

Mathematical Modeling and Control of an Unmanned Aerial System with a Cable Suspended Payload

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Abstract—This paper focuses on the mathematical modeling and control of an unmanned aerial system (UAS) with a payload suspended using a cable. The motion of the payload induces disturbances on the aerial platform and needs to be mitigated for stable operation. A passivity based controller is used to control the position of the UAS. Assuming only the orientation angles of the payload with respect to a UAS are measured, a disturbance attenuating nonlinear attitude and position controller is designed. The implementation of the controller requires the knowledge of higher time derivatives of the payload oscillations. These states (primarily the angular velocity) are estimated using a continuous-discrete Kalman Filter. The efficacy of the combined estimator and controller is demonstrated using a high fidelity numerical simulation.

I. INTRODUCTION

The problem involving unmanned aerial vehicles for transporting cable suspended payloads has been studied extensively in recent years. The recent advancements in the sensor technology and the increase in the computation power has led to the availability of the inexpensive aerial robots capable of performing aggressive maneuvers and dynamic trajectory generation and tracking. This has opened up the possibility of deploying aerial vehicles to supply aid in disaster situations like floods and earthquakes. The cable suspended payload is an alternative solution to the problem of transporting payload using robotic grippers attached to the aerial vehicles [1]. The primary advantages of using cable suspended payload is the reduced inertia, when compared to the use of robotic grippers while still being able to lift and transport the payload [2] thus providing better response to the change in attitude. The problem of stabilizing the swinging payload attached to an underactuated system has been solved using feedback linearization [3], Interconnection and Damping Assignment Passivity Based Control [4], [5] and nested saturation [6]. This problem is similar to the problem of stabilizing the payload attached to an overhead crane or an inverted pendulum with horizontal and vertical motion [7]. In the present work, a passivity based control framework motivated by [8] and [4] is employed for a Quadcopter like UAS. The governing equations of motion of the combined system (Quadcopter + payload) are derived first. Assuming the motion of the quadcopter and the payload in the longitudinal plane, the governing equations for the coupled motion with the longitudinal plane restriction are developed. To control the position and the attitude, a desired control thrust for position tracking

is derived first which leads to the synthesis of a desired pitch attitude. A nonlinear back-stepping like control law is designed to track the desired pitch angle to complete the control design. Since the controller design depends upon the knowledge of the payload swing angle and its derivatives, a state estimation filter is utilized to synthesize the angular velocity and angular acceleration of the payload assuming that the payload swing angle is measured.

The paper is organized as follows. Section II develops a mathematical model for a quadcopter with a cable suspended payload. Section III presents the passivity based control design of the system to control the position of the quadcopter and an error tracking based controller to control the attitude dynamics of the quadcopter. Finally, section IV demonstrates the effectiveness of the control law in the numerical simulation. A continuous-discrete Kalman Filter is implemented to estimate the angular velocity and angular acceleration of the swinging payload, which are used by the controller.

II. DYNAMIC MODEL

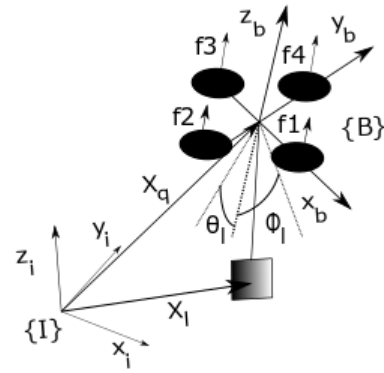


Fig. 1: Description of coordinate frames associated with the quadcopter

The following assumptions are made for the dynamic analysis of the quadrotor with a payload attached to it by a cable.

- The cable is massless and has no slack.
- Aerodynamic effects acting on the payload and the quadcopter are neglected.
- Suspension point is same as the center of mass of the quadcopter.
- Suspension is frictionless.

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The figure 1 describes the kinematic relations of the quadcopter with a cable attached payload. Consider an inertial coordinate frame $\{\mathbf{I}\}$ fixed to the ground and a body fixed frame attached to the center of mass of the quadcopter $\{\mathbf{B}\}$. The generalized coordinates $\mathbf{q}$ and the generalized velocities $\dot{\mathbf{q}}$ are given by, $\mathbf{q} = [x_Q \ y_Q \ z_Q \ \theta_l \ \phi_l \ \psi]^T$ and $\dot{\mathbf{q}} = [\dot{x}_Q \ \dot{y}_Q \ \dot{z}_Q \ \dot{\theta}_l \ \dot{\phi}_l \ \dot{\psi}]^T$ respectively. Let $\mathbf{X}_Q \in \mathbb{R}^3$, where $\mathbf{X}_Q = [x_Q, y_Q, z_Q]^T \in \mathbb{R}^3$ denotes the position of the quadcopter in the inertial frame; $[\phi, \theta, \psi]^T$ denote the Euler angles (roll, pitch, and yaw); $[\theta_l, \phi_l]^T$ denotes the swing angle of the cable. l is the length of the cable.

The kinematic relation between the center of mass of the payload $\mathbf{X}_l = [x_l, y_l, z_l]^T$ and the center of mass of the quadcopter in the inertial frame is obtained as,

$$\mathbf{X}_l = [x_Q \ y_Q \ z_Q]^T + \mathbf{R}_y(\phi_l) \mathbf{R}_x(\theta_l) [0 \ 0 \ -l]^T, \quad (1)$$

where $\mathbf{R}_x(\theta_l)$ and $\mathbf{R}_y(\phi_l)$ are the rotation matrices defined as,

$$\mathbf{R}_y(\phi_l) = \begin{bmatrix} \cos(\phi_l) & 0 & \sin(\phi_l) \\ 0 & 1 & 0 \\ -\sin(\phi_l) & 0 & \cos(\phi_l) \end{bmatrix}$$

$$\mathbf{R}_x(\theta_l) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_l) & -\sin(\theta_l) \\ 0 & \sin(\theta_l) & \cos(\theta_l) \end{bmatrix}$$

Using equation (1), the coordinates of the center of mass of the payload can be expressed in terms of the quadcopter position and the swing angle of the cable as,

$$\begin{aligned} x_l &= x_Q - l \cos(\theta_l) \sin(\phi_l), \\ y_l &= y_Q + l \sin(\theta_l), \\ z_l &= z_Q - l \cos(\theta_l) \cos(\phi_l) \end{aligned} \quad (2)$$

Using equation (2), the velocity kinematics for the payload is given as

$$\dot{\mathbf{X}}_l = \mathbf{J}_1(\theta_l, \phi_l) [\dot{x}_Q \ \dot{y}_Q \ \dot{z}_Q \ \dot{\theta}_l \ \dot{\phi}_l]^T \quad (3)$$

where $\mathbf{J}_1$ is,

$$\mathbf{J}_1 = \begin{bmatrix} 1 & 0 & 0 & -l \sin(\phi_l) \sin(\theta_l) & l \cos(\theta_l) \cos(\phi_l) \\ 0 & 1 & 0 & l \cos(\theta_l) & 0 \\ 0 & 0 & 1 & l \cos(\phi_l) \sin(\theta_l) & l \cos(\theta_l) \sin(\phi_l) \end{bmatrix} \quad (4)$$

The body angular velocity of the quadcopter, $\boldsymbol{\omega} = [\dot{\omega}_x, \dot{\omega}_y, \dot{\omega}_z]^T$ is related to the rate of change of Euler angles as,

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin(\theta) \\ 0 & \cos(\phi) & \cos(\theta) \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\theta) \cos(\phi) \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}. \quad (5)$$

and we denote,

$$\mathbf{J}_2 = \begin{bmatrix} 1 & 0 & \sin(\theta) \\ 0 & \cos(\phi) & \cos(\theta) \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\theta) \cos(\phi) \end{bmatrix}. \quad (6)$$

This relation will be used further in the derivation of the equations of motion of the quadcopter with cable suspended load using Lagrange-Euler formulation [9].

A. Euler-Lagrange Equations

The total kinetic energy of the quadcopter with a cable suspended load can be partitioned as sum of the quadcopter kinetic energy (T_Q),

$$T_Q = \frac{1}{2} m_Q \dot{x}_Q^2 + \frac{1}{2} m_Q \dot{y}_Q^2 + \frac{1}{2} m_Q \dot{z}_Q^2 + \frac{1}{2} \boldsymbol{\omega}^T \mathbf{J}_Q \boldsymbol{\omega}$$

and the cable suspended payload kinetic energy (T_l),

$$T_l = \frac{1}{2} m_l \dot{x}_l^2 + \frac{1}{2} m_l \dot{y}_l^2 + \frac{1}{2} m_l \dot{z}_l^2 + \frac{1}{2} I_l (\dot{\theta}_l^2 + \dot{\phi}_l^2)$$

where $\mathbf{J}_Q$ is the inertia matrix of the quadcopter and I_l is the moment of inertia of the payload.

Using the Jacobians derived in equations (4) and (6), the total kinetic energy $T(\mathbf{q}, \dot{\mathbf{q}})$ can be written in terms of generalized coordinates as, $T(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{J}(\mathbf{q})^T \mathbf{M} \mathbf{J}(\mathbf{q}) \dot{\mathbf{q}}$, with

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_Q & \mathbf{0}_{3 \times 2} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{M}_l & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 2} & \mathbf{J}_Q \end{bmatrix} \quad \& \quad \mathbf{J} = \begin{bmatrix} \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 2} & \mathbf{0}_{3 \times 3} \\ \mathbf{I}_{3 \times 3} & \mathbf{J}_1 & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 2} & \mathbf{J}_2 \end{bmatrix}$$

where $\mathbf{M}_Q = \text{diag}(m_Q, m_Q, m_Q)$ (m_Q is the mass of the quadcopter) $\mathbf{M}_l = \text{diag}(m_l, m_l, m_l)$ (m_l is the mass of the payload) and $\mathbf{I}_l = \text{diag}(I_l, I_l)$.

The total potential energy function $V(\mathbf{q})$ of the system is the sum of the potential energy of the quadcopter and the payload and is given as, $V(\mathbf{q}) = m_Q g z_Q + m_l g (z_Q - l \cos(\theta_l) \cos(\phi_l))$ where g is the acceleration due to gravity.

Using the expressions for the kinetic and potential energy, the Lagrangian is formulated as, $\mathcal{L} = T(\mathbf{q}, \dot{\mathbf{q}}) - V(\mathbf{q})$

Using the Euler-Lagrange formulation, the mathematical model for the quadcopter with cable suspended load is obtained in the form of

$$\mathbf{M}(\mathbf{q}) \ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \mathbf{f}_{ext} \quad (7)$$

where, $\mathbf{f}_{ext} = [F_x \ F_y \ F_z \ 0 \ 0 \ \tau_x \ \tau_y \ \tau_z]^T$ denotes the control input to the quadcopter in the inertial frame. It is easily seen that,

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \mathbf{R}(\phi, \theta, \psi) \begin{bmatrix} 0 \\ 0 \\ u_1 \end{bmatrix} \quad (8)$$

where, $\mathbf{R}(\phi, \theta, \psi)$ is the rotation matrix translating the thrust and the moment inputs from the quadcopter body frame $\{\mathbf{B}\}$ to the inertial frame $\{\mathbf{I}\}$. In terms of the individual thrust forces produced by the quadcopter (f_1, f_2, f_3 and f_4), the control inputs are determined as

$$\begin{aligned} u_1 &= f_1 + f_2 + f_3 + f_4, \\ \tau_x &= L(f_4 - f_2), \\ \tau_y &= L(f_3 - f_1), \\ \tau_z &= (-Q_1 + Q_2 - Q_3 + Q_4), \end{aligned}$$

where, Q_1, Q_2, Q_3, Q_4 are the moments generated by each of the rotors and L is the arm length of the quadcopter. Given the combined dynamics of the quadcopter and the payload in equation 7, assuming the quadcopter in a hover configuration, control functions f_i are sought such that

$\theta_l, \phi_l, \dot{\theta}_l, \dot{\phi}_l \rightarrow 0$, as $t \rightarrow \infty$ for initial condition disturbances $\theta_l(0), \phi_l(0), \dot{\theta}_l(0), \dot{\phi}_l(0)$. Additionally, $\Delta x_Q, \Delta z_Q \rightarrow 0$ as $t \rightarrow 0$ where $\Delta x_Q, \Delta z_Q$ are perturbations of the quadcopter position from the initial hover configuration.

III. SOLUTION METHODOLOGY (CONTROLLER DESIGN)

Consider the two dimensional motion (longitudinal plane restriction) of the quadcopter with cable suspended payload in $x_l - z_l$ inertial plane. The simplified model for the 2D case can be obtained by applying constraints on the dynamic model obtained in equation 7. The system dynamics for the quadcopter with cable suspended load in 2D plane is described as,

$$(m_Q + m_l)(\ddot{z}_Q + g) + m_l l (\cos(\phi_l) \dot{\phi}_l^2 - \sin(\phi_l) \ddot{\phi}_l) = F_z \quad (9)$$

$$(m_Q + m_l) \ddot{x}_Q + m_l l (\sin(\phi_l) \dot{\phi}_l^2 + \cos(\phi_l) \ddot{\phi}_l) = F_x \quad (10)$$

$$m_l l^2 \ddot{\phi}_l + m_l l \sin(\phi_l) \ddot{z} - m_l l \cos(\phi_l) \ddot{x} + m_l g l \sin(\phi_l) = 0 \quad (11)$$

$$I_{yy} \dot{\omega}_y = \tau_y \quad (12)$$

where,

$$\begin{aligned} F_x &= u_1 \sin(\theta), \\ F_z &= u_1 \cos(\theta), \end{aligned} \quad (13)$$

are the control inputs. In this paper, we adopt a similar control strategy as in [8] i.e. design a passivity based controller to stabilize the system by controlling the overall energy of the system. The system is decoupled into an inner control loop to stabilize the attitude dynamics of the quadcopter and an outer loop to stabilize the translational dynamics of the quadcopter along with the pendulum dynamics [6]. The outer loop is further decoupled to stabilize the altitude of the quadcopter (z_Q), longitudinal position of the quadcopter (x_Q) and the swing of the pendulum. From equations (9) and (10)

$$\ddot{z}_Q = \frac{u_1 \cos(\theta)}{m_Q + m_l} - g - \frac{m_l l}{m_Q + m_l} [\cos(\phi_l) \dot{\phi}_l^2 + \sin(\phi_l) \ddot{\phi}_l] \quad (14)$$

$$\ddot{x}_Q = \frac{u_1 \sin(\theta)}{m_Q + m_l} - \frac{m_l l}{m_Q + m_l} [\sin(\phi_l) \dot{\phi}_l^2 - \cos(\phi_l) \ddot{\phi}_l] \quad (15)$$

Using equations (11), (14), and (15),

$$\ddot{\phi}_l = \frac{-\sin(\phi_l - \theta) u_1}{m_Q l} \quad (16)$$

Substituting equation (16) in equations (14) and (15),

$$\begin{aligned} \ddot{z}_Q &= \frac{1}{m_Q + m_l} \left[\cos(\theta) + \frac{m_l}{2m_Q} (\cos(\theta) - \cos(\theta - 2\phi_l)) \right] u_1 \\ &\quad - g - \frac{m_l l \cos(\phi_l) \dot{\phi}_l^2}{m_Q + m_l} \end{aligned} \quad (17)$$

$$\begin{aligned} \ddot{x}_Q &= -\frac{m_l l \sin(\phi_l) \dot{\phi}_l^2}{m_Q + m_l} \\ &\quad + \frac{1}{m_Q + m_l} \left[\sin(\theta) + \frac{m_l}{2m_Q} (\sin(\theta) - \sin(\theta - 2\phi_l)) \right] u_1 \end{aligned} \quad (18)$$

For hover conditions and small pitch angles of the quadcopter, the attitude dynamics can be approximated using $\cos(\theta) \approx 1$ and $\sin(\theta) \approx \theta$ and treating θ as the control input θ_d results into following dynamics for $\ddot{x}_Q$ and $\ddot{z}_Q$

$$\begin{aligned} \ddot{z}_Q &= \frac{1}{m_Q + m_l} \left[1 + \frac{m_l}{2m_Q} (1 - \cos(2\phi_l) - \theta_d \sin(2\phi_l)) \right] u_1 \\ &\quad - g - \frac{m_l l \cos(\phi_l) \dot{\phi}_l^2}{m_Q + m_l} \end{aligned} \quad (19)$$

$$\begin{aligned} \ddot{x}_Q &= -\frac{m_l l \sin(\phi_l) \dot{\phi}_l^2}{m_Q + m_l} + \frac{\theta_d}{m_Q + m_l} \\ &\quad + \frac{1}{m_Q + m_l} \left[\frac{m_l}{2m_Q} (\theta_d - \theta_d \cos(2\phi_l) - \sin(2\phi_l)) \right] u_1 \end{aligned} \quad (20)$$

Near hovering condition, $u_1 \approx (m_Q + m_l)g$ and longitudinal dynamics can only be controlled by θ_d . This results into the following for $\ddot{x}_Q$,

$$\begin{aligned} \ddot{x}_Q &= \theta_d \left[g + \frac{gm_l}{2m_Q} (1 - \cos(2\phi_l)) \right] - \frac{m_l l \sin(\phi_l) \dot{\phi}_l^2}{m_l + m_Q} \\ &\quad - \frac{m_l g \sin(2\phi_l)}{2m_Q} \end{aligned} \quad (21)$$

Thus, choosing θ_d and u_1 as,

$$\theta_d = \frac{u_x + \frac{m_l l \sin(\phi_l) \dot{\phi}_l^2}{m_l + m_Q} + \frac{m_l g \sin(2\phi_l)}{2m_Q}}{\left[g + \frac{gm_l}{2m_Q} (1 - \cos(2\phi_l)) \right]} \quad (22)$$

$$u_1 = \frac{(m_Q + m_l) \left(g + \frac{m_l l \cos(\phi_l) \dot{\phi}_l^2}{m_Q + m_l} + u_z \right)}{\left[1 + \frac{m_l}{2m_Q} (1 - \cos(2\phi_l) - \theta_d \sin(2\phi_l)) \right]} \quad (23)$$

and with

$$\begin{aligned} u_z &= -k_p^z (z_Q - z_d) - k_d^z (\dot{z}_Q - \dot{z}_d) + \ddot{z}_d \\ u_x &= -k_p^x (x_Q - x_d) - k_d^x (\dot{x}_Q - \dot{x}_d) + \ddot{x}_d \end{aligned}$$

the closed loop position tracking dynamics of the quadcopter are reduced to,

$$\begin{aligned} \ddot{z}_Q &= -k_p^z (z_Q - z_d) - k_d^z (\dot{z}_Q - \dot{z}_d) + \ddot{z}_d \\ \ddot{x}_Q &= -k_p^x (x_Q - x_d) - k_d^x (\dot{x}_Q - \dot{x}_d) + \ddot{x}_d \end{aligned}$$

which ensures decay of the $x_Q \rightarrow 0$ and $z_Q \rightarrow 0$. The rate of decay and other transient characteristics of the tracking errors are controlled by tuning the positive gains k_p^z, k_d^z, k_p^x , and k_d^x .

Remarks

- The nonlinear control laws in equations (22) and (23) hinge upon the approximation $u_1 \approx (m_Q + m_l)g$. This is remedied later.
- The control laws require the knowledge of the swing angles and the angular velocities.

In what follows, the attitude control design procedure can either use θ_d as synthesized above or in any other manner such as the one discussed hereon.

From equations (14) and (15) the quadcopter dynamics can be expressed in the following form,

$$(m_Q + m_l) \ddot{z}_Q = u_1 \cos(\theta) - (m_Q + m_l) (g + \lambda f_1 (\phi_l, \dot{\phi}_l, \ddot{\phi}_l)) \quad (24)$$

$$(m_Q + m_l) \ddot{x}_Q = u_1 \sin(\theta) - (m_Q + m_l) (\lambda f_2 (\phi_l, \dot{\phi}_l, \ddot{\phi}_l)) \quad (25)$$

where, $\lambda = \frac{m_l l}{m_Q + m_l}$, $f_1 = \cos(\phi_l) \dot{\phi}_l^2 + \sin(\phi_l) \ddot{\phi}_l$, and $f_2 = \sin(\phi_l) \dot{\phi}_l^2 - \cos(\phi_l) \ddot{\phi}_l$.

At the hover condition, the desired pitch angle is derived as, $\theta_d = \tan^{-1} \left(\frac{\lambda f_2 (\phi_l, \dot{\phi}_l, \ddot{\phi}_l)}{g + \lambda f_1 (\phi_l, \dot{\phi}_l, \ddot{\phi}_l)} \right)$ and the desired total thrust is obtained as,

$$u_1 = \sqrt{(\lambda f_2 (\phi_l, \dot{\phi}_l, \ddot{\phi}_l))^2 + (g + \lambda f_1 (\phi_l, \dot{\phi}_l, \ddot{\phi}_l))^2}$$

The pitch controller can be designed using the pitch tracking error e_θ which is defined as, $e_\theta = \theta - \theta_d$, and $\dot{e}_\theta = \dot{\theta} - \dot{\theta}_d$. We seek a very tightly controlled pitch loop, so the pitch errors are prescribed to converge to zero exponentially with a decay rate of λ_θ . The pitch error dynamics takes the form, $\dot{e}_\theta = -\lambda_\theta e_\theta$. Thus $\omega_{yd} = -\lambda_\theta e_\theta + \dot{\theta}_d$ is the desired pitch rate that ensures $e_\theta \rightarrow 0$, as $t \rightarrow \infty$. The pitch rate error (e_ω) is then obtained as, $e_\omega = \omega_y - \omega_{yd}$. The pitch rate errors are also prescribed to converge to zero exponentially with a decay rate of λ_ω . Thus, the desired pitch rate error dynamics takes the form, $\dot{e}_\omega = -\lambda_\omega e_\omega$. Thus, $\dot{\omega}_y - \dot{\omega}_{yd} = -\lambda_\omega e_\omega$ and, the control law for the pitch dynamics is determined as,

$$\tau_y = I_{yy} (-\lambda_\omega e_\omega + \dot{\omega}_{yd})$$

It is noted that the above procedure is a nested backstepping process [10].

Clearly, for the backstepping process to be followed, the controller requires several higher derivatives of the payload swing angle. The measurement of all swing angle and all its higher order derivatives is a challenging task. To address this, we observe the following:

Specifically at the hover state, the dynamics of the swinging payload can be assumed to be equivalent to the dynamics of a simple pendulum linearized about some angle ϕ_l is given by, $\ddot{\phi}_l \approx -\frac{g}{l} \phi_l$. Thus, if the swing angle is measured, using this approximation, the angular velocity as well as the acceleration of the swinging payload can be estimated. Also, under these assumptions all higher order derivatives are successively obtained, making the backstepping control approach feasible. Note, assuming that the angle measurements for the cable are available, the angular velocity of the swinging cable can be estimated using a Continuous-Discrete Kalman Filter (CDKF) driven by the cable angle measurements. In our specific implementation, the propagation is continuous while the measurement updates happen at discrete instants (every 0.01 s). The CDKF utilizes the simplified one degree-of-freedom dynamics, $\ddot{\phi}_l = -\frac{g}{l} \phi_l + w_\phi(t)$, where $w_\phi(t)$ is a zero mean white noise process with covariance $0.25 \text{ rad}^2/\text{s}^4$. The measurement equation is $\dot{\phi}_l = \phi_l + v_\phi(t)$. The measurement error $v_\phi(t)$ is also assumed to be a zero-mean white noise process with variance 0.01 rad^2 . Additionally, $w_\phi(t)$, and $v_\phi(t)$ are assumed to be uncorrelated. The derivation of the CDKF is straightforward and can be found in [11].

IV. SIMULATION RESULTS

The following model and controller parameters were used to simulate the quadcopter with a cable suspended payload.

Parameter	Value
m_Q (kg)	1
m_l (kg)	0.5
l (m)	0.75
g (m/s ²)	9.81

A. Case 1: Quadcopter in Hover Mode and the Payload is Perturbed

The quadcopter is in a hovering state at the start of the simulation. The payload cable angle at the start of the

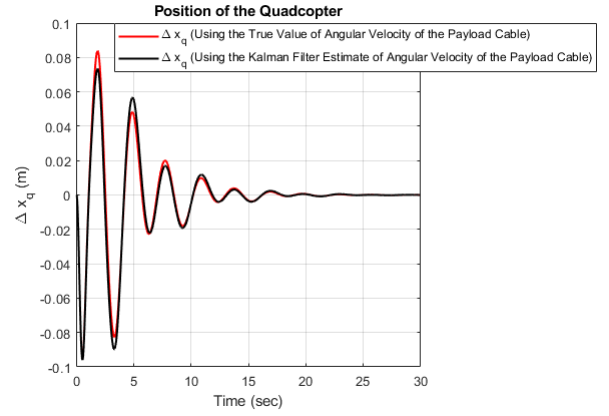


Fig. 2: Longitudinal position of the quadcopter

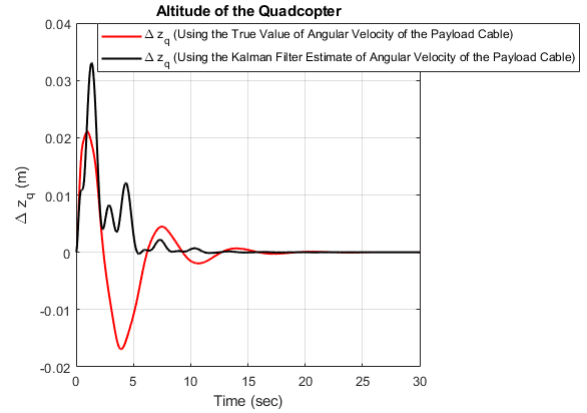


Fig. 3: Altitude of the quadcopter

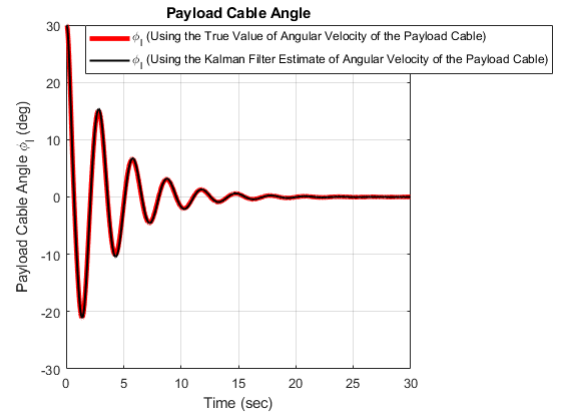


Fig. 4: Payload Cable angle history

simulation is 30° . It can be seen from the figures 2, 3, and 4 that as the simulation begins, there are disturbances introduced in the system due to the oscillation of the payload. The controller tries to reject these disturbances and gradually tries to bring the quadcopter to its initial starting position. The amplitude of the oscillations of the payload is observed to decay. From figure 5 it can be observed that the attitude controller tries to maintain the quadcopter position along x_i direction by changing the pitch angle of the quadcopter. The

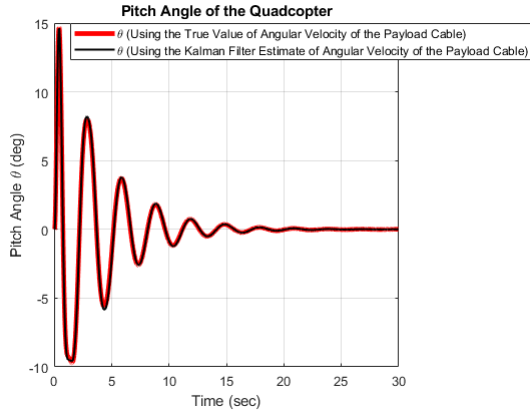


Fig. 5: Pitch angle of the quadcopter

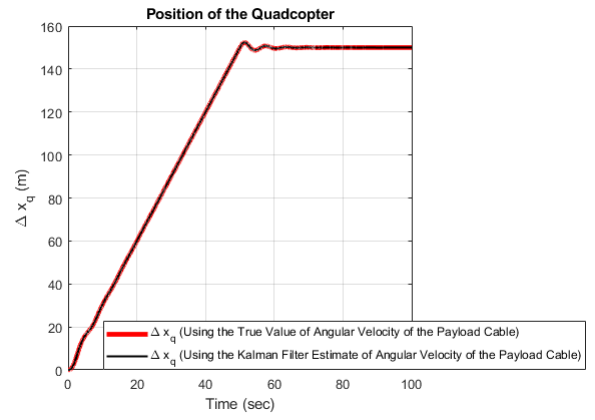


Fig. 7: Longitudinal position of the quadcopter

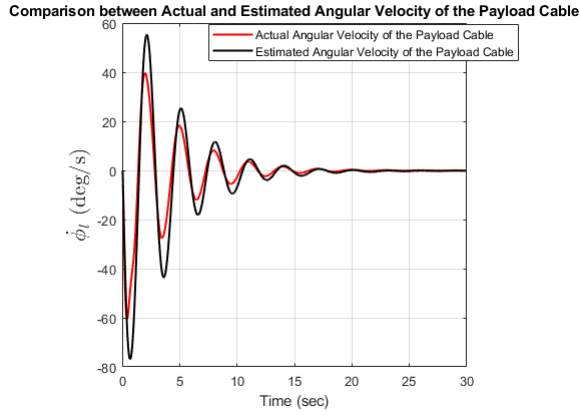


Fig. 6: Comparison between the Actual and the Estimated Angular Velocity of the Payload Cable Using Kalman Filter

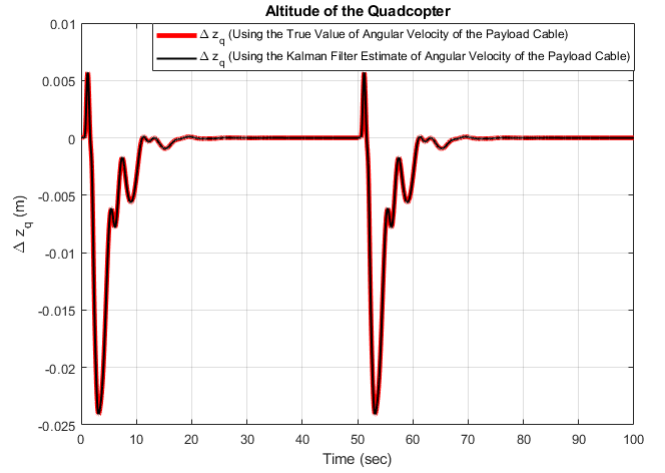


Fig. 8: Altitude of the quadcopter

pitch angle has an oscillatory behavior as well and it decays to zero as the quadcopter goes back to the initial hover pose and the payload oscillations decay. It can be seen from figure 6 that the estimator is able to track the angular velocity states of the payload cable and the controller provides a reasonable performance as compared to the case when the angular velocity of the payload cable was available as a measurement. The tuning of the parameters for the estimator as well as the controller is preliminary and requires careful consideration.

B. Case 2: Quadcopter Moving with a Constant Speed along x_i axis and is suddenly Commanded to go in the Hover Mode

The quadcopter is in hover mode and the payload is in a stable configuration at the start of the simulation. The quadcopter is commanded to follow a straight line trajectory along x_i axis with a constant speed of 10.8 kmph (3 m/s) and constant altitude. At 50 s, the quadcopter is commanded to go back to the hover mode. From figure 9, it can be seen that the payload is perturbed due to sudden changes in the quadcopter states at the start of the simulation and the controller is able to stabilize the payload when the quadcopter is in motion. The payload is perturbed again when the quadcopter goes

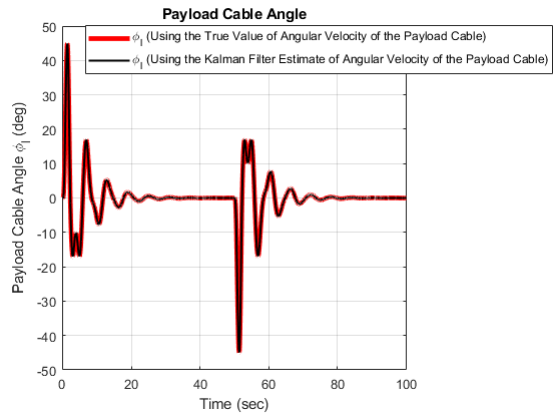


Fig. 9: Payload Cable angle history

back to the hover mode and the combined estimator and controller is able to stabilize the system. From figure 7 and 8, it can be observed that the controller is able to maintain the quadcopter altitude as well as the trajectory along x_i axis and desired position when commanded to go back to the hover mode. The quadcopter position and velocity along x_i axis is maintained by changing the pitch angle. From figure 10, it is

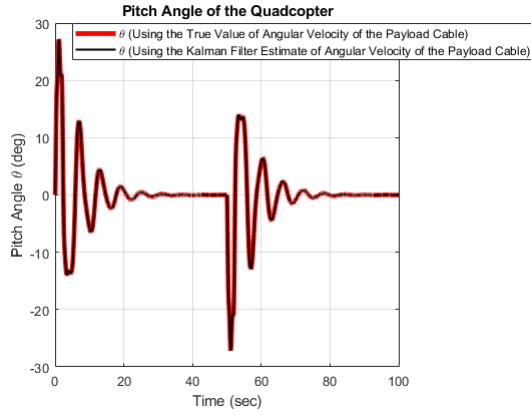


Fig. 10: Pitch angle of the quadcopter

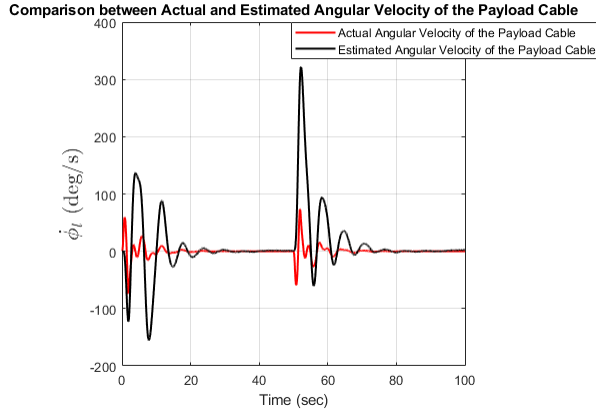


Fig. 11: Comparison between the Actual and the Estimated Angular Velocity of the Payload Cable Using Kalman Filter

observed that the pitch angle has an oscillatory behavior as the quadcopter is commanded to follow a constant speed straight line trajectory and the oscillations decay as the quadcopter attains the desired speed. When the quadcopter is commanded to go back to the hover mode at 50 s, the controller tries to maintain the position of the quadcopter along the x_i by changing the pitch angle. A decaying oscillatory behavior in the pitch angle of the quadcopter is observed again as the quadcopter goes back to the hover pose. It can be seen from figure 11 that the estimator overestimates the angular velocity states of the payload cable and the controller provides a reasonable performance as compared to the case when the angular velocity of the payload cable was available as a measurement.

V. SUMMARY, CONCLUSIONS, AND FUTURE WORK

Mathematical modeling was done for a quadcopter with a cable suspended payload using Euler-Lagrange formulation. A passivity based control law was used to control the position of the quadrotor thus attenuating the oscillations of the payload. A Continuous-Discrete Kalman Filter was implemented in the simulations to estimate angular velocity of the oscillating payload. These estimates were used by the passivity based controller. A pitch controller was designed

to control the pitch of the quadcopter, thus providing the necessary force to control the position of the quadcopter along x_i direction. The future work includes extending this framework to 3D and comparing with other techniques such as the extended state observer for disturbance estimation. Experiments will be planned to test the performance of the control law on a physical quadcopter platform.

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