given measurements that indicate an ongoing disruption.

4 Relevant Prior Work

Dynamical stochastic network models have been extensively studied, and have found applications in applications ranging from particle physics to communication networks and the social sciences [22]. Within this broad literature, one focus has been on models which allow for statistical analysis using linear moment-closure properties. The study of such quasi-linear models originated in the work on the *voter model*, which was later generalized in defining the influence model [8, 9, 23, 24]. Similar quasi-linear models also include infinite-server queueing networks, as well as some contact-process models [22]. The influence model was introduced in the work of Asavathiratham and co-workers [8, 9], and since then has been generalized in several ways and also specialized for various applications [1, 7, 10, 11, 25, 26, 29].

Within the body of research on the influence model, two directions are particularly germane to the modeling framework introduced here. First, our work continues and extends an effort to represent severe weather impact on air traffic resources using the influence model [10, 11, 26, 27]. Influence models were developed for convective weather impact on airspace capacities as a supplement to public-domain weather forecasts (e.g. ensemble forecasts), with the goal of representing the small-scale variabilities in weather propagation that significantly impact air traffic but are not captured by public-domain forecasts. Techniques for parameterizing these influence models from snapshot data originating from public-domain forecasts were also introduced, and validation efforts showed that the parameterized influence models could aptly capture correlations and uncertainties in convective weather impact at multi-hour lookahead horizons. Relative to the earlier studies, here we propose using the influence model to represent heterogeneous disruption modalities for the air transportation, including weather, cyber, and human-operator disruptions. The broader formulation then requires parameterization and learning of the model for multiple partial-observation paradigms (not only from snapshot data), as described above.

Second, our formulation has a close connection to several studies on learning and inference for influence models. Learning of influence models has been considered in the context of modeling social dynamics in human groups [7]. These studies were concerned with learning influence models from time-course data, both for the case where all nodes in the influence network are observed and the case where only a subset are observed. The authors applied standard heuristics for learning, and demonstrated effectiveness of the learning techniques via simulation. Next, the thesis [29] pursued development of estimation and learning algorithms for fully and partially-observed influence models from first principles, with the goal of obtaining formal guarantees and performance characterizations in addition to estimation and learning heuristics. The thesis introduced several promising algorithms and insights into estimation and also approached the learning or parameter estimation problem, although recognizing the difficulty of the problem for the partially-observed case. The important recent work [13] studied the fundamental question of identifiability of the partially-observed influence model. The authors obtained a general negative result on identifiability for the partially-observed model; this result has an important bearing on the hidden modeling framework developed here, since it indicates that the framework cannot be used to learn and monitor hidden disruption processes. To the best of our understanding, however, the result in [13] is incorrect, see our comment [28]. Hence, hidden disruption processes potentially can be learned or inferred from data, hence allowing monitoring of these processes. Broadly, the prior work on learning and monitoring of the influence model provides a starting point for solving the three problems defined for the air traffic disruptions model.

5 Algorithm Development: Preliminaries

The main focus of this study has been to introduce a framework for modeling partially-hidden disruptions in the air transportation system. We largely leave the concrete development of learning, monitoring, and statistical analysis algorithms to future work. We anticipate that analysis techniques for influence models and partially-observed influence models can be leverages to help develop these algorithms, however further advances are also needed in assessing identifiability of influence models and building computationally-attractive algorithms for learning/identification. With this goal in mind, we pursue two preliminary explorations related to the development of learning algorithms for the partially-observed influence model. First, via am example, we explore whether identification of partially-observed influence models is ever possible. Second, we briefly posit a computationally-attractive expectation-maximization algorithm for estimating the influence-model parameters.

First, we discuss our efforts on identification of the partially observed influence-model identification for a small-scale example, as a means to gauge the feasibility of disruption modeling using the proposed approach. The small-scale example captures a network with two nodes or sites with binary statuses, one of which is hidden and the other observed. For instance, the hidden node may represent an airport capacity state (high vs low) or local airport environmental state that is not directly forecasted/monitored, while the observed node represents an environmental variable that is forecasted or measured (e.g., regional presence of convection per an ensemble forecast, or wind-shear measurements from a nearby weather-monitoring facility). Specifically, a homogeneous influence model with the network matrix $D = [d_{ij}] = \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix}$ and local transition matrix $A = A_{ji} = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$ is considered.

As a preliminary study of the example, we have ascertained that the observed site's status (labeled site 1) is not Markov, even though the whole influence model is Markov. In particular, given that the model is initiated in steady-state, one can check that the conditional probability $P(s_1[2] = 1 \,|\, s_1[1] = 1) = 0.8355$, while $P(s_1[2] = 1 \,|\, s_1[1] = 1, s_1[0] = 0) = 0.7687$. The non-Markovianity of the observation sequence is important, since it indicates that the observations encode the dynamics of the hidden site, and hence that model identification and real-time monitoring may be possible.

Indeed, a partially-observed influence model can generically be formulated as a hidden Markov model with a high dimensional hidden Markov chain (specifically, a master Markov chain representing all $\prod_i m_i$ configurations of the network). In consequence, standard algorithms for learning of hidden Markov models such as the Baum-Welch algorithm [15] can be applied to learn the equivalent hidden Markov model, and in turn to estimate the influence model's parameters. For the example considered here, we have been able to use the Baum-Welch algorithm to identify the master Markov chain for the model (given by $G = \begin{bmatrix} .81 & .09 & .09 & .01 \\ .2542 & .3658 & .1558 & .2242 \\ .3312 & .1488 & .3588 & .1612 \\ .04 & .16 & .16 & .64 \end{bmatrix}$). In turn, the influence model parameters can be inferred: it is worth noting that influence models may exhibit redundancies wherein several parameterizations have the same master Markov chain, however at least one of these parameterizations can be found. As a verification, it is easy to check that the hidden Markov model equivalent for this example meets the criteria for identifiability (modulo a labeling of the states) [14, 16], and hence it is unsurprising that the model can be identified.

Upon identification, standard techniques for filtering in hidden Markov models can be used for real-time monitoring of the hidden states. While the example is very simplistic, it demonstrates that identification and monitoring of hidden disruption processes is possible using the influence-modeling framework. In addition, the example illustrates general procedures for identification and monitoring of hidden disruption dynamics using the influence-modeling framework. We note that the example also exposes an error in the identifiability analysis of the partially-observed model given in [13], as it shows that the non-Markovianity of the observation sequence can be exploited for identification and monitoring of the hidden dynamics. It is important to note that effective identification of the partially-observed influence model may require considerable data, and hence identification of models for rare disruption processes may remain challenging.

In the example above, we have exploited an equivalence to a hidden Markov model to learn the influence model. While this approach is appealing theoretically for verifying identifiability, it is not a practical means for identification for an influence model with even a moderate number of components. This is because the master Markov chain in this case has a very high dimensional state space, and hence a direct identification of the chain would require enormous amounts of data. A preferable alternative is to develop a method wherein the influence model parameters are directly identified. We posit that this can be done using an expectation-maximization algorithm which is a variant on the Baum-Welch algorithm. Specifically, the "expectation" step of the algorithm would be completed in exactly the same way as in the Baum-Welch algorithm: given a current guess for the influence model, the hidden states of the model would be estimated by applying the state estimation technique for the hidden Markov model equivalent. For the "maximization" step, however, the optimal influence model parameters (i.e., the set of parameters maximizing the likelihood of the estimated state sequence) would be directly computed.

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