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HYBRID-BISTABLE VIBRATION ENERGY HARVESTER WITH ADAPTIVE POTENTIAL WELL

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ABSTRACT

Many common environmental vibration sources exhibit low and broad frequency spectra. In order to exploit such excitations, energy harvesting architectures utilizing nonlinearity, especially bistability, have been widely studied since the energetic interwell oscillations between their stable equilibria can provide enhanced power harvesting capability over a wider bandwidth compared to the linear counterpart. However, one of the limitations of these nonlinear architectures is that the interwell oscillation regime may not be activated for a low excitation level that is not strong enough to overcome the potential energy barrier, thus resulting in low amplitude intrawell response which provides poor energy harvesting performance. While the strategic integration of bistability and additional dynamic elements has shown potential to improve broadband energy harvesting performance by lowering the potential barrier, there is a clear opportunity to further improve the energy harvesting performance by extracting electrical power from the kinetic energy in the additional element that is induced when the potential barrier is lowered. To explore this opportunity and advance the state of the art, this research develops a novel hybrid bistable vibration energy harvesting system with a passive mechanism that not only adaptively lowers the potential energy barrier level to improve broadband performance but also exploits additional means to capture more usable electrical power. The proposed harvester is comprised of a cantilever beam with repulsive magnets, one attached at the free end and the other attached to a linear spring that is axially aligned with the cantilever (a spring-loaded magnet oscillator). This new approach capitalizes on the adaptive bistable potential that is passively realized by the spring-loaded magnet oscillator, which lowers the double-well potential energy barrier thereby facilitating the interwell oscillations of the cantilever across a broad range of excitation conditions, especially for low excitation amplitudes and frequencies. The interwell oscillation of the cantilever beam enhances not only the piezoelectric energy

harvesting from the beam but also the electromagnetic energy harvesting from the spring-loaded magnet oscillator by inducing large amplitude vibrations of the magnet oscillator. Numerical investigations found that the proposed architecture yields significantly enhanced energy harvesting performance compared to the conventional bistable harvester with fixed magnet.

Keywords: energy harvesting, bistable potential, adaptive, Duffing oscillator, piezoelectric, electromagnetic

1 INTRODUCTION AND MOTIVATION

The relatively slow rate of technological advancement and high maintenance costs associated with conventional battery technology have led to the exploration of energy harvesters as an alternative power source for small-scale electronic devices [1], [2]. A significant amount of research has focused on vibration energy harvesting with the ubiquity of energy sources that exist in practical application environments. Conventional linear vibration energy harvesters exploit the concept of linear resonance, thus delivering the most power when exposed to an excitation frequency that exists in a narrow frequency band. However, most ambient vibration sources are broadband, and this has inspired a substantial amount of research into methods for improvement of effective harvesting bandwidth [3]–[10].

Current broadband energy harvesting techniques include (but are not limited to): resonance tuning [11], [12], frequency up-conversion [13], [14], multimodality [15], [16], and introduction of nonlinearities [4], [17]. Bistable nonlinear energy harvesters have been of particular interest, due to their potential for high-energy *interwell* (or snap-through) oscillations in a wide-frequency bandwidth [8], [17]. Bistable harvesters exhibit a double-well potential, which includes two stable equilibrium positions that bracket an unstable equilibrium (Figure 2). Intrawell oscillations constitute a low energy state which occurs

when the system oscillates about one of the stable equilibria, while interwell oscillations, where the harvester oscillates between the two stable equilibria, may provide high energy output [18]. Previous studies have revealed that bistable harvesters may outperform their linear counterparts over a wide frequency region when subjected to harmonic [6], [19], [20] or stochastic excitations [21], [22], provided that interwell response is achieved. In general, bistable harvesters with a higher potential energy barrier may provide larger power outputs since the distance between stable equilibria increases, resulting in larger response amplitudes. On the other hand, if the excitation is not sufficient to overcome the potential barrier height and activate interwell response, the bistable harvester will operate in the intrawell oscillation regime, which may yield low energy harvesting performance. Therefore, in order to maximize the performance of bistable harvesters, it is critical to carefully design the restoring force potential to facilitate interwell oscillations over a wide variety of excitation conditions [9], [23]–[25].

One avenue to facilitate interwell oscillations is to design the bistable system with shallow potential wells, making interwell oscillations possible across a broader range of excitation frequencies and amplitudes. To achieve shallow potential wells, researchers have explored platforms with fixed (static) restoring force potential, such as modified magnetic buckling loads or multi-stability (tri- or quad-stable systems) [26]–[30]. In these cases, the invariant restoring force potential is designed to have a potential energy barrier height that is more favorable to interwell oscillations (shallow well), thus improving energy harvesting performance, especially for low-intensity ambient excitations. In an effort to achieve even greater energy harvesting performance over a broad excitation frequency bandwidth, dynamically changing the restoring force potential has also been explored. Adaptive energy wells have been introduced through active methods via electromagnetic control of the magnetic potential [31], [32]. Passive methods have been explored as well, which avoid the power and sophisticated control requirements associated with active approaches. It has been found that strategically integrating a magnetically coupled oscillator with monostable and bistable energy harvesters may enhance the energy harvesting performance [33]–[39]. While these advancements have shown substantial potential to improve broadband energy harvesting performance by lowering the potential barrier (thus, reducing the excitation intensity threshold for activating interwell oscillation), there is a clear but unexplored opportunity to further enhance system energy harvesting capability by exploiting the usable power that is redistributed when lowering the potential energy barrier and leveraging the dynamic characteristics of the coupled oscillator to further enhance the interwell response of the bistable cantilevered beam.

To advance the state of the art, this research explores the passive and adaptive potential well concept in a new light toward the development of a novel multimodal-hybrid energy harvesting platform comprised of a magnetic repulsive bistable cantilever beam and a spring-loaded magnet oscillator aligned with the

cantilever. The proposed architecture exhibits a hybrid energy harvesting capability in that the passive mechanism (the spring-loaded magnet oscillator), developed to lower the potential energy barrier by adaptively reducing the magnetic potential energy, concurrently extracts electrical power from the redistributed energy. This electrical power is extracted by harnessing the kinetic energy of the magnetically coupled oscillator that is induced when adaptively lowering the magnetic potential energy. In addition to the extra power directly captured by the electromagnetic conversion, the architecture is configured in such a way that the dynamic characteristics of the linear magnet oscillator, especially the resonance and higher harmonics, can be exploited to further enhance the interwell response of the bistable cantilevered beam. Thus, the unique dynamic characteristics of the proposed bistable oscillator and adaptive well mechanism can notably enhance the energy harvesting capability by not only more easily facilitating the high-energy interwell oscillations of the bistable harvester but also capturing energy from the magnetically coupled oscillator. In the following section, the working principle and mathematical model behind the newly proposed architecture is presented. Next, numerical investigation results are introduced, which demonstrate the effectiveness of the proposed method for a wide range of operating conditions. Finally, concluding remarks are provided to summarize the potential of the new platform.

2 MATHEMATICAL MODELING AND WORKING PRINCIPLE

A schematic of the nonlinear hybrid energy harvester proposed in this research is shown in Figure 1. The energy harvester is comprised of a cantilever beam with a permanent magnet (magnet 1) attached at the free end and a spring-loaded permanent magnet (magnet 2) that is attached to the base and is aligned with the cantilever beam. The polarities of the magnets are oriented in the opposite direction to exhibit repulsive magnetic force. The magnets are located closer than the critical separation distance where the magnetic repulsion exceeds the elastic restoring force of the beam [40], such that the cantilever beam exhibits bistable dynamics with double-well potential [8], [18]. The cantilever beam is deflected into one of the two stable equilibria that are separated by the potential energy barrier at the center which is induced by the repulsive magnetic force between

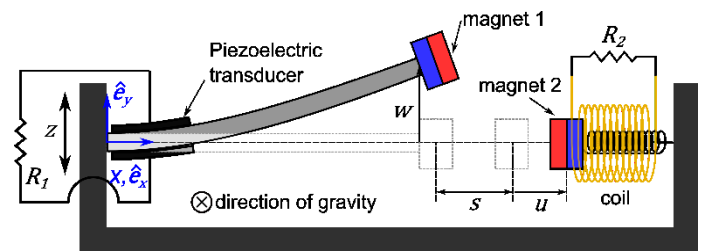


FIGURE 1. SCHEMATIC OF THE PROPOSED NONLINEAR ENERGY HARVESTER WITH ADAPTIVE POTENTIAL WELL.

these two magnets as shown in Fig. 2. Two piezoelectric transducers attached at the root of the cantilever beam and an electromagnetic coil fixed around the movable magnet are connected to electrical loads R_1 and R_2 , respectively, for harvesting vibration energy from the base excitation.

In this research, the governing equations of the proposed nonlinear energy harvester in Fig. 1 are derived by applying the extended Hamilton's principle and utilizing the assumed mode method for discretization. Considering a trial function that consists of the dominant dynamics of the fundamental mode $\Phi(x)$ [40], the transverse displacement of the cantilever beam relative to the base to which it is clamped is assumed as

$$w(x, t) = \Phi(x)q(t) \quad (1)$$

where $q(t)$ is the generalized displacement. The discretized governing equations of the proposed energy harvester can be expressed using [3], [8]

$$m_1 \ddot{q}(t) + c_1 \dot{q}(t) + k_1^b q(t) + F_w + \Theta_p V_p(t) = -m_t \ddot{z}(t) \quad (2a)$$

$$\Theta_p \dot{q}(t) - \frac{1}{2} C_p \dot{V}_p(t) = \frac{V_p(t)}{R_1} \quad (2b)$$

$$m_2 \ddot{u}(t) + c_2 \dot{u}(t) + k_2 u(t) + F_u + \Theta_c I_2(t) = 0 \quad (2c)$$

$$\Theta_c \dot{u}(t) - (R_L + R_2) I_2(t) = 0 \quad (2d)$$

where $\ddot{z}(t) = p \sin(\omega t)$ is the base acceleration with amplitude p and frequency ω ; m_1 , c_1 , and k_1^b are the effective mass, damping coefficient, and stiffness of the cantilever, respectively; m_t is the effective mass of the tip magnet attached to the cantilever; Θ_p and C_p respectively indicate the electromechanical coupling coefficient and the clamped capacitance of the piezoelectric transducer; V_p is the voltage across the piezoelectric transducer; $u(t)$, m_2 , c_2 , and k_2 are the displacement, mass, damping coefficient, and stiffness of the magnet oscillator, respectively; F_w and F_u are the magnetic forces applied to magnet 1 and 2, respectively; Θ_c and R_L are the electromagnetic coupling coefficient and internal resistance of the induction coil; and I_2 is the current induced by the magnet and coil. The overdot indicates a time derivative. Since the excitation frequency is relatively low, the coil inductance is ignored in this analysis [41].

In this research, the permanent magnets are modeled as point dipoles with magnetic moment vectors $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ for magnet 1 and 2, respectively. By the geometrical location of the magnets shown in Fig. 1, the distance vector $\mathbf{r}_{12}$ from the center of magnet 1 to magnet 2 is given by

$$\mathbf{r}_{12} = [s + u] \hat{\mathbf{e}}_x - \left[w(L) + \frac{1}{2} l_m \theta(L) \right] \hat{\mathbf{e}}_y \quad (3)$$

where s is the separation distance between the magnet dipoles, l_m is the length of magnet 2, $w(L)$ is the tip displacement of the beam, and $\theta(L)$ is the deflection angle at the beam tip. The magnetic field generated by magnet 1 on magnet 2 is obtained as [40].

$$\mathbf{B}_{12} = -\frac{\mu_0}{4\pi} \left(\frac{\boldsymbol{\mu}_1}{|\mathbf{r}_{12}|^3} - \frac{3\mathbf{r}_{12}(\boldsymbol{\mu}_1 \cdot \mathbf{r}_{12})}{|\mathbf{r}_{12}|^5} \right) \quad (4)$$

where $\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$ is the permeability in a vacuum. In this study, $s = 34 \text{ mm}$, $l_m = 12 \text{ mm}$, and $|\boldsymbol{\mu}_1| = |\boldsymbol{\mu}_2| = 1.215 \text{ Am}^2$. The magnetic potential energy U_m is described by

$$U_m(u, q) = -\mathbf{B}_{12} \cdot \boldsymbol{\mu}_2. \quad (5)$$

The magnetic forces F_u and F_w can be derived by differentiating Eq. (5) with respect to u and q , respectively,

$$F_u(u, q) = \frac{\partial U_m}{\partial u}, \quad F_w(u, q) = \frac{\partial U_m}{\partial q}. \quad (6a, b)$$

The output power harvested by the piezoelectric transducer and electromagnetic coil can be respectively obtained as

$$P_p = \frac{1}{T} \int_0^T \frac{V_p^2}{R_1} dt, \quad P_e = \frac{1}{T} \int_0^T \left(\frac{\Theta_c \dot{u}}{R_2 + R_L} \right)^2 R_2 dt \quad (7a, b)$$

where T is a sufficiently long time period compared to the base excitation period ($2\pi/\omega$). Thus, the total output power of the proposed energy harvester is

$$P = P_p + P_e. \quad (8)$$

Fig. 2(a) illustrates the total potential energy, a summation of the beam's elastic potential and the magnetic potential of the proposed energy harvester, as a function of the displacements of the cantilever tip $w(L)$ and movable magnet u . Relevant parameters used in the potential energy calculations are provided in Table 1. The blue colored trajectory in the three dimensions is the potential energy of the proposed platform obtained by quasi-statically changing the displacement $w(L)$, whereas the projection in the $w(L) - u$ plane shows the trajectory of the movable magnet. The black and red curves on the $w(L) - u$ potential energy plane show the potential energy of the conventional bistable energy harvester with fixed magnet (black) and the proposed harvester (red), respectively. Fig. 2(b) presents

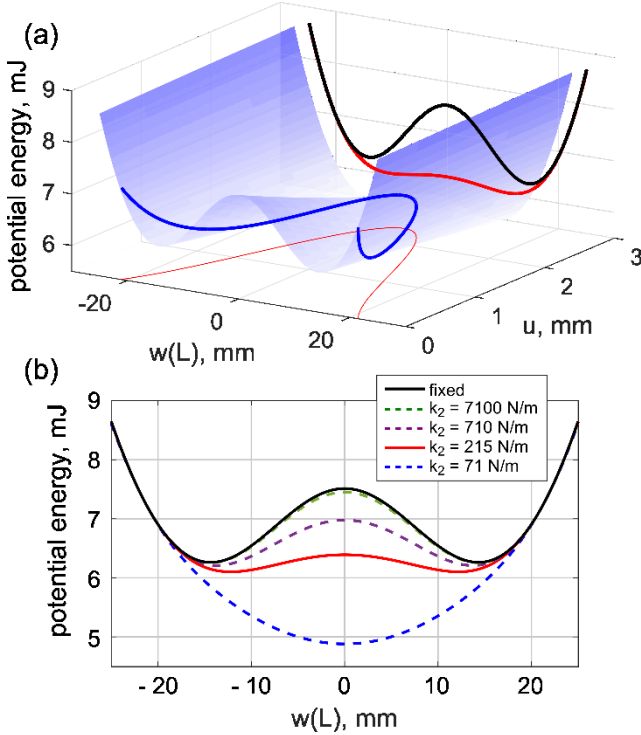


FIGURE 2. (a) STATIC DOUBLE-WELL POTENTIAL ENERGY FUNCTION DEPENDING ON THE DISPLACEMENTS OF THE CANTILEVER TIP($w(L)$) AND MOVABLE MAGNET(u). (b) THE POTENTIAL ENERGY OF THE PROPOSED HARVESTER WITH DIFFERENT STIFFNESS VALUES FOR THE AUXILIARY OSCILLATOR.

the potential energy of the proposed platform with different stiffness values of the auxiliary oscillator (the spring-loaded magnet oscillator). It is clearly observed that the potential energy depends on the stiffness of the auxiliary oscillator. In contrast to conventional nonlinear energy harvesters with a magnet fixed ($u = 0$) to the base [40], in the proposed harvester, the spring-loaded magnet oscillates in response to the cantilever oscillation. As the tip magnet (magnet 1) of the cantilever approaches the center position ($w(L) = 0$), the repulsive magnetic force compresses the spring and magnet 2 moves away from the tip magnet. As a result, the potential energy barrier level decreases, which facilitates the energetic interwell oscillations between the two stable equilibria. It is worth noting that while the potential energy of the auxiliary oscillator with relatively high stiffness ($k_2 = 7100$ N/m) exhibits negligible difference from that of the conventional harvester with fixed magnet, the bistability of the proposed platform could be lost with relatively soft, compliant axial support ($k_2 = 71$ N/m). Thus, the stiffness k_2 needs to be carefully designed to effectively lower the potential energy barrier of the bistable system.

Overall, the newly proposed energy harvester enables an adaptive potential energy well in the bistable system without active controls [31,32]. This facilitates large amplitude interwell oscillations of the cantilever beam for a lower base excitation level when compared to a conventional bistable energy harvester

with a fixed magnet. In addition, when the cantilever exhibits interwell oscillations, the magnet oscillator not only moves with large amplitudes but also at twice the frequency of the cantilever due to the configuration of the proposed harvester, which significantly enhances the power harvested by the electromagnetic coil. Moreover, the resonance and higher harmonics that can be observed from the magnet oscillator response can be strategically exploited to further enhance the dynamic behavior of the bistable cantilever and thus the power harvesting performance of the proposed architecture.

3 NUMERICAL INVESTIGATIONS

This section presents a numerical case study to illustrate the effectiveness of the proposed nonlinear vibration energy harvester. The case study is conducted by numerically solving the governing equation system shown in Eq. (2). Relevant parameters used in the model are presented in Table 1.

Figure 3 shows a comparison of the excitation conditions where the cantilever beam exhibits interwell oscillations for the conventional bistable energy harvester with a fixed magnet and that for the newly proposed hybrid harvester with magnet oscillator. Numerical simulations are conducted for 10 runs with random initial conditions encompassing the two stable equilibria of the cantilever beam for each base excitation condition with various frequencies and amplitudes. The horizontal axis indicates the base excitation frequency and the excitation

TABLE 1. SYSTEM PARAMETERS

Parameter	Value
Cantilever beam structure	
Effective Mass, m_1	13.6 g
Effective stiffness, k_1^b	102 N/m
Damping ratio, c_1	0.015 Ns/m
Electromechanical coupling coefficient, Θ_p	111 μ N/V
Capacitance of piezoelectric transducer, C_p	37.2 nF
Effective Mass of tip magnet, m_t	52 g
Load resistance, R_1	150 k Ω
Spring-loaded magnet oscillator	
Effective Mass, m_2	30 g
Effective stiffness, k_2	213 N/m
Damping ratio, c_2	0.051 Ns/m
Electromagnetic coupling coefficient, Θ_c	1.54 N/A
Load resistance, R_2	100 Ω
Internal resistance, R_L	50 Ω

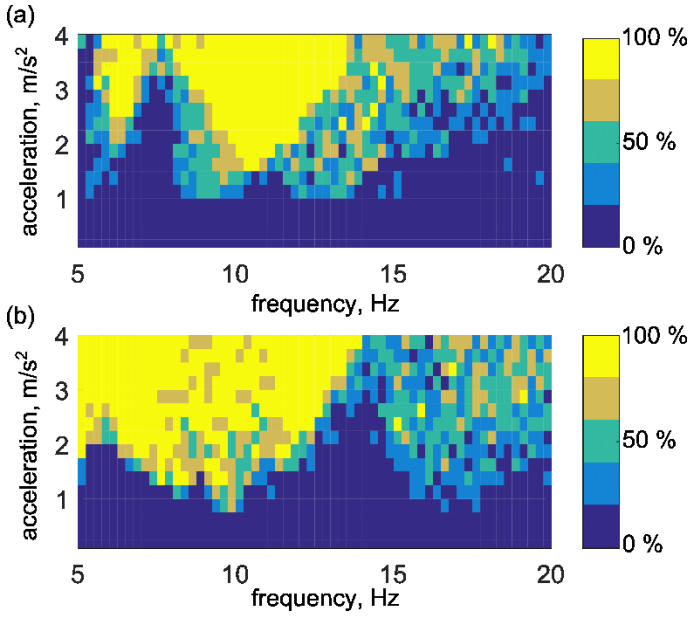


FIGURE 3. COMPARISON OF REGIONS WHERE THE CANTILEVER BEAM EXHIBITS INTERWELL RESPONSE FOR (A) CONVENTIONAL BISTABLE ENERGY HARVESTER AND THAT FOR (B) THE PROPOSED HYBRID HARVESTER WITH MAGNET OSCILLATOR. THE PERCENTAGE INDICATES THE PROPORTION WHERE INTERWELL OSCILLATIONS WERE ACHIEVED OUT OF THE MULTIPLE SIMULATIONS WITH RANDOM INITIAL CONDITIONS.

acceleration amplitude is presented along the vertical axis. The regions where the cantilever beam more frequently exhibited interwell oscillations are represented by increasing brightness. From Fig. 3, it is clearly observed that the proposed platform induces interwell oscillations for a significantly broader range of excitation frequencies and amplitudes. In particular, the proposed harvester facilitates interwell oscillations for relatively low excitation amplitudes and frequencies compared to the conventional counterpart.

Figure 4 compares the predicted average harvested power (Eq. (8)) by the conventional bistable energy harvester with fixed magnet and that by the proposed hybrid harvester as a function of base excitation frequency and acceleration amplitude. The increasing brightness of the contour represents a greater magnitude of harvested power. As can be observed from Fig. 4, the proposed energy harvester substantially enhances the energy harvesting performance for a broad range of excitation conditions. Figure 5 presents a collection of average power outputs for two excitation frequencies, which are respectively obtained by the conventional bistable harvester, the piezoelectric transducer, and electromagnetic coil of the new platform as a function of base excitation acceleration amplitude. When the base excitation frequency is 7.5 Hz, which is lower than the linear resonance frequency of the cantilever beam ($\sqrt{k_1/m_1}$), the

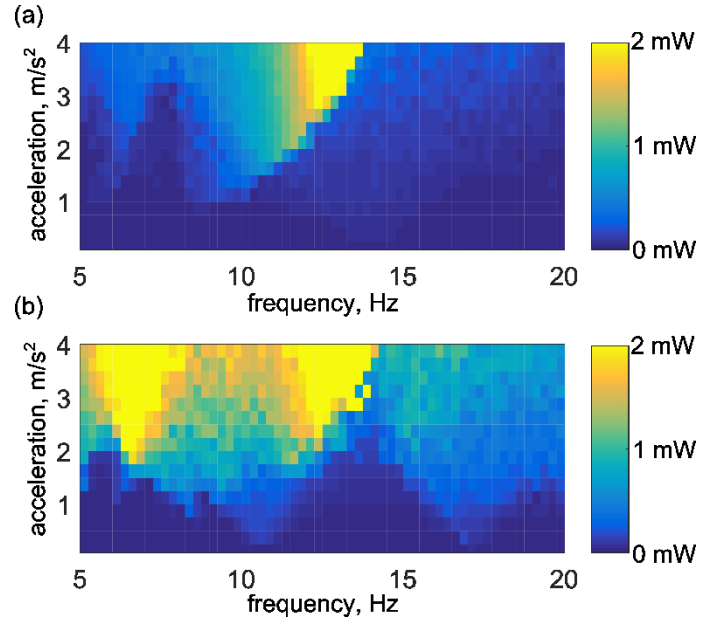


FIGURE 4. COMPARISON OF AVERAGE POWER HARVESTED BY (A) CONVENTIONAL BISTABLE ENERGY HARVESTER AND (B) THE PROPOSED HYBRID HARVESTER WITH MAGNET OSCILLATOR.

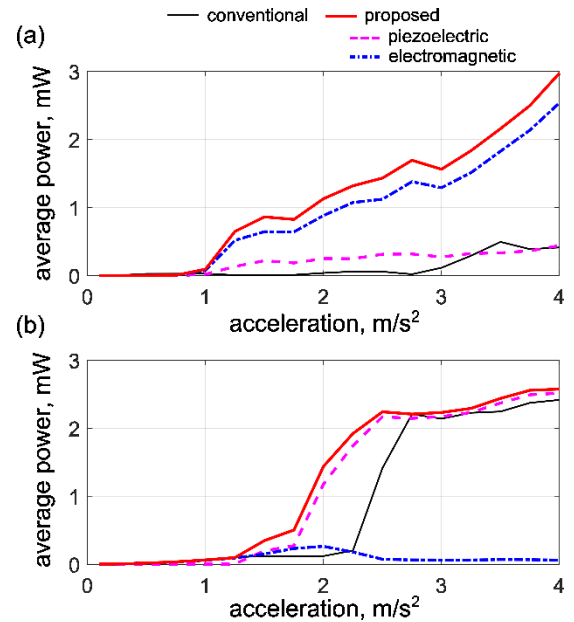


FIGURE 5. AVERAGE POWER HARVESTED BY CONVENTIONAL BISTABLE ENERGY HARVESTER (THIN BLACK), PIEZOELECTRIC TRANSDUCER (PINK DASH), ELECTROMAGNETIC COIL (BLUE DASH-DOT) AND THEIR SUM (THICK RED) OF THE PROPOSED HARVESTER. BASE EXCITATION FREQUENCIES ARE 7.5 HZ AND 12.5 HZ FOR (a) AND (b), RESPECTIVELY.

electromagnetic harvesting mechanism significantly contributes to the total harvested power compared to the power harvested from the piezoelectric transducer as shown in Fig. 5(a). On the other hand, when the excitation frequency is 12.5 Hz, which is near the resonance of the cantilever (Fig. 5(b)), the piezoelectric harvester is the primary contributor to the total harvester power.

For both excitation frequencies, it is shown that the proposed system provides significantly increased power output for lower excitation amplitudes compared to the conventional bistable harvester. Furthermore, it is evident that the hybrid energy harvesting mechanism exploiting the additional electromagnetic harvester substantially enhances the energy harvesting performance of the proposed platform.

One of the notable improvements of the proposed harvester over the conventional counterpart is that a significant amount of energy can be harvested in a low frequency range, from approximately 5 to 7 Hz in this case study (Fig. 4(b)). This improvement is mainly attributed to the additional magnetic oscillator introduced in the new harvester architecture, which can be clearly observed from Figure 6. Similar to Fig. 5, a collection of average harvested power is presented in Fig. 6 but as a function of base excitation frequency. For a low excitation level ($p=0.25 \text{ m/s}^2$) where the cantilever beam is likely to exhibit intrawell oscillations, the proposed harvesting platform behaves similarly to a two degrees-of-freedom linear oscillator. The proposed system exhibits two resonance peaks on both sides of the single resonance peak of the conventional system, which enhances the frequency bandwidth. For a high excitation level ($p=3.5 \text{ m/s}^2$) that mostly induces interwell cantilever oscillations, the resonance frequency of the cantilever beam of the proposed platform remains similar to that of the conventional system. On the other hand, an additional resonance peak is introduced at 6.5 Hz, which is approximately half of the magnet oscillator resonance frequency ($\sqrt{k_2/m_2}$). Due to the geometry of the proposed harvester, the spring-loaded magnet oscillates at twice the frequency of the cantilever when the cantilever exhibits interwell oscillation. Thus, when the proposed system is harmonically excited at 6.5 Hz and the cantilever exhibits interwell oscillations, the magnet oscillator is excited at its resonance, resulting in a significant amount of power harvesting enhancement.

4 SUMMARY AND CONCLUSION

This research proposes and investigates a novel hybrid vibration energy harvester that leverages bistability and additional axial magnet oscillator to improve the vibration energy harvesting performance. The newly developed platform is a bistable oscillator comprised of a cantilever beam with a magnet at the free end and a spring-loaded magnet oscillator axially aligned with the cantilever, which passively realizes an adaptive bistable potential. When compared to the conventional bistable harvester with fixed magnet, it is observed that the proposed platform facilitates the energetic interwell oscillations of the cantilever for a wider range of excitation amplitudes and frequencies. In addition to the energy harvested by the

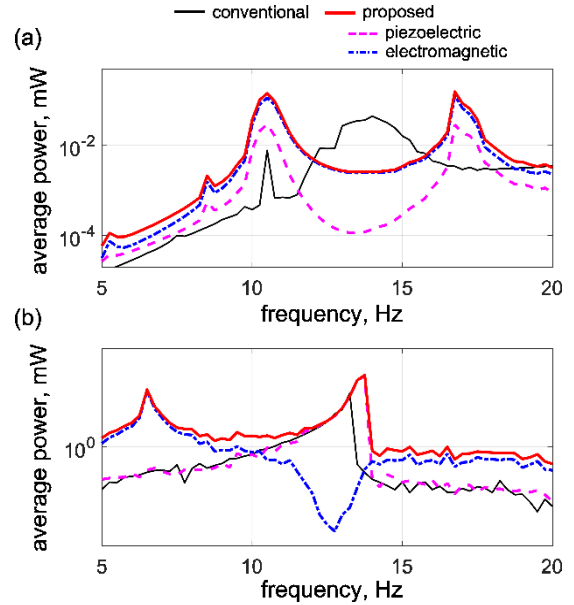


FIGURE 6. AVERAGE POWER HARVESTED BY CONVENTIONAL BISTABLE ENERGY HARVESTER (THIN BLACK), PIEZOELECTRIC TRANSDUCER (PINK DASH), ELECTROMAGNETIC COIL (BLUE DASH-DOT) AND THEIR SUM (THICK RED) OF THE PROPOSED HARVESTER. BASE EXCITATION ACCELERATION AMPLITUDES ARE 0.25 m/s^2 AND 3.5 m/s^2 FOR (a) AND (b), RESPECTIVELY.

piezoelectric transducer attached to the cantilever, the electromagnetic coil directly captures vibration energy from the magnet oscillator. As a result, the newly proposed architecture is found to provide a substantial increase in energy harvesting, especially for relatively low excitation amplitudes and frequencies, by lowering the double-well potential energy barrier and leveraging the dynamic characteristics of the magnetic oscillator. Numerical results illustrate that the energy harvesting performance is significantly enhanced by utilizing the proposed approach. Overall, the results of this study show the promising potential of implementing the proposed coupled bistable-linear architecture with hybrid energy harvesting mechanism for enhancing the performance under broadband environmental vibrations commonly encountered in practice. In light of previously investigated magnetically-coupled systems [35], [36], various potential physical implementations of this system will be considered in future work for experimental investigations. Care will need to be taken to consider and minimize damping related to constraining the magnet oscillator motion to the axis of the inductor coil.

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