

1 Fabrication and experimental demonstration of a hybrid resonant acoustic 2 gradient index metasurface at 40kHz

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Over the past few years, gradient index metasurfaces(GIMs) have been voraciously studied for numerous wave control capabilities that they facilitate. In this regard, a hybrid structure consisting of shunted Helmholtz resonators and a straight channel is often chosen as building blocks of the metasurfaces. Prior research, however, has primarily focused on GIMs that operate in audible frequency range, due to the difficulties in fabricating such intricate structures at the millimeter and sub-millimeter scales, for ultrasonic applications. In this paper, we design, fabricate and experimentally realize a GIM for airborne ultrasound at 40kHz. The fabrication of such a GIM is made possible by projection micro-stereolithography, an emerging additive manufacturing technique capable of micro-scale, high aspect-ratio features over a wide area. Numerical simulations were first conducted to verify the metasurface design. Experiments were subsequently performed to corroborate the simulations and the theory. The thermoviscous effects associated with ultrasonic frequencies, their potential applications as well as optimal design strategies for minimal dissipations are discussed.

11 The emergence of metasurfaces¹⁻⁸ has bolstered interests
12 of various engineering communities in compact wave-based
13 devices. Several studies over the past decade have thus pro-
14 posed sub-wavelength structures that facilitate passive wave-
15 front manipulation⁹⁻¹². In acoustics, not only has this un-
16 folded the realization of novel designs¹³⁻¹⁵, but also has re-
17 visited classical structures that could now be redesigned to
18 provide unusual functionalities^{4,11,12}. Inspired by the di-
19 verse features that could be further explored, recent works in
20 this area have revolved around employing such structures to
21 contemporary design strategies. In this context, GIMs have
22 been ardently examined to propose several features such as
23 focusing¹⁴, bending¹¹, retro-reflection^{16,17}, and holographic
24 rendering¹⁸. Additionally, some recent works have examined
25 and embraced the inherent dissipation¹⁹⁻²¹ in these GIMs,
26 as not just a loss but an avenue to more unconventional
27 applications such as asymmetric transmission^{7,22} and wide-
28 angle absorption²³. A hybrid design consisting of shunted
29 Helmholtz resonators and a straight channel is a frequent can-
30 didate for the building blocks of these structures, due to their
31 relative simplicity and high sound transmission. The fabri-
32 cation of such structures has been enabled by conventional
33 3D printing techniques such as fused deposition modeling⁷
34 or stereo-lithography^{24,25}. The precision of these manufac-
35 turing methods, however, has rendered its implementation in the
36 ultrasound range rather scarce^{6,26}. Metasurface-based designs
37 operating at ultrasound frequencies would be advantageous
38 for a variety of applications such as levitation^{6,27}, sensing, and
39 imaging²⁸, owing to the better precision brought about by the
40 smaller wavelengths.

41 Ongoing developments in the field of mechanical metama-
42 terials is accompanied by the ~~rapid evolution(Z: advances)~~
43 of manufacturing approaches that help put forward artificial
44 materials with exceptional mechanical properties. Such ma-
45 terials possess complex three dimensional micro- and nano-
46 architectures and hence require sophisticated manufacturing
47 capabilities. Projection micro-stereolithography(P μ SL)^{29,30},
48 in this regard, is an emerging additive manufacturing tech-
49 nique that is capable of fabricating samples with high struc-
50 tural complexity and feature size ranging from a few microm-
51 eters to tens of centimeters - a characteristic that could benefit
52 the fabrication of acoustic metasurfaces/metamaterials at ul-
53 trasound frequencies.

54 In this work, we design, fabricate and experimentally
55 demonstrate the performance of a hybrid resonant acoustic
56 gradient index metasurface (HRAGIM), that operates at 40
57 kHz. The fabrication of the designed prototype is made possi-
58 ble by the aforementioned additive manufacturing technique.
59 **The role of thermoviscous dissipation in its elementary units**
60 **is discussed and full wave simulations that incorporate these**
61 **losses are put forward. The results from these simulations**
62 **are then validated via experimental measurements that are ob-**
63 **tained from a two-dimensional experimental platform. The**
64 **challenges at ultrasound frequencies such as stronger dissipa-**
65 **tion are highlighted.**

66 The HRAGIM has elementary units which consist of a se-
67 ries of four sub-wavelength Helmholtz resonators(HRs) and
68 an open channel. While the shunted resonators provide the
69 reactance to shift the phase of the incident wave, the open
70 channels serve as sub-wavelength slits that enhance the rate
71 of transmission due to Fabry-Pérot resonance. By varying the
72 parameter h_1 , shown in Fig 1(a), a full range of phase shifts
73 from 0 to 2π , can be obtained. Numerical simulations were
74 first performed using the pressure acoustics module on COM-

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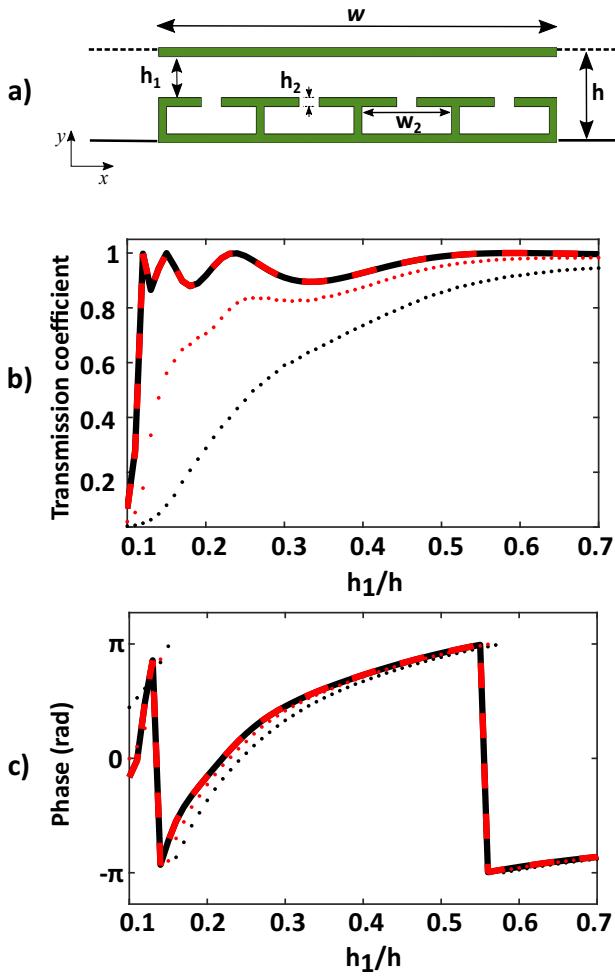


FIG. 1. (a) The elementary unit of the hybrid resonant GIM. To operate at 40 kHz, the dimensions of the unit are: $w = 3.428\text{mm}$, $h = 0.857\text{mm}$, $w_2 = 0.9875\text{mm}$, $h_2 = 0.0857\text{mm}$. The values of h_1 are chosen to be 0.130264mm , 0.152546mm , 0.187683mm , 0.232247mm , 0.318804mm , 0.47635mm to achieve a discretized phase distribution. Numerical prediction of the transmission coefficient (b) and phase shift (c) through the unit cell as a function of h_1/h . The solid black lines and dashed red lines indicates the result at 40 kHz and 3.4 kHz, respectively, for the case without dissipation. Similarly the black and red dots are the results shown when thermoviscous dissipation is included.

75 SOL Multiphysics 5.3a, a commercial finite element package.
 76 The unit cell has fixed height, $h = 0.1\lambda$ and width $w = 0.4\lambda$,
 77 where λ is the wavelength of the incident wave. This was
 78 done in order to obtain values of h_1 , whose phase shifts(ϕ)
 79 are equally spaced. As will be discussed later, the material
 80 used for fabrication was a high stiffness UV curable acry-
 81 late polymer and it is thus safe to assume that the walls of
 82 the resonators are acoustically hard, since their impedance
 83 is much greater than that of air (Speed of sound, $c \propto \sqrt{E}$,
 84 Young's Modulus). The phase gradient, ξ , is then engineered,
 85 by arranging the appropriate unit cells in the required pattern
 86 where,

$$\xi = \frac{d\phi_s}{dx} = (\sin\theta_t - \sin\theta_i)k_0, \quad (1)$$

87 here θ_t is the angle of transmission, θ_i is the angle of incidence
 88 and k_0 is the wavenumber. In this case, $\xi = 97.2 (2\pi \cdot \text{rad} \cdot \text{m}^{-1})$,
 89 where the array period, γ , of the metasurface is 10.284mm .
 90 It should be noted that such a GIM for airborne ultrasound
 91 can also be formed utilizing other types of unit cells. In this
 92 paper, however, we employ the above-mentioned design due
 93 to the versatility of the associated hybrid resonances and the
 94 firm theoretical framework that prior research in such struc-
 95 tures has offered. Here, it is worthwhile mentioning that it
 96 is due to this reason that such a design can be scaled as a
 97 function of frequency: the GIM in this paper, is an adapta-
 98 tion from the works of Li et al^{7,11,12}. The solid black lines and
 99 the dashed red lines in Figs. 1(b) and (c) are from pressure
 100 acoustics simulations for unit cell designs that operate at 40
 101 kHz and 3430 Hz respectively. This illustrates that the dimen-
 102 sions of the structure that was previously designed to operate
 103 at a lower frequency, can be scaled down to work for airborne
 104 ultrasound. However, it is pivotal to examine the prospec-
 105 tive challenges that such a metasurface could face while op-
 106 erating at higher frequencies. Foremost, as discussed in prior
 107 literature,^{20,21} the prominent dissipation in such structures is
 108 due to the influence of wall friction, owing to the thermovis-
 109 cious dissipation. This is characterized by, $\Delta = h_1/\delta$, where,
 110 h_1 , is the slit's transverse dimension (as seen in Fig.1(a))
 111 and $\delta = \sqrt{2\nu/\omega}$, the viscous boundary layer thickness. Here
 112 $\nu = 1.45 \times 10^{-5} \text{m}^2/\text{s}$, is the kinematic viscosity of air, ω is the
 113 angular frequency and it is apparent that $\Delta \propto \sqrt{\omega}$. Hence,
 114 the largest value of Δ at 3430 Hz, is only 3.67%, while it is
 115 12.53%, for the same value of h_1/h (scaled). This was further
 116 examined by performing numerical simulations that solve the
 117 thermoviscous acoustics equations in the unit cell region and
 118 pressure acoustics equations in the incident and transmitted
 119 regions. No-slip and isothermal conditions were imposed on
 120 the solid boundaries to incorporate dissipation. The dotted
 121 black and red lines, compare the results from these calcula-
 122 tions to those carried out on the hybrid resonant structure de-
 123 signed to operate at 3430Hz. It can be seen, that with the
 124 incorporation of themoviscous effects, the decrease in trans-
 125 mission coefficient is as expected, much larger in the case of
 126 the 40 kHz design than in that of 3430 Hz, as seen in Fig. 1(b).
 127 The low rate of transmission for smaller values of h_1/h is also
 128 as anticipated, as $\Delta \propto 1/h_1$. In addition to the boundary layer
 129 effects, the attenuation of sound, α_s , in free space due the ther-
 130 mal and viscous losses in air, is directly proportional to the
 131 square of the frequency, f^2 . In the audible range, this attenu-
 132 ation is rather negligible and can be ignored. The higher fre-
 133 quencies however, it becomes more important to take this at-
 134 tenuation into account. In our case, at 40 kHz, $\alpha_s \approx 0.2 \text{Np/m}$
 135 and it's incorporation in our simulations(both unit cell and the
 136 upcoming full wave), was still found to have a negligible ef-
 137 fect. This may not be the case for frequencies higher than
 138 40 kHz. It is hence important to note that although hybrid
 139 resonant GIMs can be scaled as a function of frequency, the
 140 effect of losses(Δ and α_s) are more profound at higher fre-
 141 quencies, are not directly scalable. Furthermore, Fig. 1(c)
 142 indicates thermoviscous dissipation has a relatively weak in-
 143 fluence on phase shift at both 3.4 kHz and 40 kHz. This is
 144 true as dissipation dominates the resistance part of impedance

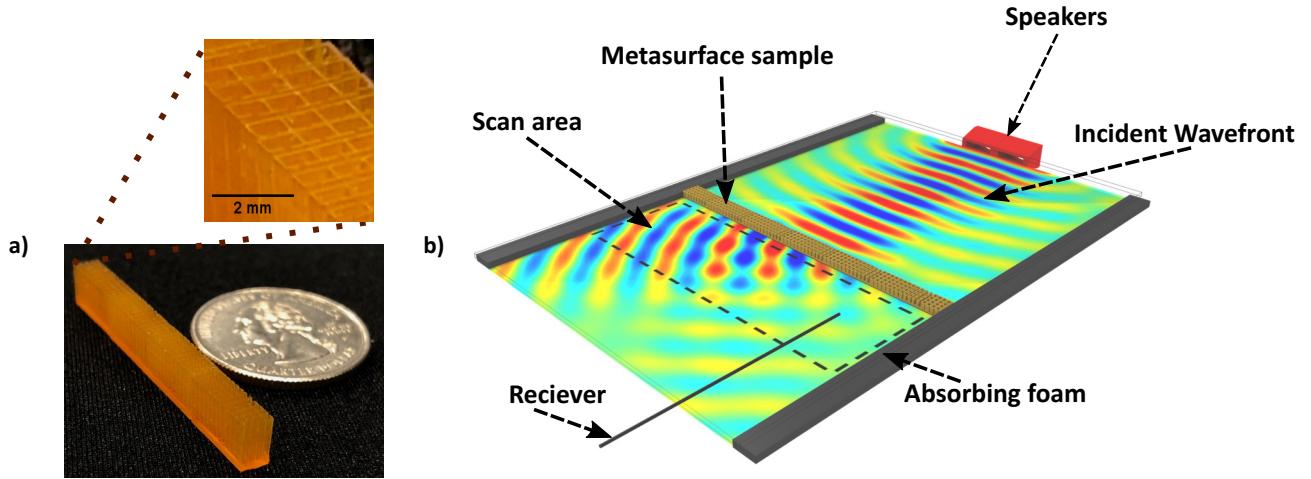


FIG. 2. (a) A sample with thickness 3.428mm and length 61.704mm. The inset shows the zoom of the sample, where the shunted Helmholtz resonators and straight channels can be seen. (b) The scaled down experimental set up of the two-dimensional linear scan stage. Two ultrasonic speakers are connected to tapered horn-type wave guide adapters, that are shown on the right (red), to ensure the incident wavefront. The tube is represented by the black line on the left, which maps the scan area and guides the sound to a receiver.

more than the reactance. The GIM can thus provide the desired wave bending effect for airborne ultrasound, however with a considerable decrease in transmission.

The fabrication of such a minute design is made possible by a high resolution, large area projection microstereolithography system capable of fabricating micro-scale, high-aspect ratio features over a wide area. In contrast to other methods such as fused deposition modeling and UV projection waveguide system, this approach is ideal for samples with high structural complexity and with a feature size ranging from microns to centimeters. A three-dimensional CAD model is first made of the metasurface, which is then sliced into 2D patterns. These patterns are projected via a UV digital micro-mirror device (DMD), focused onto the surface of a photosensitive monomer, which cures under UV exposure. The cured layer in the shape of the 2D slice pattern is then lowered to resupply liquid resin on its surface. The pattern projection is repeated to form the subsequent layer. To improve the resolution of the as-printed metasurfaces, a reduction lens is embedded in the UV light path. To expand the scalability of the printed metasurface, the projection system moves with an optical scanning system to project pattern on multiple areas of the liquid surface, producing large scale metasurfaces with micro-scale resolution. **Z: High printing resolution is achieved by embedding a series of reduction lenses along the UV light path.**

The sample has a width of 3.428mm, is made up of 6 periods and is thus 61.124mm in length, as seen in Fig. 2(a). The material used for fabrication was a custom formulated UV-cured 1,6 – Hexanediol diacrylate polymer, with a low dosage of photo-absorber (add the full name of Sudan), which has a young's modulus, E of 512 MPa and Density, 1.1 g/cm³. The working of this minuscule prototype is then experimentally validated in a 2D waveguide of height 6mm, to ensure that only the fundamental mode can propagate inside (this is hence, also the depth of the third dimension of the sample as seen in Fig. 2(a)). The waveguide shown in Fig. 2(b), is

made with laser-cut acrylic plates to confine the transmitted ultrasonic wave in a quasi-two-dimensional space. The entire experimental setup is essentially a scaled-down version of the scan stage used for audible sound in previous works^{7-9,31}. It would thus be ideal to use a speaker array to generate a plane wave with a Gaussian amplitude profile to impinge on the metasurface. However, for the convenience of the scaled down measurement platform, the source and the receiver are modified to increase the signal-to-noise ratio associated with the low operating wavelength. Full wave pressure acoustic simulations were performed to help determine the suitable experimental set up. Two single frequency (40 kHz) Murata speakers (MA40S4S) were employed as the source – fitted with tapered horn-type waveguide adapters, which were 3D printed for this purpose. As can be seen in the simulation of this set up on Fig. 2(b), these adapters ensure a planar wavefront to be incident on the metasurface of engineered phase gradient. Perfectly matched layers (PML) were used in the simulations to minimize reflections from the boundaries, which were imitated in experiments by using absorbing foam on both top and bottom. A two-dimensional linear scan stage was programmed to map the field on the transmissive side of the metasurface. This is done by translating a glass tube of diameter 1mm over a rectangular region, with a step size of 2mm. It should be noted that the scattering due to the tube, is negligible as it is much smaller than the wavelength of the propagating sound. At every step, the glass tube guides the sound to a microphone (Murata MA40S4R). An LM358-based operational amplifier was used as the pre-amplification system, from where the signal is transmitted to an NI PCI-6251 data-acquisition board. The collected time-domain signals were then Fourier transformed to the frequency domain to generate the complex field pattern at 40 kHz. Figure 3, shows the results from the measurement in comparison to two sets of full wave simulations - with and without dissipation. The 'without dissipation' case in Fig. 3(A) reaffirms the scalabil-

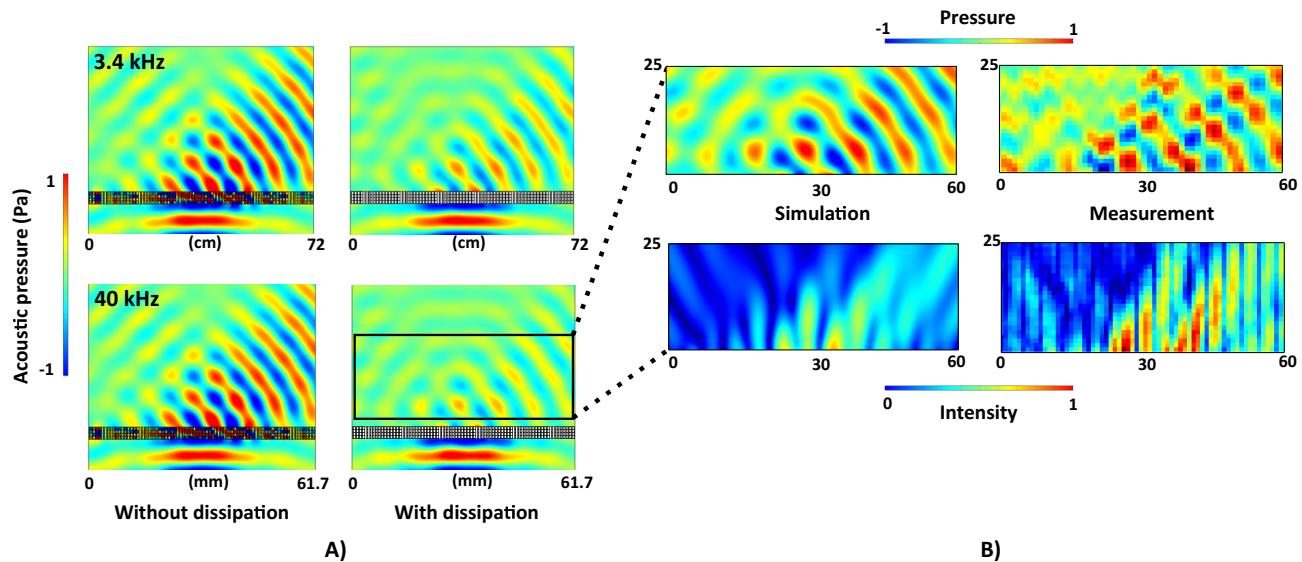


FIG. 3. (a) Simulated acoustic fields that show wavefront bending at both 3.4 kHz (top) and 40 kHz (bottom), for cases with (right) and without (left) dissipation. (b) Comparison of the normalized acoustic pressure and intensity fields for the "with dissipation" case and measured results.

218 ity of the GIM: Similar wavefront bending is seen for both the
 219 3.4 kHz (top) and 40 kHz (bottom). However, when the ther-
 220 moviscous effects are incorporated, the effect of loss is more
 221 significant in the case of 40 kHz, as discussed previously. The
 222 normalized wave field in the 'with dissipation' case in Fig
 223 3(A), is then compared with our measurement results shown
 224 in Fig 3(B). A reasonable agreement is seen in the numeri-
 225 cal and experimental results, for both complex pressure and
 226 intensity. Minor deviations can be attributed to experimental
 227 factors such as acoustic structure interaction, that are not taken
 228 into account in our numerical simulations. Additionally, the
 229 surface roughness/waviness of the sample and the stiffness of
 230 the polymer used for fabrication can be further optimized for
 231 more precise phase modulation.

232 In conclusion, we have designed, fabricated and experi-
 233 mentally characterized a minuscule gradient index metasur-
 234 face that operates at 40 kHz. The scalability of such a design
 235 is demonstrated and the role of thermoviscous interactions are
 236 discussed. It is clearly seen that dissipation has a greater ef-
 237 fect on transmission at higher frequencies. However, its effect
 238 on phase is rather negligible and wavefront modulation can
 239 therefore be realized. Although the presence of thermovis-
 240 ous dissipation can act as a limitation for transmissive appli-
 241 cations such as bending and focusing, it can be fruitful to en-
 242 gineer compact devices (Z: for) with features such as tunable
 243 asymmetric transmission^{7,22} and sound absorption²³. In addi-
 244 tion, the diffractive acoustics and the role of multiple internal
 245 reflections in the presence of dissipation in such a GIM can
 246 be further explored for airborne ultrasound. It is hoped that
 247 this study will bring about new possibilities to the research in
 248 acoustic metasurfaces, especially in miniaturized acoustic de-
 249 vices. Such a metasurface based design and their realization
 250 through new additive manufacturing techniques, can hence be
 251 readily scaled down to operate at much higher frequencies,
 252 to find applications as compact acoustic devices for sensing,

253 levitation, non-contact ultrasonic imaging and therapeutic ul-
 254 trasound.

255 1 B. Assouar, B. Liang, Y. Wu, Y. Li, J. C. Cheng, and Y. Jing, *Nature Re-*
 256 *views Materials* **3** (2018), 10.1038/s41578-018-0061-4.
 257 2 H. Ge, M. Yang, C. Ma, M. H. Lu, Y. F. Chen, N. Fang, and P. Sheng, *National Science Review* **5**, 159 (2018).
 258 3 S. A. Cummer, J. Christensen, and A. Alù, *Nature Reviews Materials* **1** (2016), 10.1038/natrevmats.2016.1.
 259 4 Y. Zhu, X. Fan, B. Liang, J. Cheng, and Y. Jing, *Physical Review X* **7**, 1 (2017).
 260 5 C. Shen, J. Xu, N. X. Fang, and Y. Jing, *Physical Review X* **4**, 1 (2014).
 261 6 G. Memoli, M. Caleap, M. Asakawa, D. R. Sahoo, B. W. Drinkwater, and S. Subramanian, *Nature Communications* **8**, Article 14608 (2017).
 262 7 Y. Li, C. Shen, Y. Xie, J. Li, W. Wang, S. A. Cummer, and Y. Jing, *Physical Review Letters* **119**, 035501 (2017).
 263 8 J. Li, C. Shen, A. Díaz-Rubio, S. A. Tretyakov, and S. A. Cummer, *Nature Communications* **9**, 1 (2018).
 264 9 Y. Xie, W. Wang, H. Chen, A. Konneker, B. I. Popa, and S. A. Cummer, *Nature Communications* **5**, 1 (2014).
 265 10 W. Wang, Y. Xie, B. I. Popa, and S. A. Cummer, *Journal of Applied Physics* **120** (2016), 10.1063/1.4967738.
 266 11 Y. Li, S. Qi, and M. B. Assouar, *New Journal of Physics* **18** (2016), 10.1088/1367-2630/18/4/043024.
 267 12 Y. Li, X. Jiang, B. Liang, J. C. Cheng, and L. Zhang, *Physical Review Applied* **4**, 1 (2015).
 268 13 Y. Xie, A. Konneker, B. I. Popa, and S. A. Cummer, *Applied Physics Letters* **103**, 1 (2013).
 269 14 Y. Li, B. Liang, X. Tao, X. F. Zhu, X. Y. Zou, and J. C. Cheng, *Applied Physics Letters* **101** (2012), 10.1063/1.4769984.
 270 15 K. Song, J. Kim, S. Hur, J.-H. Kwak, S.-H. Lee, and T. Kim, *Scientific Reports* **6**, 32300 (2016).
 271 16 C. Shen, A. Díaz-Rubio, J. Li, and S. A. Cummer, *Applied Physics Letters* **112** (2018), 10.1063/1.5025481.
 272 17 G. Y. Song, Q. Cheng, T. J. Cui, and Y. Jing, *Physical Review Materials* **2**, 065201 (2018).
 273 18 Y. Jing and S. A. Cummer, *Scientific reports*, 1 (2016).
 274 19 D. C. Skigin, M. Lester, S. Collin, M. Molerón, M. Serra-garcia, and C. Daraio, *New Journal of Physics* (2016), 10.1088/1367-2630/18/3/033003.
 275 20 X. Jiang, Y. Li, and L. Zhang, *The Journal of the Acoustical Society of America* **141**, EL363 (2017).

294 ²¹N. J. Gerard, Y. Li, and Y. Jing, *Journal of Applied Physics* **123** (2018), 312 ³⁰E. a. R. Ch, X. Y. Zheng, H. Lee, T. H. Weisgraber, M. Shusteff, J. DeOtte,
295 [10.1063/1.5007863](https://doi.org/10.1063/1.5007863). 313 E. B. Duoss, J. D. Kuntz, M. M. Biener, Q. Ge, J. A. Jackson, S. O.
296 ²²F. Ju, Y. Tian, Y. Cheng, and X. Liu, *Applied Physics Letters* **113** (2018), 314 Kucheyev, N. X. Fang, and C. M. Spadaccini, *Science* **344**, 1373 (2014).
297 [10.1063/1.5032263](https://doi.org/10.1063/1.5032263). 315 ³¹C. Shen, Y. Xie, J. Li, S. A. Cummer, and Y. Jing, *Applied Physics Letters*
298 ²³C. Shen and S. A. Cummer, *Physical Review Applied* **9**, 54009 (2018). 316 **108**, 2 (2016).
299 ²⁴L. Zigoneanu, B. I. Popa, and S. A. Cummer, *Physical Review B - Con-* 317 ³²N. Yu, P. Genevet, M. a. Kats, F. Aieta, J.-P. Tetienne, F. Capasso, and
300 *densed Matter and Materials Physics* **84**, 1 (2011). 318 Z. Gaburro, *Science* **334**, 333 (2011).
301 ²⁵B. Liu, J. Zhao, X. Xu, W. Zhao, and Y. Jiang, *Scientific Reports* **7**, 1 319 ³³Y. Li, B. Liang, Z.-m. Gu, X.-y. Zou, and J.-c. Cheng, *Scientific reports* **3**,
302 (2017). 320 **2546** (2013).
303 ²⁶Y. Xie, Y. Fu, Z. Jia, J. Li, C. Shen, Y. Xu, H. Chen, and S. A. Cummer, 321 ³⁴J. Qian, J.-p. Xia, H.-x. Sun, S.-q. Yuan, Y. Ge, and X.-z. Yu, *Journal of
304 *Scientific Reports* **8**, 1 (2018). Applied Physics* **122**, 244501 (2017).
305 ²⁷A. Marzo, S. A. Seah, B. W. Drinkwater, D. R. Sahoo, B. Long, and S. Sub- 322 ³⁵C. Shen, Y. Xie, J. Li, S. A. Cummer, and Y. Jing, *Journal of Applied
306 *ramanian, Nature Communications* **6**, 1 (2015). Physics **123** (2018), 10.1063/1.5009441.
307 ²⁸G. T. Clement, H. Nomura, H. Adachi, and T. Kamakura, *Physics in 323 ³⁶T. Liu, S. Liang, F. Chen, and J. Zhu, *Journal of Applied Physics* **123**
308 *Medicine and Biology* **58**, 6263 (2013). 324 (2018), 10.1063/1.4997631.
309 ²⁹X. Zheng, W. Smith, J. Jackson, B. Moran, H. Cui, D. Chen, J. Ye, N. Fang, 325 ³⁷Y. Zhu, J. Hu, X. Fan, J. Yang, B. Liang, X. Zhu, and J. Cheng, *Nature
310 N. Rodriguez, T. Weisgraber, and C. M. Spadaccini, *Nature Materials* **15**, 326 Communications* **9**, 1 (2018).
311 [1100 \(2016\).](https://doi.org/10.1038/nmat4890) 327 ³⁸Z. Tian, C. Shen, J. Li, E. Reit, Y. Gu, H. Fu, S. A. Cummer, and T. J.
330 Huang, *Advanced Functional Materials* **1808489**, 1808489 (2019).**