Fully Distributed Resilience for Adaptive Exponential Synchronization of Heterogeneous Multiagent Systems Against Actuator Faults

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***Abstract*—Cooperative control of multiagent systems (MAS) on communication networks has received a great deal of attention, mostly for the case of homogeneous agents, which all have the same dynamics. An advantage of cooperative synchronization mechanisms is their local distributed nature, which makes them scalable to large networks. However, most existing design mechanisms require some global information, such as the leader's dynamics or global graph information, so that the control protocols are technically not fully distributed. Moreover, the distributed nature of the control protocols makes them susceptible to faults or uncertainties. In this paper, we study heterogeneous MAS, where all agents may have different dynamics. We provide adaptive resilience mechanisms for rejecting actuator faults, and guarantee exponential convergence of synchronization errors, whereas most existing results on actuator faults guarantee only boundedness of errors. Finally, we provide algorithms that are fully distributed, requiring no knowledge of either the leader's dynamics or of graph properties.**

***Index Terms*—Actuator faults, adaptive exponential control, fully distributed, heterogeneous multiagent systems (MAS), output synchronization.**

I. Introduction

The cooperative control of multiagent systems (MAS) has been rapidly developed (see [1]–[4] for surveys), and now is undergoing a period of major change from homogeneous to heterogeneous MAS, which allows the agents' dynamics and even their state dimensions to be nonidentical [5]–[9]. What remains unchanged is that networked MAS have compelling advantages due to the distributed nature of control protocols, which only depend on neighbors in a communication network. However, networked MAS are susceptible to faults that can induce interruptions, and can even result in synchronization degradation or instability.

To enhance the resilience, notable research has been triggered by introducing detection and isolation mechanism into the control algorithms [10]–[14]. As for the control of a system with actuator faults, adaptive-control based resilient approaches are reported, i.e., [15]–[17]. Most recently, actuator faults are addressed with the combination of event-triggered input control [18], and input quantization control [19]. In addition, resilience is provided for MAS to address unknown control directions, which can be considered as parts of actuator faults, (see [20]–[23]). Despite many advances, these resilient protocols generally only result in the ultimate boundedness, cannot guarantee the transient performance or convergence rate after faults are injected into the synchronization control of MAS.

To achieve the synchronization of MAS, much research has been conducted, e.g., [24]–[29]. However, most of these protocols require the global knowledge of the graph topology, especially about the spectrum of the network. To remove such global information, a remedy has been proposed by dynamically updating the coupling gain, e.g., [30]–[32]. However, these distributed protocols are applicable to only homogeneous MAS, but not to heterogeneous MAS. In recent years, many interesting works have been reported based on cooperative output regulation, e.g., [6], [7], [21], [33]–[46]. Note that all of these protocols require the global information to complete the control design, including one or both of the graph topology and the leader dynamics for solving the output regulator equations. This means that the existing heterogeneous MAS protocols do not work in a distributed manner.

Inspired by the above literature, this paper aims to provide algorithms for the fully distributed resilience of heterogeneous MAS against actuator faults. By using adaptive control techniques, we achieve exponential output synchronization under a directed graph. The contributions of this paper are stated as follows.

1) We give a complete solution to the fully distributed control design problem for output synchronization of heterogeneous MAS. Our solution has a feature that it is independent of any global information of the communication graph or the leader states/dynamics, and only relies on the local agent dynamics and the relative neighborhood information.
2) We enhance the resilience of heterogeneous MAS exponential synchronization by allowing actuators under unknown faults.

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II. Problem Formulation and Preliminaries

In this section, we formulate the exponential synchronization control problems of MAS under unknown actuator faults.

Notations: $I_n \in \mathbb{R}^{n\times n}$ denotes an identity matrix. $\mathbf{1}_n \in \mathbb{R}^n$ denotes a vector with all the components being one. $\otimes$ denotes the Kronecker product. $[x_{ij}]$ denotes a matrix with x_{ij} being an entry in the ith row and the jth column. $\text{diag}\{x_i\}$ is a diagonal matrix with a vector x_i on the main diagonal. $X > 0$ ($X \geq 0$) denotes that a matrix X is positive definite (semipositive definite). Likewise, $X < 0$ ($X \leq 0$) is negative definite (seminegative definite). For $\lambda_i \in \mathbb{C}$ and $X \in \mathbb{R}^{n\times n}$, let λ_i denote an eigenvalue of X for $i = 1, 2, \ldots, n$. $\sigma_{\min}(X)$ and $\sigma_{\max}(X)$ denote minimum and maximum singular values of X, respectively. $\|\cdot\|$ is the Euclidean norm of a vector and $\|\cdot\|_F$ is the Frobenius norm of a matrix.

A. Problem Formulation

Consider a group of N agents with system dynamics as follows:

$$\dot{x}_i = A_i x_i + B_i \bar{u}_i, \quad y_i = C_i x_i \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$ is the system state, $\bar{u}_i \in \mathbb{R}^{m_i}$ is the system input under faults to be detailed later, $y_i \in \mathbb{R}^p$ is the measured output with $i = 1, 2, \ldots, N$, and A_i, B_i, and C_i are system dynamics with appropriate dimensions. In addition, the dynamics of the leader agent, labeled 0, is given by the following:

$$\dot{\zeta}_0 = S\zeta_0, \quad y_0 = R\zeta_0 \tag{2}$$

where $S \in \mathbb{R}^{q\times q}$, $R \in \mathbb{R}^{p\times q}$ are constant system dynamics, $\zeta_0 \in \mathbb{R}^q$ is the system state, and $y_0 \in \mathbb{R}^p$ is the reference output. The leader agent (2) can be considered as an exosystem or a command generator, which generates a desired trajectory to be followed by all N agents (1). Note that the agents given in (1) and (2) are connected by a distributed communication graph. This implies that both the leader state and its dynamics (2) are observed only by a small group of N agents (1).

In this paper, the system input $\bar{u}_i$, for $i = 1, 2, \ldots, N$, are under unknown faults, namely under actuator faults. We model actuator faults as follows:

$$\bar{u}_i = \mu_i(t) u_i + \delta_i^a(t) \tag{3}$$

where $\mu_i(t)$ denotes a nonzero bounded scaling coefficient caused by actuator faults; $\delta_i^a(t)$ denotes a bounded fault disturbance vector caused in actuator channels. Only the corrupted control $\bar{u}_i$ enters the dynamics (1).

Remark 1: The fault model in (3) is from [15]–[18], [23]. •

Denote the global states as $y = [y_1^T, y_2^T, \ldots, y_N^T]^T \in \mathbb{R}^{pN}$ and $\underline{y}_0 = \mathbf{1}_N \otimes y_0 \in \mathbb{R}^{pN}$. We define the global output synchronization error as follows:

$$\epsilon_y = y - \underline{y}_0 \in \mathbb{R}^{pN} \tag{4}$$

with $\epsilon_{yi} = y_i - y_0$ for local agent.

We define actuator fault problems for the output synchronization of heterogeneous MAS.

Exponential Synchronization Problem: Under the actuator faults (3), this synchronization problem is to design fully distributed resilient protocols u_i for heterogeneous MAS (1) such that the output synchronization error ϵ_y exponentially converges to the zero, i.e., $||\epsilon_y|| \leq V_\epsilon \exp(-\alpha_\epsilon t)$, where V_ϵ and α_ϵ are certain positive constants.

B. Preliminaries

In what follows, we give some preliminaries related to graph theory.

Consider a class of directed graphs described as $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ denotes a set of nodes, $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ denotes a set of edges, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ denotes an adjacency matrix. The flow of information in the graph $\mathcal{G}$ is denoted by a weight a_{ij} and an edge (v_j, v_i) satisfying $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$, otherwise $a_{ij} = 0$. Here, we assume that no repeated edges or no self-loops are allowed in $\mathcal{G}$. We define $\mathcal{N}_i = \{j | (v_j, v_i) \in \mathcal{E}\}$ as a set of neighbors of node i, and $H = \text{diag}\{h_i\} \in \mathbb{R}^{N\times N}$ be an in-degree matrix with $h_i = \sum_{j\in\mathcal{N}_i} a_{ij}$. Hence, the Laplacian matrix is given as $L = H - \mathcal{A}$. A direct path from node i to node j is captured by a sequence of successive edges satisfying $\{(v_i, v_k), (v_k, v_l), \ldots, (v_m, v_j)\}$. A graph is said to have a spanning tree, if there exists a directed path from a node to every other nodes. If the leader node is a neighbor of node i, then an edge (v_0, v_i) exists with a weighting gain g_i being positive. Considering N nodes in the graph, we define the positive gain g_i in a global form as the pinning matrix $G = \text{diag}\{g_i\} \in \mathbb{R}^{N\times N}$. Throughout this paper, the following assumption of the graph topology holds.

Assumption 1: The directed graph $\mathcal{G}$ contains a spanning tree with the leader as its root.

Note that this is a standard assumption for the distributed control of MAS and will be used in the main result.

To solve the above cooperative output regulation problem, the following assumptions are needed.

Assumption 2: The pairs (A_i, B_i), $i = 1, 2, \ldots, N$ are stabilizable.

Assumption 3: For all $\lambda \in \Omega(S)$, where $\Omega(S)$ denotes the spectrum of S, $\text{rank}(\begin{bmatrix} A_i - \lambda I, & B_i \\ C_i, & 0 \end{bmatrix}) = n_i + p$.

Assumption 4: The matrix S has no eigenvalues with negative real parts.

These assumptions are standard in the classic output regulation problem [47]. *Assumption 4* is meant to rule out the trivial case where the leader's dynamics are stable [47]. It is important to allow also unstable leader's dynamics. Unstable leader dynamics can be found in flocking networks [48], [49], in which flock centering, collision avoidance, and velocity matching are required.

Lemma 1 ([50], Bellman–Gronwall Lemma): Supposing that there are some constants $\varrho \geq 0$ and $t_b \geq t_a$, a nonnegative piecewise continuous function $\alpha : [t_a, t_b] \to \mathbb{R}$, and a continuous function $w : [t_a, t_b] \to \mathbb{R}$ satisfying $w(t) \leq \varrho + \int_{t_a}^{t} \alpha(\tau) w(\tau) d\tau, t \in [t_a, t_b]$, then we have $w(t) \leq \varrho \exp(\int_{t_a}^{t} \alpha(\tau) d\tau), t \in [t_a, t_b]$. □

III. Fully Distributed Design for Heterogeneous MAS With Actuator Faults

In this section, we give an algorithm to guarantee the exponential output synchronization of heterogeneous MAS with actuator faults. To accomplish this objective, each agent is endowed in this section with a distributed leader state observer, termed as $\zeta_i \in \mathbb{R}^q$. We define the distributed leader state error as follows:

$$\xi_i = \sum_{j=1}^{N} a_{ij}(\zeta_i - \zeta_j) + g_i(\zeta_i - \zeta_0) \tag{5}$$

and its global form is rewritten as $\xi = [\xi_1^T, \ldots, \xi_N^T]^T$, where $g_i \geq 0$ is a pinning gain with $g_i > 0$ only if agent i can get information directly from the leader node. Then, one has the following:

$$\xi = (L_G \otimes I_q)\tilde{\zeta} \tag{6}$$

where $L_G \equiv L + G$ is related to the graph topology and $\tilde{\zeta} \equiv [\tilde{\zeta}_1^T, \ldots, \tilde{\zeta}_N^T] \equiv [\zeta_1^T - \zeta_0^T, \ldots, \zeta_N^T - \zeta_0^T]^T$. From (6) and *Assumption 1*, one has the following:

$$||\xi_i|| \leq \sigma_{\max}(L_G \otimes I_q)\|\tilde{\zeta}\|. \tag{7}$$

From *Assumption 3*, one obtains that the following output regulator equations:

$$A_i\Pi_i + B_i\Gamma_i = \Pi_i S \tag{8a}$$

$$C_i\Pi_i = R \tag{8b}$$

have solution matrices $\Pi_i \in \mathbb{R}^{n_i\times m}$ and $\Gamma_i \in \mathbb{R}^{m_i\times m}$ for $i = 1, 2, \ldots, N$ [47].

In what follows, three results, including leader state/dynamics observers, output regulator equation solvers, and adaptive exponential controls, are bought together in Section III-C to solve *exponential synchronization problem*.

A. Fully Distributed Observers to Estimate Leader States and Dynamics

In this section, we design distributed leader states and dynamics observers that are independent of the global graph topology and the global leader information.

To facilitate the analysis, let the leader dynamics in (2) be rewritten as follows:

$$\Upsilon = \begin{bmatrix} S \\ R \end{bmatrix} \in \mathbb{R}^{(p+q)\times q} \tag{9}$$

and its estimations be splited in two parts as follows:

$$\hat{\Upsilon}_{0i} = \begin{bmatrix} \hat{S}_{0i} \\ \hat{R}_{0i} \end{bmatrix} \in \mathbb{R}^{(p+q)\times q} \tag{10}$$

$$\hat{\Upsilon}_{i} = \begin{bmatrix} \hat{S}_{i} \\ \hat{R}_{i} \end{bmatrix} \in \mathbb{R}^{(p+q)\times q} \tag{11}$$

where $\hat{\Upsilon}_{0i}$ and $\hat{\Upsilon}_i$ will be updated by (14) and (15) and converge to Υ at different rates.

Lemma 2 ([3], Theorem 4.25): Under *Assumption 1*, there exists a positive-definite diagonal matrix Q such that $QL_G + L_G^T Q$ is positive-definite. □

Now, we give fully distributed state and dynamics observers for heterogeneous MAS, as summarized in the following theorem. Different from the existing distributed controls where homogeneous MAS are considered in [30] or the leader dynamics are globally known to each follower [46], our leader observers have a feature that the leader information including leader states and dynamics are exchanged using distributed communication networks. More importantly, our observers have a unique structure that contains two splited communication networks as shown in (10) and (11), and this helps us inherit the property in *Assumption 4* that allows the leader dynamical state be unbounded.

Theorem 1: Suppose that *Assumption 1* holds true. The distributed leader state observer with the estimated leader dynamics is chosen as follows:

$$\dot{\zeta}_i = \hat{S}_i\zeta_i - (c_i + \dot{c}_i)\xi_i \tag{12}$$

where $\hat{S}_i$ is the estimation of the leader dynamics S and the design gain c_i is chosen as follows:

$$\dot{c}_i = \xi_i^T\xi_i \tag{13}$$

with its initial value $c_i(0) \geq 1$. Let the dynamic estimates $\hat{\Upsilon}_{0i}$ and $\hat{\Upsilon}_i$ in (10) and (11) be updated as follows:

$$\dot{\hat{\Upsilon}}_{0i} = \sum_{j=1}^{N} a_{ij}\left(\hat{\Upsilon}_{0j} - \hat{\Upsilon}_{0i}\right) + g_i\left(\Upsilon - \hat{\Upsilon}_{0i}\right) \tag{14}$$

$$\dot{\hat{\Upsilon}}_{i} = ||\hat{\Upsilon}_{0i}||_F\left(\hat{\Upsilon}_{0i} - \hat{\Upsilon}_i\right) + \sum_{j=1}^{N} a_{ij}\left(\hat{\Upsilon}_{j} - \hat{\Upsilon}_{i}\right) + g_i\left(\Upsilon - \hat{\Upsilon}_{i}\right). \tag{15}$$

Then, all the signals in the distributed leader state and dynamics observers (12)–(15) are globally bounded. Moreover, the estimated leader states and dynamics, ζ_i, $\hat{\Upsilon}_{0i}$, and $\hat{\Upsilon}$, exponentially converge to the actual leader states and dynamics, ζ_0 and Υ_i, for $i = 1, \ldots, N$. □

Proof: The results can be obtained by extending [30] and [38]. We omit the proof due to the limited space. ■

In contrast to [38], the key idea is that we propose adaptive mechanisms in (14) and (15) for estimating the leader dynamics so that the convergence rate of the leader dynamics estimation error $\tilde{\Upsilon}_i = \Upsilon - \hat{\Upsilon}_i$ satisfies *Lemma 3*.

In what follows, we define partial parts of $\tilde{\Upsilon}_i$ as $\tilde{R}_i \equiv R - \hat{R}_i$, where R is given by (9) and $\hat{R}_i$ is given by (11). We detail more features of the proposed distributed observer in *Lemma 3*.

Lemma 3: The adaptive distributed leader dynamics observers in (15) ensure $||\tilde{\Upsilon}_i||_F\,||\zeta_0||$ exponentially converges to zero. □

Proof: The results can be derived from *Theorem 1*. We omit the proof due to the limited space. ■

Note that [38] does not guarantee the result in *Lemma 3* by only using the distributed information. The result in *Lemma 3* reveals that, given certain time t, there exist constants $\beta_{\Upsilon_i\zeta_0}$ and $\alpha_{\Upsilon_i\zeta_0}$ so that one has the following:

$$\left\|\tilde{\Upsilon}_i\right\|_F ||\zeta_0|| \leq \beta_{\Upsilon_i\zeta_0}\exp(-\alpha_{\Upsilon_i\zeta_0}t) \tag{16}$$

where ζ_0 is the leader state generated by (2). Thus, (16) implies that the convergence rate of the estimation error $\tilde{\Upsilon}_i$ is forced to be faster than the change of the leader states due to the use of our updating laws (14) and (15). This unique feature plays a key role in handling the synchronization of heterogeneous MAS in Section III-C.

B. Fully Distributed Output Regulator Equation Solvers

Since the leader dynamics are only distributively available to the followers, it requires output regulator equations in (8) to be solved based on distributed leader dynamics. In this section, we propose fully distributed solvers for output regulator equations. Note that the output regulator equation solvers in [38] cannot be used here, because of the following reasons.

1) That approach strongly depends on the global graph topology.

2) No system disturbance is allowed in the synchronization control of [38], which means that no resilience is provided and, thus, actuator faults cannot be handled adaptively.

With the estimated leader dynamics $\hat{\Upsilon}_i$ in (15), the output regulator equations in (8) are rewritten as follows:

$$A_i\hat{\Pi}_{0i} + B_i\hat{\Gamma}_{0i} = \hat{\Pi}_{0i}\hat{S}_i \tag{17a}$$

$$C_i\hat{\Pi}_{0i} = \hat{R}_i \tag{17b}$$

where $\hat{\Pi}_{0i}$ and $\hat{\Gamma}_{0i}$ denote the solutions matrices driven by the estimated dynamics $\hat{S}_i$ and $\hat{R}_i$. Rewriting (17) yields the following:

$$A_{1i}\hat{Y}_{0i}I_q - A_{2i}\hat{Y}_{0i}\hat{S}_i = \hat{R}_i^* \tag{18}$$

where $A_{1i} = \left[\begin{smallmatrix} A_i, & B_i \\ C_i, & 0 \end{smallmatrix}\right]$, $A_{2i} = \left[\begin{smallmatrix} I_{n_i}, & 0 \\ 0, & 0 \end{smallmatrix}\right]$, $\hat{Y}_{0i} = [\hat{\Pi}_{0i}^T \quad \hat{\Gamma}_{0i}^T]^T$, and $\hat{R}_i^* = [0 \quad \hat{R}_i^T]^T$. Then, a standard form of linear equations in (18) can be reformulated as follows:

$$\hat{\Phi}_i\hat{\Delta}_{0i} = \hat{\Re}_i \tag{19}$$

where $\hat{\Phi}_i = (I_q \otimes A_{1i} - \hat{S}_i^T \otimes A_{2i})$, $\hat{\Delta}_{0i} = \text{vec}(\hat{Y}_{0i})$, and $\hat{\Re}_i = \text{vec}(\hat{R}_i^*)$. Accordingly, we define the solutions of the output regulator equations in (8) as $\Delta_i = \text{vec}(Y_i) = \text{vec}([\Pi_i^T \quad \Gamma_i^T]^T)$, a vector $\Re_i = \text{vec}(R_i^*) = \text{vec}([0 \quad R_i^T]^T)$, and a matrix $\Phi_i = (I_q \otimes A_{1i} - S^T \otimes A_{2i})$. Given a unique solution Δ_i as [46] does, Φ_i is nonsingular and $\Phi_i^T \Phi_i$ is positive-definite. Moreover, we define another two estimated solutions to the output regulator equations in (8), and label them as follows:

$$\hat{\Delta}_{1i} \equiv \text{vec}(\hat{Y}_{1i}) \quad \text{and} \quad \hat{\Delta}_i \equiv \text{vec}(\hat{Y}_i) \tag{20}$$

with $\hat{Y}_{1i} = [\hat{\Pi}_{1i}^T \quad \hat{\Gamma}_{1i}^T]^T$ and $\hat{Y}_i = [\hat{\Pi}_i^T \quad \hat{\Gamma}_i^T]^T$. We define errors of the estimated solutions to output regulator equations in (8) as $\tilde{\Delta}_{ji} \equiv \text{vec}(Y_i) - \text{vec}(\hat{Y}_{ji}) \equiv \Delta - \hat{\Delta}_{ji}$ for $j = 0, 1$, and $\tilde{\Delta}_i \equiv \text{vec}(Y_i) - \text{vec}(\hat{Y}_i) \equiv \Delta - \hat{\Delta}_i$. The estimated solutions to the output regulator equations satisfy the convergence property as shown in the following theorem.

Theorem 2: Suppose that *Assumptions 1*, *3*, and *4* hold true. If the estimated solutions to the output regulator equations in (8), including $\hat{\Delta}_{ji}$ for $j = 0, 1$, and $\hat{\Delta}_i$, are adaptively solved as follows:

$$\dot{\hat{\Delta}}_{0i} = -\hat{\Phi}_i^T \left(\hat{\Phi}_i \hat{\Delta}_{0i} - \hat{\Re}_i\right) \tag{21}$$

$$\dot{\hat{\Delta}}_{1i} = ||\hat{\Upsilon}_i||_F \left(\hat{\Delta}_{0i} - \hat{\Delta}_{1i}\right) - \hat{\Phi}_i^T \left(\hat{\Phi}_i \hat{\Delta}_{1i} - \hat{\Re}_i\right) \tag{22}$$

$$\dot{\hat{\Delta}}_i = ||\hat{\Upsilon}_i||_F \left(\hat{\Delta}_{1i} - \hat{\Delta}_i\right) - \hat{\Phi}_i^T \left(\hat{\Phi}_i \hat{\Delta}_i - \hat{\Re}_i\right) \tag{23}$$

where $\hat{\Upsilon}_i$ is distributively solved in (14) and is used to calculate $\hat{\Phi}_i$ and $\hat{\Re}_i$ in (19), then, $\tilde{\Delta}_{ji}$ for $j = 0, 1$, and $\tilde{\Delta}_i(t)$ exponentially converge to zero. □

Proof: The results can be obtained by extending [38]. We omit the proof due to the limited space. ■

The key idea is that we propose adaptive mechanisms in (21)–(23) for estimating the solution to the output regulator equations so that *Lemma 4* holds true.

Lemma 4: The adaptive distributed leader dynamics observers in (15) ensure $||\tilde{\Delta}_i||||\zeta_0||$ and $\dot{\hat{\Pi}}_i \zeta_0$ exponentially converge to zero. □

Proof: The results can be derived from *Theorem 2*. We omit the proof due to the limited space. ■

Note that *Lemma 4* is a unique feature obtained by the use of the updating law (23). From *Lemma 3*, an inequality that is similar to (16) can be found. Control design in [38] does not guarantee *Lemma 4* by only using the distributed information.

C. Fully Distributed Resilience for Exponential Synchronization of MAS With Actuator Faults

From *Theorems 1 and 2*, we can build a fully distributed control scheme to achieve the synchronization of heterogeneous MAS based on the distributed leader states/dynamics. Moreover, the leader dynamics Matrix S is allowed to have positive real eigenvalues, and thus covers a case that the leader states are even unbounded. Note that such a fully distributed property is realized under a situation that the proposed observers and solvers (see *Theorems 1 and 2*) exponentially converge to the desired trajectories, which establishes the foundation for the exponential synchronization of heterogeneous MAS. In what follows, we show how our distributed observers and solvers combined with adaptive control to achieve the synchronization resilience of heterogeneous MAS against unknown faults on actuators (3).

Before presenting the proposed control scheme, we define the estimated state error as follows:

$$z_i = x_i - \hat{\Pi}_i \zeta_i \tag{24}$$

and we define its statelike error as follows:

$$\bar{z}_i = B_i^T P_{ci} z_i \tag{25}$$

where P_{ci} is a positive-definite matrix to be discussed in detail later. Let $\bar{z}_{ij}$ be the jth element of vector $\bar{z}_i$, and let $\text{diag}(\frac{\bar{z}_{ij}}{\sqrt{\bar{z}_{ij}^2 + \eta^2}})$ denote a diagonal matrix with main diagonal being $\frac{\bar{z}_{ij}}{\sqrt{\bar{z}_{ij}^2 + \eta^2}}$, where $\eta = \exp(-\alpha_\eta t)$ with α_η being a positive constant. The notation $\hat{d}_i$ denotes the estimate of $d_i = \sup_{t \geq 0} |\delta_i^a(t)|$ and its error $\tilde{d}_i = d_i - \hat{d}_i$. The notation $N_i(\chi_i)$ denotes a diagonal matrix with the main diagonal being Nussbaum functions. Typically, many functions belong to the definition of Nussbaum function, as shown in [20]–[23]. Throughout this paper, we choose $N_i(\chi_i) = \exp(\chi_i^2) \sin(\chi_i)$. We design $d_{\max} = \max\{||d_i||\}$ for $i = 1, \ldots, N$ be a bounded constant, φ_i be a positive constant, and d_{ij} be the jth element of vector d_i.

Now, we present the exponential synchronization result for heterogeneous MAS with unknown actuator faults.

Theorem 3: Consider heterogeneous MAS consisting of N followers (1) and one leader (2) with faults (3) on actuators. Under *Assumptions 1–4*, let the adaptive control scheme consist of the fully distributed leader state/dynamics observers in (12)–(15), output regulator equation solvers in (21)–(23), and local control protocols as follows:

$$u_i = N_i(\chi_i) \bar{\tau}_i \tag{26}$$

$$\bar{\tau}_i = \hat{\Gamma}_i \zeta_i + K_i z_i - \text{diag}\left(\frac{\bar{z}_{ij}}{\sqrt{\bar{z}_{ij}^2 + \eta^2}}\right) \hat{d}_i \tag{27}$$

where the design controller gain K_i is chosen as follows:

$$K_i = -R_i^{-1} B_i^T P_i \tag{28}$$

where P_i is a solution of the following control algebraic Riccati equation:

$$A_i^T P_i + P_i A_i + Q_i - P_i B_i R_i^{-1} B_i^T P_i = 0. \tag{29}$$

Let the adaptive laws for updating parameters $\hat{d}_i$ and χ_i for $i = 1, 2, \ldots, N$ in (26) and (27) as follows:

$$\dot{\hat{d}}_i = \text{diag}\left(\frac{\bar{z}_{ij}}{\sqrt{\bar{z}_{ij}^2 + \eta^2}}\right) \bar{z}_i \tag{30}$$

$$\dot{\chi}_i = -\varphi_i \bar{\tau}_i^T \bar{z}_i. \tag{31}$$

Then, *exponential synchronization problem* is solved. □

Proof: From (28) and (29), it is clear that $A_{ci} = A_i + B_i K_i$ is a Hurwitz matrix. Thus, there exists a positive-definite matrix Q_{ci} such that the following hold true:

$$A_{ci}^T P_{ci} + P_{ci} A_{ci} = -Q_{ci} \tag{32}$$

where P_{ci} is a positive-definite matrix. Taking the derivative of (24) yields the following:

$$\begin{aligned}\dot{z}_i &= A_{ci} z_i + \Pi_i \tilde{S}_i \zeta_i - A_i \tilde{\Pi}_i \zeta_i - B_i \tilde{\Gamma}_i \zeta_i + \tilde{\Gamma}_i \hat{S}_i \zeta_i + B_i \delta_i^a(t) \\ &\quad - B_i \mathrm{diag}\left(\frac{\bar{z}_{ij}}{\sqrt{\bar{z}_{ij}^2 + \eta^2}}\right) \hat{d}_i + B_i(\mu_i(t) N_i(\chi_i) - 1)\bar{\tau}_i \\ &\quad - \dot{\hat{\Pi}}_i \zeta_i + \hat{\Pi}_i (c_i + \dot{c}_i)\xi_i \end{aligned} \tag{33}$$

where (12), (26), and (32) are employed. Consider the following Lyapunov function candidate:

$$V_i = z_i^T P_{ci} z_i + \tilde{d}_i^T \tilde{d}_i \tag{34}$$

and its time derivative is given as follows:

$$\begin{aligned}\dot{V}_i &= 2 z_i^T P_{ci} \dot{z}_i - 2\tilde{d}_i^T \dot{\hat{d}}_i \\ &\le -z_i^T \sigma_{\min}(Q_{ci}) z_i + 2|\bar{z}_i|^T d_i - 2\bar{z}_i^T \mathrm{diag}\left(\frac{\bar{z}_{ij}}{\sqrt{\bar{z}_{ij}^2 + \eta^2}}\right) d_i \\ &\quad + 2\bar{z}_i^T(\mu_i(t)N_i(\chi_i) - 1)\bar{\tau}_i + 2 z_i^T P_{ci}\Big(\Pi_i \tilde{S}_i \zeta_i - A_i \tilde{\Pi}_i \zeta_i \\ &\quad - B_i \tilde{\Gamma}_i \zeta_i + \tilde{\Gamma}_i \hat{S}_i \zeta_i\Big) - 2 z_i^T P_{ci} \dot{\hat{\Pi}}_i \zeta_0 \\ &\quad - 2 z_i^T P_{ci} \dot{\hat{\Pi}}_i \tilde{\zeta}_i - 2 z_i^T P_{ci} \hat{\Pi}_i (c_i + \xi_i^T \xi_i)\xi_i \\ &\le -z_i^T \sigma_{\min}(Q_{ci}) z_i + 2\bar{z}_i^T(\mu_i(t)N_i(\chi_i) - 1)\bar{\tau}_i \\ &\quad + 2\eta d_{\max} + 2 z_i^T P_{ci}\Big(\Pi_i \tilde{S}_i - A_i \tilde{\Pi}_i - B_i \tilde{\Gamma}_i + \tilde{\Gamma}_i \hat{S}_i\Big)\zeta_0 \\ &\quad + 2 z_i^T P_{ci}\Big(\Pi_i \tilde{S}_i - A_i \tilde{\Pi}_i - B_i \tilde{\Gamma}_i + \tilde{\Gamma}_i \hat{S}_i\Big)\tilde{\zeta}_i \\ &\quad + 2||z_i||||P_{ci}||_F \left\|\dot{\hat{\Pi}}_i \zeta_0\right\| + 2||z_i||||P_{ci}||_F \left\|\dot{\hat{\Pi}}_i\right\|_F \|\tilde{\zeta}_i\| \\ &\quad + 2\left(c_i + \xi_i^T \xi_i\right)\sigma_{\max}(L_G \otimes I_n)||z_i P_{ci} \hat{\Pi}_i|| \|\tilde{\zeta}\| \end{aligned} \tag{35}$$

where (12), (13), (25), and (33) are used to obtain the first equation; $|\bar{z}_{ij}| d_{ij} - \bar{z}_{ij} \frac{\bar{z}_{ij}}{\sqrt{\bar{z}_{ij}^2 + \eta^2}} d_{ij} \le \eta d_{ij} \le \eta d_{\max}$ and (7) are used to obtain the second inequality. Moreover, *Lemma 4* guarantees that there exist positive constants $V_{\Pi\zeta}$ and $\alpha_{V\zeta}$ such that the following holds true:

$$\left\|\dot{\hat{\Pi}}_i \zeta_0\right\| \le V_{\Pi\zeta} \exp(-\alpha_{V\zeta} t). \tag{36}$$

■

Let β_{V1} be a constant satisfying $0 < \beta_{V1} < \frac{1}{2}\sigma_{\min}(Q_{ci})$. Substituting (36) into $2||z_i||||P_{ci}||_F ||\dot{\hat{\Pi}}_i \zeta_0||$ in (35) yields the following:

$$\begin{aligned} 2||z_i||||P_{ci}||_F \left\|\dot{\hat{\Pi}}_i \zeta_0\right\| &\le \left(\frac{1}{4}\sigma_{\min}(Q_{ci}) - \frac{1}{2}\beta_{V1}\right)||z_i||^2 \\ &\quad + \frac{||P_{ci}||_F^2 V_{\Pi\zeta}^2}{\frac{1}{4}\sigma_{\min}(Q_{ci}) - \frac{1}{2}\beta_{V1}} \times \exp(-2\alpha_{V\zeta} t) \\ &\equiv \left(\frac{1}{4}\sigma_{\min}(Q_{ci}) - \frac{1}{2}\beta_{V1}\right)||z_i||^2 \\ &\quad + \beta_{V21} \exp(-2\alpha_{V\zeta} t) \end{aligned} \tag{37}$$

where Young's inequality is used. It is clear that, from *Lemma 4*, there exist positive constants V_Π and α_Π such that the following holds true:

$$\left\|\dot{\hat{\Pi}}_i\right\|_F \le V_\Pi \exp(-\alpha_\Pi t). \tag{38}$$

From *Theorem 1*, there exist positive constants V_ξ and α_ξ such that the following holds true:

$$\|\tilde{\zeta}_i\| \le \sqrt{V_\xi} \exp(-\alpha_\xi t). \tag{39}$$

With (38) and (39), $2||z_i||||P_{ci}||_F \|\dot{\hat{\Pi}}_i\|_F \|\tilde{\zeta}_i\|$ in (35) is rewritten as follows:

$$\begin{aligned} &2||z_i||||P_{ci}||_F \left\|\dot{\hat{\Pi}}_i\right\|_F \|\tilde{\zeta}_i\| \\ &\le 2||z_i||||P_{ci}||_F V_\Pi \exp(-\alpha_\Pi t)\sqrt{V_\xi}\exp(-\alpha_\xi t) \\ &\le \left(\frac{1}{4}\sigma_{\min}(Q_{ci}) - \frac{1}{2}\beta_{V1}\right)||z_i||^2 + ||P_{ci}||_F^2 V_\Pi^2 \\ &\quad \times \frac{V_\xi \exp(-2\alpha_\xi t - 2\alpha_\Pi t)}{\left(\frac{1}{4}\sigma_{\min}(Q_{ci}) - \frac{1}{2}\beta_{V1}\right)} \\ &\equiv \left(\frac{1}{4}\sigma_{\min}(Q_{ci}) - \frac{1}{2}\beta_{V1}\right)||z_i||^2 + \beta_{V22}\exp(-2\alpha_\xi t - 2\alpha_\Pi t). \end{aligned} \tag{40}$$

Substituting (31), (37), and (40) into (35) yields the following:

$$\begin{aligned}\dot{V}_i &\le -\left(\beta_{V1} + \frac{\sigma_{\min}(Q_{ci})}{2}\right) z_i^T z_i - \frac{2}{\varphi_i}(\mu_i(t)N_i(\chi_i) - 1)\dot{\chi}_i \\ &\quad + 2\eta d_{\max} + \beta_{V2}\exp(-\alpha_V t) \\ &\quad + 2 z_i^T P_{ci}\Big(\Pi_i \tilde{S}_i - A_i \tilde{\Pi}_i - B_i \tilde{\Gamma}_i + \tilde{\Gamma}_i \hat{S}_i\Big)\zeta_0 \\ &\quad + 2 z_i^T P_{ci}\Big(\Pi_i \tilde{S}_i - A_i \tilde{\Pi}_i - B_i \tilde{\Gamma}_i + \tilde{\Gamma}_i \hat{S}_i\Big)\tilde{\zeta}_i \\ &\quad + 2(c_i + \xi_i^T \xi_i)\sigma_{\max}(L_G \otimes I_n)||z_i P_{ci} \hat{\Pi}_i|| \|\tilde{\zeta}\| \end{aligned} \tag{41}$$

where $\beta_{V2} \equiv \max\{\beta_{V21}, \beta_{V22}\}$ and $\alpha_V \equiv \min\{2\alpha_\xi + 2\alpha_\Pi, 2\alpha_{V\zeta}\}$. By using Young's inequality and following the results in *Theorems 1 and 2* and *Lemmas 3 and 4*, the last three terms at the right-hand side of (41) can be further changed into forms similar to (37) and (40). Thus, there exist positive constants β_{V3}, β_{V4}, and α_{V1} such that (41) is changed into the following:

$$\begin{aligned}\dot{V}_i &\le -\beta_{V3} z_i^T z_i - \frac{2}{\varphi_i}(\mu_i(t)N_i(\chi_i) - 1)\dot{\chi}_i \\ &\quad + 2\eta d_{\max} + \beta_{V4}\exp(-\alpha_{V1} t). \end{aligned} \tag{42}$$

Solving (42) yields the following:

$$V_i(t) \le V_i(0) - \int_0^t \beta_{V3} z_i^T z_i d\tau - \frac{2}{\varphi_i} N_i^B(t) + N_i^A(t) \tag{43}$$

where $N_i^B(t) = \int_0^t (\mu_i(t)N_i(\chi_i) - 1)\dot{\chi}_i d\tau$ and $N_i^A(t) = \int_0^t (2\eta d_{\max} + \beta_{V4}\exp(-\alpha_{V1}\tau))d\tau$. Note that $N_i^A(t)$ in (43) is bounded. Moreover, the boundedness of $N_i^B(t)$ can also be obtained by seeking a contradiction argument as shown in [23]. Thus, all of the signals in (34) are bounded. Moreover, from (43), one concludes that

$$z_i^T z_i \le -\int_0^t \frac{\beta_{V3}}{\sigma_{\min}(P_{ci})} z_i^T z_i d\tau + \bar{B}_i \tag{44}$$

where $\bar{B}_i = \frac{1}{\sigma_{\min}(P_{ci})}(V_i(0) + \sup_{t\ge 0}(\frac{2}{\varphi_i}|N_i^B(t)| + |N_i^A(t)|))$ is defined as a bounded constant. Recalling *Bellman–Gronwall Lemma*, (44) is thus rewritten as follows:

$$||z_i|| \le \sqrt{\bar{B}_i}\exp\left(-\frac{\beta_{V3}}{2\sigma_{\min}(P_{ci})} t\right). \tag{45}$$

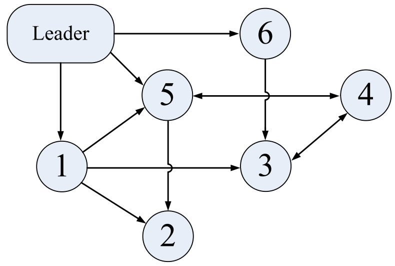

Fig. 1. Graph $\mathcal{G}$ used for output synchronization.

Hence, the output synchronization error for each local agent satisfies the following:

$$||\epsilon_{yi}|| \leq \left\|C_i x_i - C_i\hat{\Pi}_i\zeta_i\right\| + \left\|C_i\hat{\Pi}_i\tilde{\zeta}_i\right\| + ||C_i\hat{\Pi}_i\zeta_0 - R\zeta_0||$$
$$\leq ||C_i||_F \left(||z_i|| + ||\hat{\Pi}_i||_F \left\|\tilde{\zeta}_i\right\| + \left\|\tilde{\Pi}_i\zeta_0\right\|\right) \quad (46)$$

where the exponential convergences of z_i, $\tilde{\zeta}_i$, and $\tilde{\Pi}_i\zeta_0$ are, respectively, ensured in (45), *Theorem 1*, and *Lemma 4*. From (46), it is revealed that the output synchronization error ϵ_{yi}, as well as ϵ_y in (4), exponentially converges to zero. As a result, *exponential synchronization problem* is solved. ∎

Remark 2: The exiting works on controlling a system with faults such as [15]–[17] usually result in the ultimate bounded control or the asymptotic control paradigms. Here, we extend them to an exponential control. Moreover, benefited from the features provided by Nussbaum function based adaptive approach, the proposed control frame in *Theorem 3* also covers a situation that the signs of coefficients $\mu_i(t)$ are unknown and nonidentical. Note that most of the related literature such as [20]–[23] are restricted to a category where unknown control directions are required identical. •

IV. Simulation Study

In this section, we present numerical examples to demonstrate the resilience of the proposed fully distributed protocols against faults on actuators. Specifically, the simulated heterogeneous MAS has six followers and one leader subject to a directed graph, $\mathcal{G}$, given in Fig. 1.

The dynamics of ith follower are given as $\dot{x}_i = \left[\begin{smallmatrix}0.8 & -1.5\\ 2 & 0.8\end{smallmatrix}\right]x_i + \left[\begin{smallmatrix}1.8 & -1\\ 1 & 1.6\end{smallmatrix}\right]\bar{u}_i$, $y_i = \left[\begin{smallmatrix}3.5 & -1.8\\ 0.6 & 4.5\end{smallmatrix}\right]x_i$ for $i = 1, 2$; $\dot{x}_i = \left[\begin{smallmatrix}0.6 & -1\\ 1 & -2\end{smallmatrix}\right]x_i + \left[\begin{smallmatrix}1 & -2\\ 1.9 & 4\end{smallmatrix}\right]\bar{u}_i$, $y_i = \left[\begin{smallmatrix}-1 & 2\\ 3 & 2.8\end{smallmatrix}\right]x_i$ for $i = 3, 4$;

$$\dot{x}_i = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & -2\end{bmatrix}x_i + \begin{bmatrix}6 & 0\\ 0 & 1\\ 1 & 0\end{bmatrix}\bar{u}_i, y_i = \begin{bmatrix}-2.5 & 5 & 2.5\\ 2.5 & 5 & -2.5\end{bmatrix}x_i,$$

for $i = 5, 6$; and the leader dynamics are given as $\dot{\zeta}_0 = \left[\begin{smallmatrix}0 & 3\\ 0 & 0\end{smallmatrix}\right]\zeta_0$ and $y_0 = \left[\begin{smallmatrix}1 & 0\\ 0 & 1\end{smallmatrix}\right]\zeta_0$. We design the actuator faults as

$$\bar{u}_i = \begin{cases}1.5u_i, & t < 15\\ 3u_i + 0.1\cos(0.1t) + 1, & t \geq 15\end{cases},$$

for $i = 1, 2, \ldots, 6$.

Here, we follow *Theorems 1–3* to implement the proposed protocols for MAS with actuator faults. To achieve the synchronization, we design the controller gain in (28) as $K_{1,2} = \left[\begin{smallmatrix}-1.416 & -0.853\\ 0.660 & -1.136\end{smallmatrix}\right]$, $K_{3,4} = \left[\begin{smallmatrix}-0.624 & -0.287\\ 1.165 & -0.560\end{smallmatrix}\right]$, and $K_{5,6} = \left[\begin{smallmatrix}-1.000 & -0.1924 & -0.084\\ 0.018 & -0.962 & -0.303\end{smallmatrix}\right]$. Moreover, we design Q_{ci} as an identity matrix for $i = 1, 2, \ldots, 6$ to solve the Lyapunov (32).

In order to implement protocols in *Theorems 1* and *3*, we design the initial value of (13) as $c_i(0) = 1$ to satisfy the requirement in

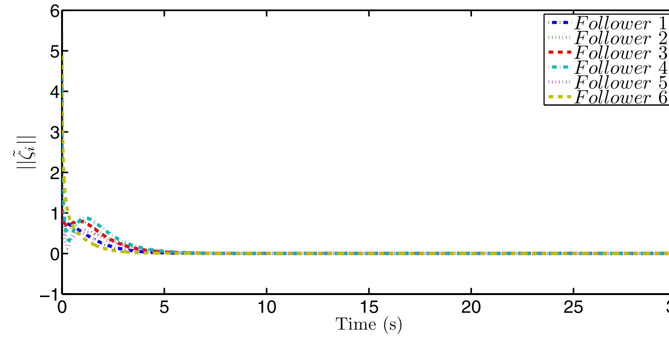

Fig. 2. $\|\tilde{\zeta}_i\|$, for $i = 1, 2, \ldots, 6$.

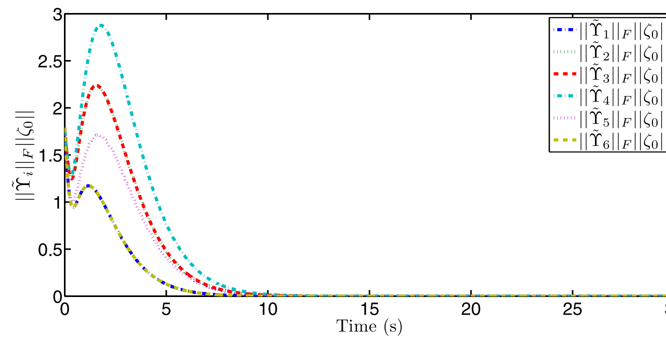

Fig. 3. $||\tilde{\Upsilon}_i||_F\,||\zeta_0||$, for $i = 1, 2, \ldots, 6$.

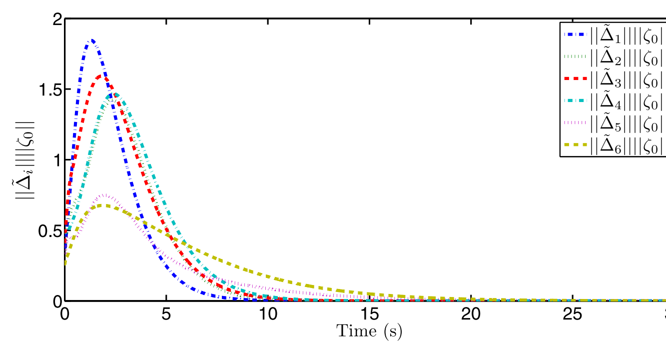

Fig. 4. $||\tilde{\Delta}_i\zeta_0||$, for $i = 1, 2, \ldots, 6$.

Theorem 1, randomly choose ζ_0 in (2), and design all the other initial values to be zero, including ζ_i in (12), $\hat{\Upsilon}_{0i}$ in (14), and $\hat{\Upsilon}_i$ in (15). The trajectories of leader state observer errors $\tilde{\zeta}_i$ for $i = 1, 2, \ldots, 6$ are given in Fig. 2. Moreover, trajectories of $||\tilde{\Upsilon}_i||_F\,||\zeta_0||$ in *Lemma 3* are shown in Fig. 3. Since ζ_0 is generated by unstable dynamics, the convergence of $||\tilde{\Upsilon}_i||_F\,||\zeta_0||$ implies that $||\tilde{\Upsilon}_i||_F$ also converges. Therefore, results in *Theorem 1* and *Lemma 3* are certified by Figs. 2 and 3.

To validate *Theorem 2*, we design the initial values of updating parameters in (21), (22), and (23) to be zero. Note that $\hat{\Upsilon}_i$ is solved by (15), whose initial values and tuning gains are same as the previous simulation. We plot trajectories of $||\tilde{\Delta}_i\zeta_0||$ and $||\dot{\hat{\Pi}}_i\zeta_0||$ in Figs. 4 and 5, respectively. Since ζ_0 is unstable, Fig. 4 reveals that the estimated solutions to the output regulator equations converge to the actual solutions, which certifies *Theorem 2*. Moreover, Fig. 5 denotes that $\dot{\hat{\Pi}}_i\zeta_0$ converges to zero so that the result in *Lemma 4* is certified.

Now, we are in a position to prove the effectiveness of *Theorem 3*. The initial values of follower agents are designed as $x_1(0) = [0, 0.5]^T$, $x_2(0) = [1, 1.5]^T$, $x_3(0) = [2, 2.5]^T$, $x_4(0) = [3, 3]^T$, $x_5(0) = [2.5, 2, 1.5]^T$, and $x_6(0) = [1, 0.5, 0]^T$. The initial

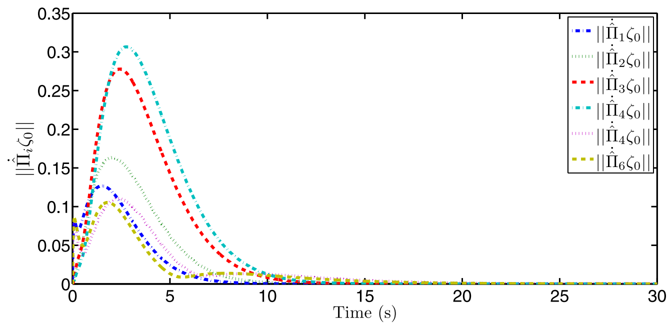

Fig. 5. $||\dot{\hat{\Pi}}_i \zeta_0||$, for $i = 1, 2, \ldots, 6$.

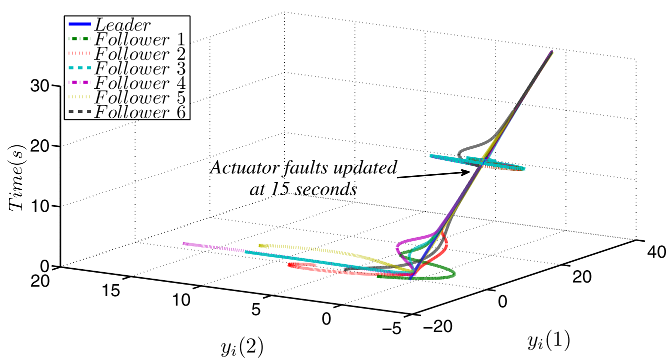

Fig. 6. Trajectories of leader states and follower states under the proposed method.

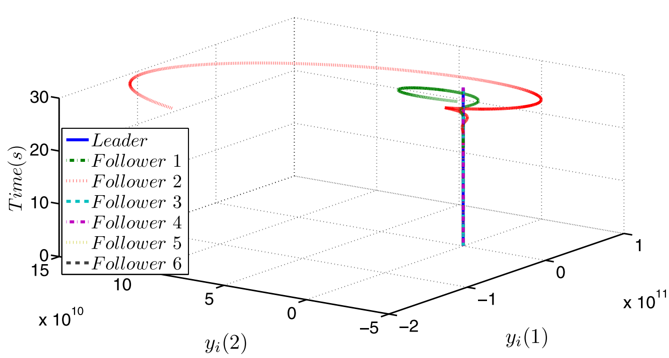

Fig. 7. Trajectories of leader states and follower states.

values for updating Nussbaum function are $\chi_i(0) = 0$ for $i = 1, \ldots, 6$. In addition, we set the initials of leader states and updating parameters in (2), (12), (13)–(15), and (21)–(23) as the same ones in the previous simulations. The trajectories of all agents are presented in Fig. 6. This reveals that the resilient synchronization of heterogeneous MAS with actuator faults is achieved after applying the proposed distributed protocol. Moreover, we use the design in [42] and preclude adaptive resilience mechanisms for rejecting actuator faults given in (26) and (27). For comparison purpose, trajectories of all agents under the same actuator faults are shown in Fig. 7, where the followers' states are unbounded. This further testifies the effectiveness of the proposed method.

V. Conclusion

This paper addresses the distributed synchronization resilience problem for MAS with actuator faults. The transient output synchronization possesses the exponential convergence for heterogeneous MAS. A complete solution is proposed to achieve the fully distributed control design for the output synchronization. Moreover, the resilience is enhanced by extending exponential synchronization under actuator faults. The effectiveness has been validated by simulation studies. As for the future research, we will extend [51] to address insecure communications in heterogeneous MAS.

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