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Modeling the cyclic response of sands for liquefaction analysis

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ABSTRACT: Constitutive relations used to describe the stress-strain-strength behavior of soils in cyclic loading are known to play a critical role on our ability to predict the response of geo-structures to seismic loading. The extent and intricacies of this role, however, are highly problem-dependent and often difficult to discern from the effects of other ingredients of a numerical simulation. Moreover, realistic assessments of constitutive models and numerical analysis techniques require detailed comparisons of their performances with reliable experimental observations. The experimental data that have been produced in the course of recent Liquefaction Experiments and Analysis Projects (LEAP-2015 and LEAP-2017) provide an opportunity for a more thorough assessment of the capabilities and limitations of constitutive models for sands over a wide range of strains. The LEAP experimental data along with a large number of cyclic element tests are used here to explore the performance of several constitutive models in numerical simulation of soil liquefaction and its effects on lateral spreading of mildly sloping grounds

1 INTRODUCTION

Constitutive modeling of liquefiable soils has been the subject of intense research in the past four decades. A primary motivation for the development of constitutive model for sands is the critical role of these models in a nonlinear dynamic effective stress analysis framework that can be used for analysis of liquefaction and its effects. Early developments were focused on combining simple elastoplastic models for shear behavior with an additional component to account for the shear-induced volume change of the soil. For example, Ghaboussi & Dikmen (1978) used a combination of a simple plasticity model for shear deformations, an empirical rule to define the change in the mean effective stress, and a revised form of Masing rules to account for the degradation of soil stiffness during cyclic loading. Similarly, Zienkiewicz et al. (1978) adopted a non-associated Mohr-Coulomb elastoplastic model with an additional expression determining the progressive increase of the volumetric strain. Both models were successfully used to simulate liquefaction in saturated soil layers subjected to seismic loading.

A more comprehensive set of constitutive models that specifically dealt with the modeling of cyclic response emerged in the 1980s. The constitutive models proposed by Aubry et al. (1982) combined a multi-mechanism approach with the concept of field of hardening moduli for modeling cyclic response (Mroz, 1967, Mroz et al., 1978). Aubry et al. (1982) presented realistic simulations of sand cyclic response in both drained and undrained conditions and for the cases where cyclic mobility emerged as the result of significant reduction in the mean effective stress. However, no example of the application of the model in boundary value problem was reported at the time.

Zienkiewicz et al. (1985) introduced a relatively simple constitutive model for sands based on the concept of generalized plasticity (Zienkiewicz & Mroz, 1984). Almost simultaneously, Prevost (1985) proposed a simple and robust sand model within the framework of multi-surface plasticity (Mroz, 1967). These two contributions were among the first models that were formulated within a multi-dimensional framework and were shown to simulate some basic features of the cyclic response of sands. An additional key development was the implementation of these models in two well-established finite element software (SWADYNE and DNA-FLOW) which provided an excellent opportunity for the assessment of these models in various geotechnical engineering problems that involved liquefaction analysis. Both of these software packages were research software and were originally only available to the research groups under the main developers at Swansea and Princeton. Parallel to these efforts, significant development of the multi-mechanism plasticity framework was undertaken in Japan (Iai et al. 1992) where a sand model capable of simulating many features of sandy soils emerged and was used in a large number of boundary value problems involving soil liquefaction (Iai & Kameoka, 1993).

In addition to these efforts, significant modeling effort on soil liquefaction was advanced in Canada where Finn and collaborators developed and implemented relatively simple analytical models where a pore pressure generation model was linked to an elastoplastic model of soil shear behavior. The models developed by Finn and collaborators were also implemented in a research software (Finn et al., 1986). Many geotechnical engineering problems involving soil liquefaction were analyzed using this software.

During the late 1980 and early 1990's, a group of geotechnical academics in the US and UK initiated a major centrifuge testing campaign with the goal of producing reliable experimental data for validation of the existing models for soil liquefaction (Arulanandan & Scott, 1993). The project, known as VELACS, was an international collaborative effort, in which a number of key contributors to soil constitutive modeling participated in a series of blind predictions of the centrifuge tests that modeled seismic response of a number of geotechnical systems ranging from level-ground and mildly sloping saturated soil deposits to earth-dams and quay walls. Details of the project achievements and conclusions are documented in Arulanandan & Scott (1993).

A key outcome of the VELACS validation effort was the recognition of the need for further development of constitutive models for sands. A small number of the constitutive models available in early 1990s clearly had the key ingredients and capabilities that made them suitable candidates for further development and refinements. For example, Elgamal and his collaborators (Parra, 1996, Yang, 2000, Elgamal et al, 2002 & 2003, Yang & Elgamal, 2002, Yang et al., 2003) further developed and improved Prevost's multi-surface plasticity model for analysis of liquefaction-induced deformations.

As a graduate student researcher in the VELACS project, the first author had the privilege to assist the project principal investigators (Professors Arulanandan and Scott) with the evaluation of the numerical predictions and their comparisons with the centrifuge test data. A key personal observation was that the majority of the existing models at the time, while capable of modeling the essential features of sand cyclic response, required different sets of parameters for different soil densities. Moreover, the plasticity models for sands usually lacked the ability to simulate the softening part of soil response which was commonly observed in monotonic shearing of many medium dense sands. This was while similar post-peak responses were easily reproduced for over-consolidated clays by using relatively simple constitutive models such Cam clay.

Motivated by the simplicity and elegance of the two-surface plasticity framework for modeling cyclic response of metals (Dafalias & Popov, 1976), Manzari & Dafalias (1997) developed one of the first critical state model for sands that was capable of modeling monotonic and cyclic response of sands. Cubrinovski and Ishihara (1998a,b) proposed another state-dependent constitutive model based on combined isotropic and kinematic hardening formulation.

The model by Manzari & Dafalias (1997) was later revised (Dafalias & Manzari, 2004) to include a fabric-dilatancy tensor that enabled better modeling of unloading and reverse loading. A micropolar extension of the model (Manzari &Yonten, 2011a,b) was also developed

and used for analysis of soil bifurcation and shear banding in geostructural systems. In the past two decades, the model has been further developed by many researchers who have sought to improve its capabilities for a variety of applications. For example, Taiebat & Dafalias (2008) extended the model to capture the response of sands at stress paths with constant shear stress ratio. Other researchers used the model framework to develop simplified 2D models with improved simulation capabilities in cyclic loading (e.g., Boulanger & Ziotopoulou, 2013).

With the development of the critical state two-surface plasticity model for sand, a number of existing constitutive models such as the Pastor-Zienkiewicz generalized plasticity model for sand were also extended to include critical state of sand (Ling & Yang, 2006).

Despite the development of many good constitutive models in the past three decades, a large number of these models were only used in simulation of laboratory element tests (e.g., monotonic and cyclic triaxial, direct simple shear, and torsional shear) to demonstrate and assess the model capabilities and/or shortcomings. The use of many of these models in practical geotechnical engineering problems remained limited until recently. A key obstacle to their use in solution of practical geotechnical engineering problem was the lack of availability in commercial finite element/finite difference codes. It is also a common perception that many of the more capable models were difficult to calibrate and/or required a large number of element tests for calibration.

In the past few years, major progress has been made to overcome this impediment. Today, several constitutive models for sands are available through pre-compiled user routines for the use in commercial codes that are used by geotechnical engineers. A major credit for this progress goes to the researchers who implemented a number of advanced soil models in open source finite element software packages such as OpenSees (McKenna et al., 2000). For example, during the period of 2000-2004, Yang & Elgamal implemented two extended versions of the Prevost's (1985) model in OpenSees. Jeremić & Yang (2002), Jeremic et al. (2009), and Ghofrani & Arduino (2013) implemented the constitutive models by Manzari & Dafalias (1997) and Dafalias & Manzari, 2004) in OpenSees. Today, several other sand models are available in this open source software. This has provided an unprecedented opportunity for the geotechnical community across the world to experiment with these models and assess their capabilities and limitations in the solution of a variety of boundary value problems.

In this paper, the performance of several of the current constitutive model are discussed in the light of the observations from a new international collaborative research that is aimed at assessing the validity of constitutive and numerical modeling techniques for analysis of soil liquefaction and its effects.

2 OBSERVATIONS FROM LEAP-2015 AND LEAP-2017 PROJECTS

The Liquefaction Experiments and Analysis Projects (LEAP) is an international research collaboration among universities, research institutions, and geotechnical engineering industry to produce and use high quality experimental data for validation of constitutive and numerical modeling techniques that are used in the analysis of soil liquefaction and its effects on geostructures (Manzari et al., 2014). In the past four years, two LEAP projects (LEAP-2015 and LEAP-2017) with the focus on seismically-induced lateral spreading of liquefiable soils were completed (Kutter et al., 2019, Manzari et al., 2019, Goswami et al., 2019).

In each of these projects, a large number of laboratory tests were performed to characterize the physical and mechanical characteristics of Ottawa F65 sand (Vasko, 2015; Vasko et al., 2018; El Ghoraiby et al., 2018 &2019) and to evaluate liquefaction resistance of this soil when it is subjected to different stress/strain paths. The results of these tests were used in calibration of several constitutive models for sands (Manzari et al., 2017 and 2019-a) which are commonly used by researchers and practitioners in liquefaction analysis. A list of the participating teams along with the constitutive models and numerical simulation software used in the calibrations and the analyses are shown in Table 1.

An example of the performance of these model is shown in Figures 1 and 2 where a stress-controlled undrained cyclic triaxial test on Ottawa F65 sand is simulated (Manzari et al.,

Table 1. Numerical Simulation Teams

N°	Numerical Simulation Team	Constitutive Model	Analysis Platform
1	Tsinghua University	Tsinghua Constitutive Model	OpenSEES
2	Meisosha Corporation	Cocktail Glass Model	FLIP Rose
3	Shimizu Corporation	Bowl Model	HiPER
4	University of Napoli Federico II	Hypoplastic Model	Plaxis
5	UC Davis-Auburn University	PM4Sand Model	Flac-2D
6	University of Washington	Manzari-Dafalias Model/PM4Sand Model	OpenSEES
7	Kyoto University	Cocktail Glass Model	FLIP TULIP
8	Universidad del Norte	ISA-Hypoplasticity Model	ABAQUS
9	University of British Columbia	SANISand	FLAC-3D
10	University of California, San Diego	PDMY	OpenSEES
11	Fugro West	PM4Sand Model/UBCSAND	FLAC-2D
12	Hiroshima (Kansai) University	Cocktail Glass Model	FLIP Rose

2019-a). A quick review of these figures reveal that many models have produced stress paths that qualitatively similar to the test results (1-a). However reproducing the stress-strain curves appeared to much more challenging ((Figure 1-b).

While the experiment shows asymmetric stress-strain curves with noticeable ratcheting towards the extension side, except for models 1, 3, 4, 6a, and 8 the rest of the models show more or less symmetric stress-strain curves. A possible explanation for symmetric response shown by models 5, 6b, and 10 is that the constitutive models used in these simulations are formulated in plane strain condition and the simulations are cyclic biaxial rather than cyclic triaxial. Other constitutive models (e.g., 10) have the ingredients for reproducing the ratcheting phenomena, but did not reproduce such behavior with the calibrated parameters.

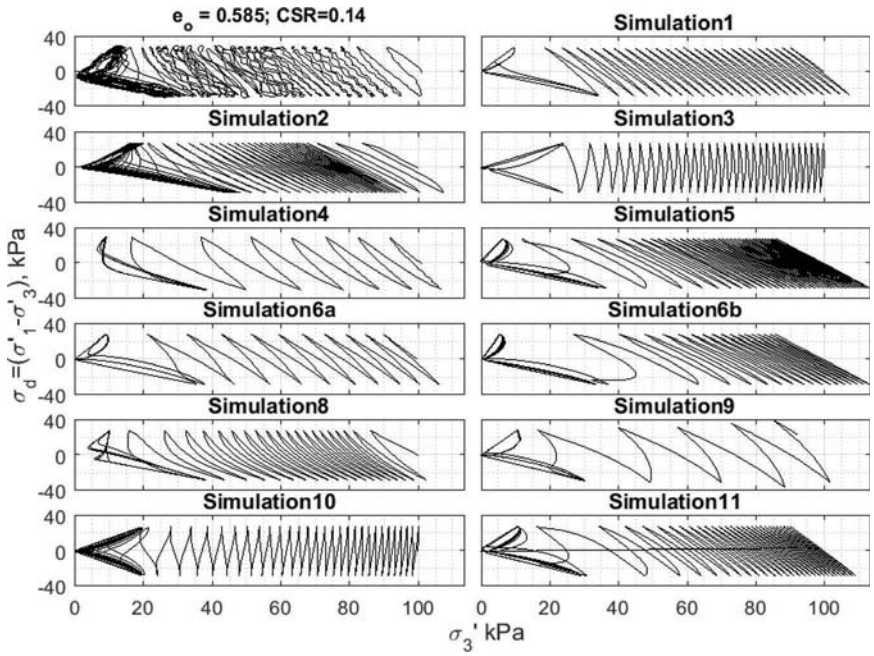

Figure 1-a. Comparison of the numerical simulations of stress paths for a cyclic stress-controlled test on Ottawa F65 sand. e= 0.585 (Dr~71.5%), p'$_0$=100 kPa, CSR=0.14.

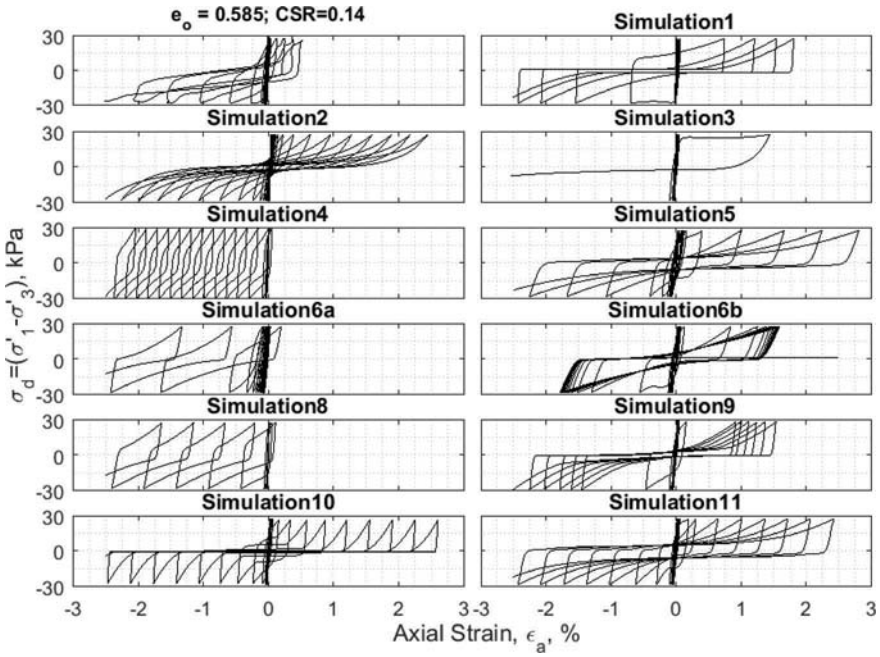

Figure 1-b. Comparison of the simulated stress-strain curves for a cyclic stress-controlled undrained cyclic triaxial test on Ottawa F65 sand. e= 0.585 (Dr~71.5%), p'_0=100 kPa, CSR=0.14.

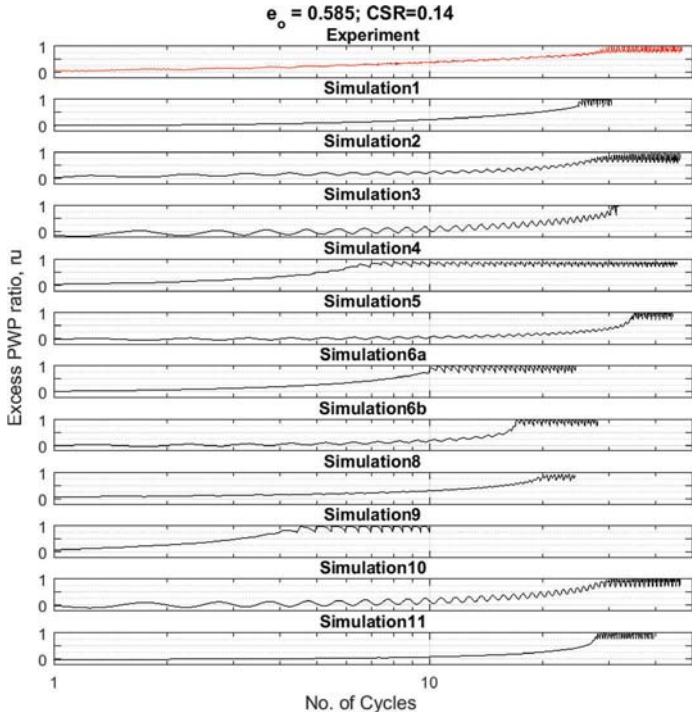

Figure 2. Comparisons of the observed versus computed excess pore water pressure ratios with number of cycles for e = 0.585 (D_r~71.5%) at CSR=0.14.

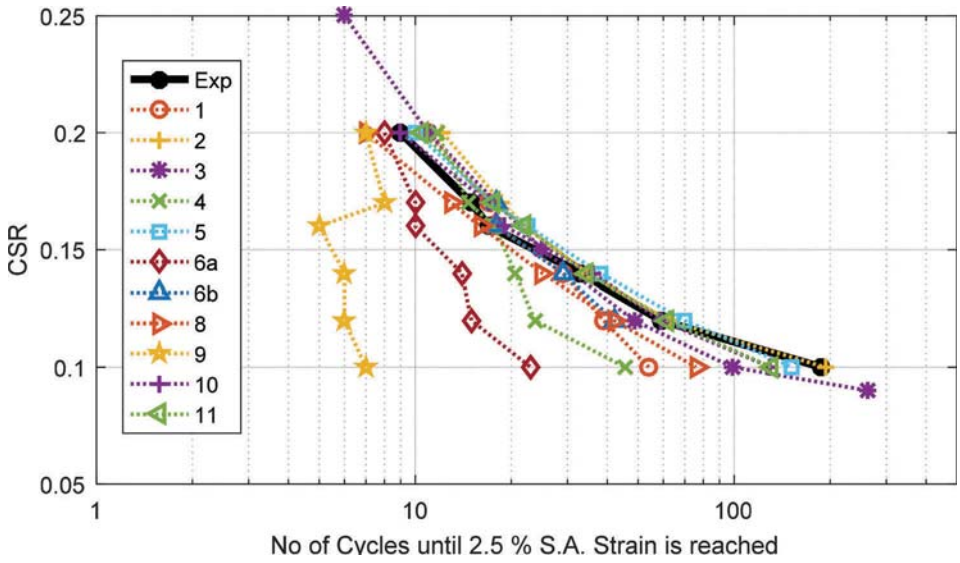

Figure 3. Comparison of the simulated liquefaction strength curves by different numerical simulations teams with the experimental results for e = 0.585 (Dr~71.5%).

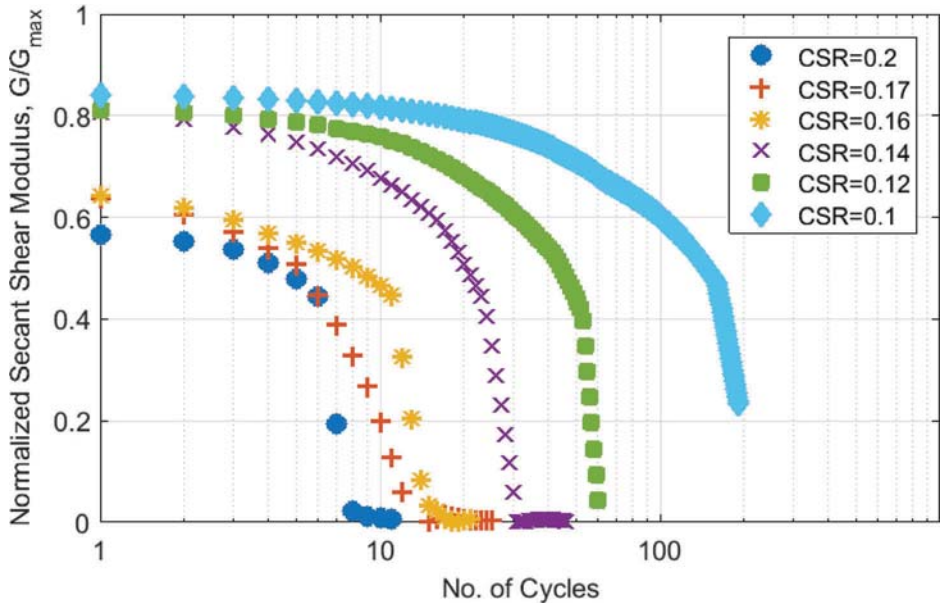

Figure 4. Evolution of normalized secant shear modulus (G/G_{max}) for Ottawa F65 sand in stress-controlled cyclic undrained triaxial tests (El Ghoraiby et al. 2018 & 2019).

Another criteria for evaluation of the performance of sand models is the ability to reproduce the liquefaction strength curves (Figure 3). The number of uniform stress cycles to cause 2.5% single axis strain for a certain cyclic stress ratio (CSR σ_d/σ'_{v0}) in the sample is used in this Figure. Figure 3 shows that several models (e.g., 2, 3, 5, 6b, 10, 11) are able to closely match the experimentally obtained results while a few others (e.g., 6a, 9) show visibly different

trends and reach the single amplitude axial strain of 2.5% in much less number of cycles than the experimentally obtained values. A key reason for this phenomena is that the models used in these simulations show overly-predicted ratcheting because of the large difference between the responses of the model in compression versus extension.

While this particular feature of the model appeared to be helpful in simulating the stress-strain response curves (Figure 1), it is clearly a drawback when it comes to the modeling of liquefaction resistance. This particular issue and its impact on the model performance in estimation of seismically induced lateral spreading is further discussed in the following.

Figure 4 shows an example of the evolution (degradation) of the normalized secant shear modulus in stress-controlled undrained cyclic triaxial tests conducted for LEAP-2017 project (El Ghoraiby et al. 2018 & 2019). The maximum shear modulus used in this Figure was obtained by shear wave velocity measurements using bender elements in the samples of Ottawa F65 sand prepared with different densities and confining pressures. As expected with larger cyclic stress ratios (CSR), the degradation of the normalized secant shear modulus (G/G_{max}) with the number of stress cycles takes a faster pace.

To further assess the performance of the participating constitutive models in the LEAP-2017 numerical simulation exercise, the evolution of normalized secant shear modulus for each simulation is compared with the experimentally obtained data for one specific value of CSR and one initial void ratio (Figure 5). Given the different values of shear modulus used in each simulation, the starting value of normalized shear modulus is different for different models. Moreover due to the relatively small value of the selected CSR, the performance of each model in smaller strain and the corresponding shear modulus degradation at smaller strain levels are more clearly observed in Figure 5. It is seen, for example, that while it takes about 10 cycles for the soil to show a degradation of about 20% in shear modulus, a number of models show a much faster (9, 6a, 6b, 2) or considerably slower rate of degradation (5). The degradation rate for a few models appear to be closer to the experimental trend (1, 11).

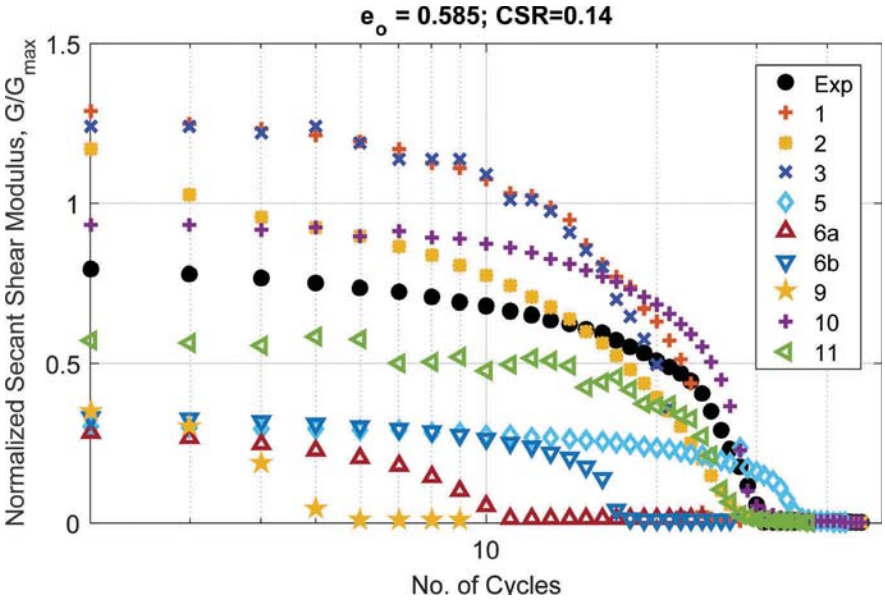

Figure 5. Comparison of the evolution of normalized secant shear modulus in stress-controlled cyclic undrained triaxial tests with the simulations of different constitutive models.

3 PERFORMANCE IN NUMERICAL SIMULATION OF LATERAL SPREADING

The constitutive models calibrated based on the laboratory element tests were later employed in numerical simulations of a large number of centrifuge tested conducted by the LEAP collaborators (Manzari et al., 2019-b). The centrifuge tests modeled the seismic response of a fully submerged deposit of Ottawa F65 sand to synthetic base excitations of various intensities (Kutter et al., 2019). The density of the soil deposit was also varied among the tests to allow for an assessment of the sensitivity of the response to the soil density and intensity of the base excitation (Goswami et al., 2019). Figure 6 shows baseline schematic of the LEAP-2017 experiment for shaking parallel to the axis of the centrifuge.

To assess the overall performance of the type-B predictions and the quality of their fit to the centrifuge test results, the following indicators are selected: 1) the maximum lateral displacement at the center of the soil surface, 2) the maximum excess pore water pressure ratio achieved at the depth of one meter (P4), and 3) a scalar representing a measure of spectral acceleration (MSA). The root mean square error (RMSE) of these indicators was computed for each numerical simulation team based on the selected centrifuge experiments as

$$RMSE = \sqrt{\frac{1}{N}\sum_{N}(P^e - P^s)^2} \tag{1}$$

where P^e and P^s are the values of an indicator for the experiments and simulations, respectively, N is the number of experiments which is equal to 9 for all the predictions except for the predictions 11a and 11b which were reported for only three centrifuge experiments.

The following measure is used to represent the spectral accelerations:

$$MSa = \int_{0.5}^{20} S_a \, df \tag{2}$$

MSa is the area under the spectral acceleration graph (Sa versus f).

Figure 7 shows a summary comparison of the numerical simulations with the experimental data obtained in the selected centrifuge tests. In this Figure, the RMSE values for the Type B simulations compared to the observed lateral surface displacements of the slope, excess pore pressure ratios near the ground surface (P4), and spectral accelerations (MSa) near ground surface (AH4) all in the central section of the sloping ground. It is observed that predictions 2, 3, 6a and 11a show reasonably small RMSEs for lateral displacements, while predictions 1, 2,

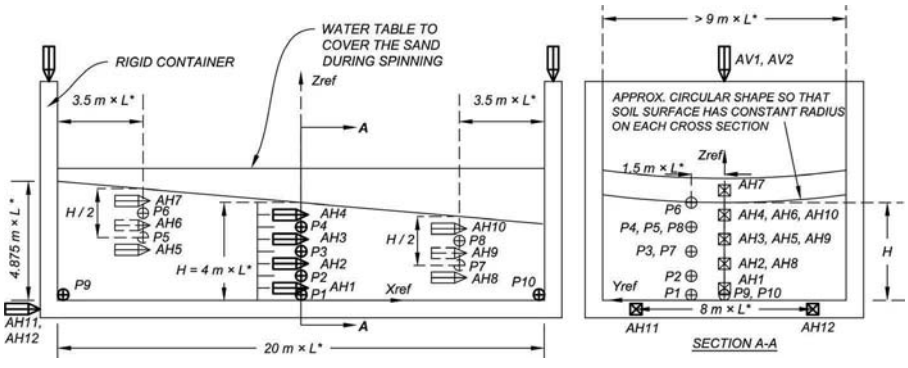

Figure 6. Baseline schematic for the LEAP-2017 experiment for shaking parallel to the axis of the centrifuge (Kutter et al., 2018).

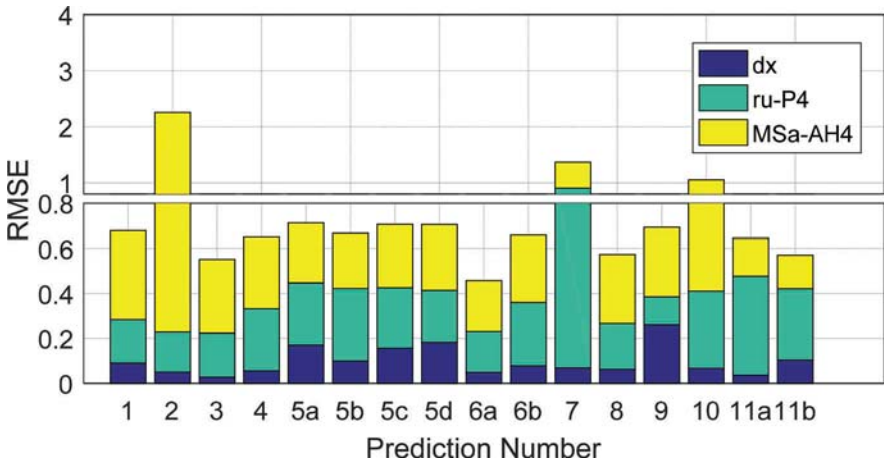

Figure 7. RMSE values for the Type B simulations compared to the observed lateral displacements, pore pressure ratios at P4, and spectral accelerations (MSa) at AH4.

3, 5a, 5d, 6a, 8, and 9 show relatively small RMSEs for excess pore pressure ratios, ru at P4. The spectral accelerations predicted by team 11 at AH4 show the lowest RMSEs.

4 CONCLUDING REMARKS

The results of laboratory tests conducted in LEAP-2015 and LEAP-2017 projects were used to assess the capabilities of several advanced constitutive models for sands. The performance of these models in capturing the seismically-induced lateral spreading of saturated mildly sloping grounds was also briefly discussed. More complete analyses of the experimental data and numerical simulations results obtained in the projects are currently ongoing and will be reported in subsequent publications.

ACKNOWLEDGEMENT

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The simulations presented in Figures 1, 2, 3, and 6 are based on the data provided by the numerical simulation teams led by Drs. Arduino, Bilotta, Elgamal, Fukutake, Mercado, Montgomery, Ozutsumi, Taiebat, Travasarou, Ueda and Ziotopoulou as part of the LEAP-2017 numerical simulation exercise. The contributions of these team leaders as well as their students and research collaborators who helped produce these simulations are gratefully acknowledged.

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