ANALYZING THE DEVELOPMENT OF MKT IN CONTENT COURSES

Yvonne Lai
University of Nebraska – Lincoln
yvonnexlai@unl.edu

Jeremy Strayer
Middle Tennessee St Univ
Jeremy.Strayer@mtsu.edu

Alyson E. Lischka
Middle Tennessee St Univ
Alyson.Lischka@mtsu.edu

To improve teaching and learning in content courses for secondary mathematics teachers, we take the approach of supporting faculty who teach these courses – often mathematics faculty – in developing their own mathematical knowledge for teaching (MKT) at the secondary level. We describe a framework that has informed the design of educative curricula for a set of these courses. This framework integrates theory for knowledge development, empirical work on dimensions of knowledge used in teaching, and findings on observable behaviors in teaching that reveal prospective secondary teachers' knowledge development.

Keywords: Mathematical Knowledge for Teaching, Teacher Education – Preservice

Mathematics teacher preparation programs for all levels aim to provide opportunities for prospective mathematics teachers (prospective secondary teachers) to develop mathematical knowledge for teaching (MKT) (e.g., CBMS, 2012). Yet, mathematicians, who often teach content courses for secondary preparation programs (Murray & Star, 2013), who may have much to offer in the way of mathematical knowledge, often do not have experience teaching in K-12 settings. They may not be positioned to notice nuances in the development of prospective secondary teachers' MKT at the secondary level (Lai, 2016), especially on tasks that situate mathematics in pedagogical contexts. This problem exacerbates existing tensions between intended outcomes of secondary preparation programs and teachers' perceptions that their mathematical preparation is irrelevant to their teaching (e.g., Goulding, Hatch, & Rodd, 2003).

One approach to this problem is developing educative curricula (Davis & Krajcik, 2005). We take this approach, focusing on curriculum supports for mathematics faculty to learn MKT at the secondary level and ways to observe and analyze the development of prospective secondary teachers' MKT, particularly in enactments of approximations of practice (Grossman et al., 2009). In this paper, we discuss a framework that has informed the design of educative curricula for mathematics faculty teaching content courses for secondary level prospective secondary teachers. This framework integrates theory for knowledge development, empirical work on dimensions of knowledge used in teaching, and findings on observable behaviors in teaching that reveal prospective secondary teachers' knowledge development. Such a framework is potentially a critical resource for supporting mathematics faculty in teaching MKT. Our purposes in this paper are to describe this novel framework, and how it can support mathematics faculty in learning MKT and providing substantive feedback to prospective secondary teachers about their MKT, and serve as a resource to refine research on learning and teaching MKT.

Conceptual Foundations

To construct the framework, we integrated Rowland and colleagues' Knowledge Quartet framework for observing MKT and Ader and Carlson's (2018) framework for analyzing and observing teaching with Silverman and Thompson's (2008) developmental framework for MKT. We take MKT to be "practice-based theory of knowledge for teaching" (Ball & Bass, 2003, p. 5) and follow Thompson, Carlson, and Silverman (2007) in taking MKT to include coherent and generative understandings of key ideas that make up the curriculum.

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The empirical analysis of mathematics teaching at the elementary and secondary level underlying the Knowledge Quartet suggests four dimensions of observable MKT: Foundation (knowledge of mathematics and its nature), Transformation (presentation of ideas to learners), Connection (sequencing of material for instruction), and Contingency (the ability to respond to unanticipated events) (Rowland, Thwaites, & Jared, 2016).

Ader and Carlson's (2018) framework for analyzing instructional interactions identifies levels of understanding and acting on student thinking in terms of teachers' mental actions and observable behaviors. They characterized teachers' executions of responses to student thinking in terms of Piaget's (1977/2001) notions of decentering and reflective abstraction.

The Knowledge Quartet and Ader and Carlson's framework can be used to articulate Silverman and Thompson's (2008) framework for use in educative curricula for teacher education, as we detail in the next section. Silverman and Thompson used Simon's (2006) idea of key developmental understandings (KDUs) in combination with decentering and reflective abstraction to construct stages of MKT development. Although they described components of development, but they did not elaborate where these components might be observed in actual teaching practice or in approximations of practice, or how one instance of decentering or reflective abstraction may be more sophisticated than another.

Framework for Observing and Analyzing the Development of MKT

The framework we use is based on the work described above and then refined based on the analyses of 15 secondary prospective secondary teachers' responses to an approximation of practice used in a mathematics content course, which has been used at three different institutions in different states in multiple years. The responses analyzed are representative.

Table 1: Framework for Observing and Analyzing the Development of MKT

Developmental component[a]	Knowledge dimension[b]	Mental actions[c]	Level (L), in terms of observable behaviors[d]
Personal KDU	Foundation	Reflective abstraction on personal math. knowledge	L1: Performs procedures within topic L2: Describes procedures accurately L3: Connects isolated features to underlying concepts L4: Connects structure of procedure to underlying concepts
Decentering applied to Activities and Analyzing Potential for Student KDU	Transformation	Reflective abstraction on student thinking	Gives explanations, representations, and examples that: L1: Describe only procedures L2: Describe own way of thinking L3: Attempt to change students' current thinking L4: Enhance students' understanding
	Connection		Poses questions that: L1: Focusing on procedures or echoing key phrases L2: May reveal student thinking, but then gives explanations while not asking students to provide reasoning L3: Attempts to move students' reasoning L4: Supports advancing students' reasoning
	Contingency		Responses to student thinking: L1: Do not act in any visible way upon the thinking L2: Evaluate but do not use the thinking in teaching L3: Directly use the thinking L4: Frames questions or explanations in terms of students' thinking, to make connections and deepen understanding
Note: Levels here depend on the KDU. This is just one possible example of how levels may appear. Constructs for each column were refined through our analysis and based on: (a) Silverman and Thompson, 2008; (b) Rowland, Thwaites, & Jared, 2016; (c) Piaget, 1977/2001; and, (d) Ader and Carlson, 2018			

Discussion: Uses of Framework in Teacher Education

The framework presented in this paper is a resource for implementing and writing educative curricula in teacher preparation, as well as for future research in the learning and teaching of MKT. In terms of implementing curricula, we used the framework to inform guides for faculty to use when interpreting and responding to prospective secondary teachers' work. Differentiating between the dimensions of MKT used can support faculty in noticing different kinds of knowledge in teachers and providing more explicit feedback. Foundation, Connection, and Transformation emphasize potential differences among a prospective secondary teacher's display of personal knowledge, providing explanations to students, and posing questions that elicit student reasoning. Although faculty may not traditionally make these distinctions in providing feedback in a mathematics course, these distinctions are also ones that may be familiar to faculty and may well be informative educative for their own teaching practice (e.g., Pascoe & Stockero, 2017).

It is worth noting that our analysis indicated that the dimensions of knowledge were independent in the context of prospective secondary teachers' enactments of approximations of practice, a context for which the dimensions had not been previously analyzed. This suggests that development of MKT may well proceed along these dimensions in different ways. One prospective secondary teacher in our dataset explained the connection between a definition and procedure as a rationale for a task they would assign to their students (Foundation, L4), but only posed questions that focused on echoing key phrases (Connection, L1), proposed only explanations of procedures to the students (Transformation, L1), and did not acknowledge any of the sample student thinking provided by approximation of practice (Contingency, L1). Another prospective secondary teacher began with using the provided sample student work to make a specific mathematical point about a definition (Contingency, L4) then did not provide any subsequent examples or explanations to connection of procedure and definition (Transformation, L1).

In our own work writing approximations of practice for use in content courses and hearing initial feedback from mathematics faculty, articulating how observables may correspond to knowledge dimensions has prompted us to think more clearly about the opportunities presented by approximations of practice. For instance, in an early draft of an approximation of practice, we asked prospective secondary teachers to respond to student thinking, but we did not give a clear mathematical goal for the teaching situation. This left unspecified the Foundation knowledge we were aiming to elicit, which impacted the Transformation and Connection knowledge visible in prospective secondary teachers' responses.

Finally, the framework supports validating and refining the articulation of the development of MKT. We view this framework as a set of testable hypotheses grounded in known results. At the same time, we have drawn from work in emergent stages. We see great opportunity in using this framework, which unifies work from different groups, to push researchers' understanding of the development of MKT. Future work made visible by this framework includes investigating how well these codes hold up to responses to approximations of practice across content courses; whether the interpretation of levels for the purpose of providing feedback to teachers improved teaching and learning outcomes; and the independence or dependence of levels and dimensions. As Morris and Hiebert (2009) argued, a professional knowledge base – such as that for teaching mathematics courses for prospective secondary teachers – can only advance with shared goals and artifacts that articulate those goals for the professional community in ways that can be observed. We propose a framework that articulates the goal of developing teachers' MKT in

terms of actions observable in approximations of practice used in content courses. Our framework simultaneously leverages theory for the development of MKT, empirical analyses of teaching, and empirical analyses of approximations of practice used in content courses.

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