Robust Topology Design Optimization Based on Dimensional Decomposition Method

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Abstract

This paper presents an efficient approach for robust topology design optimization (RTO) which is based on polynomial dimensional decomposition (PDD) method. The level-set functions are adopted to facilitate the topology changes and shape variations. The topological derivatives of the functionals of robustness root in the concept of deterministic topological derivatives and dimensional decomposition of stochastic responses of multiple random inputs. The PDD for calculating robust topological derivatives consists of only a number of evaluations of the deterministic topological derivatives at the specified points in the stochastic space and provides effective and efficient design sensitivity analyses for RTO. The numerical examples demonstrate the effectiveness of the present method.

1 Introduction

Conventional deterministic topology design techniques [1–5] do not consider the impact of uncertainties extensively existing in the manufacturing process and operational environment. When the system responses driving the design process are highly sensitive to such uncertainties, large deviations in predictions of responses of engineering structure will be resulted. Therefore deterministic topology optimization iterations driven by those system responses may lead to pseudo optimal designs with substantial performance degradation. Especially, when the deterministic design solution found located near the boundary of the feasible domain, even slight changes caused by uncertainties could produce unknowingly risky designs violating one or more constraints.

Robust topology optimization (RTO), targeted at minimizing the propagation of input uncertainty, generates insensitive topology design with the presence of uncertainty. It has been an important methodology in the past decade for topology design of aerospace, automotive, and civil structures sustaining the plague of uncertainties. The objective or constraint functions in RTO usually combine the mean and standard deviation of certain stochastic responses, describing the objective robustness or feasibility robustness of a given topology. Therefore, an RTO solution requires evaluations of statistical moments and their sensitivity with respect to topology changes. In nature, statistical moment analysis is to calculate a high-dimensional integral regarding the probability measure $f_{\mathbf{X}}(\mathbf{x})$ of $\mathbf{X}$ over $\mathbb{R}^N$, where N is the number of random variables. Generally, the analytical evaluation for such an integral is not readily available. As a consequence, it often resorts to numerical integration. However, direct numerical integration is often computational prohibitive for the cases that N exceeds three or four, especially when expensive finite element analyses (FEA) are required for the evaluation of response functions. To alleviate the computational cost, many approximate methods for statistical moment analysis were developed, including the point estimate method (PEM) [6], Taylor series expansion or perturbation method [6], tensor product quadrature (TPQ) [7], Neumann expansion method [8], polynomial chaos expansion (PCE) [9], statistically equivalent solution [10], dimension-reduction method [11, 12], and others [13]. There are two major concerns in those approaches when applied to large-scale engineering problems. First, the perturbation or Taylor series expansions, PEM, PCE, TPQ, and dimension-reduction methods often begin to be inapplicable or inadequate when the input uncertainty is large and/or stochastic responses are highly nonlinear. Second, many of the aforementioned methods are often computationally expensive for stochastic topology sensitivity analysis since many of them invoke finite-difference techniques which require repetitive stochastic analysis at both perturbed and nominal design points.

This paper builds a framework for robust topology design optimization of complex engineering structures subject to random inputs. The method roots in polynomial dimensional decomposition (PDD) of a multivariate stochastic response function, deterministic topological sensitivity analysis, and the level-set method. It generates analytical formulations of the first three moments. In addition, it can calculate both the first two moments and their topology derivatives in one stochastic analysis. Section 2 describes the PDD approximation of topological derivatives for robust topology optimization, resulting in explicit formulae for the first two moments. Section 3 briefly describes the level-set method for topology

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changes. The calculation of PDD expansion coefficients, required in sensitivity analyses of moments, is discussed in Section 4. In Section 5, two numerical examples demonstrate the effectiveness of the proposed method. Finally, conclusions are drawn in Section 6.

2 PDD for Topological Derivatives of Robust Topology Optimization

2.1 Robust Topology Design Problems

Both objective and constraint functions of RTO problems may involve the first two moment properties for the assessment of robustness. A generic RTO problem is often formulated as the following mathematical programming problem

$$\begin{aligned} \min_{\Omega} \quad & c_0(\Omega) := w_1 \frac{\mathbb{E}\left[y_0(\Omega,\mathbf{X})\right]}{\mu_0^*} + w_2 \frac{\sqrt{\mathrm{var}\left[y_0(\Omega,\mathbf{X})\right]}}{\sigma_0^*}, \\ \text{subject to} \quad & c_k(\Omega) := \alpha_k \sqrt{\mathrm{var}\left[y_k(\Omega,\mathbf{X})\right]} - \mathbb{E}\left[y_k(\Omega,\mathbf{X})\right] \le 0; \;\; k = 1,\cdots,K, \\ & \Omega \subseteq D \end{aligned} \tag{1}$$

where $D \subset \mathbb{R}^3$ is a bounded domain in which all admissible topology design Ω are included; $\mathbf{X} := (X_1,\cdots,X_N)^T \in \mathbb{R}^N$ is an N-dimensional random input vector completely defined by a family of joint probability density functions $\{f_{\mathbf{X}}(\mathbf{x}), \mathbf{x} \in \mathbb{R}^N\}$ on the probability triple $(\Omega_{\mathbf{X}}, \mathcal{F}, P)$, and $\Omega_{\mathbf{X}}$ is the sample space; $\mathcal{F}$ is the σ-field on $\Omega_{\mathbf{X}}$; P is the probability measure associated with probability density $f_{\mathbf{X}}(\mathbf{x})$; $w_1 \in \mathbb{R}_0^+$ and $w_2 \in \mathbb{R}_0^+$ are two non-negative, real-valued weights, satisfying $w_1 + w_2 = 1$, $\mu_0^* \in \mathbb{R} \setminus \{0\}$ and $\sigma_0^* \in \mathbb{R}_0^+ \setminus \{0\}$ are two non-zero, real-valued scaling factors; $\alpha_k \in \mathbb{R}_0^+$, $k = 0,1,\cdots,K$, are non-negative, real-valued constants associated with the probabilities of constraint satisfaction; $\mathbb{E}$ and var are the expectation operator and variance operator, respectively, with respect to the probability measure P. The evaluation of both $\mathbb{E}$ and var on certain random response demands statistical moment analysis.

2.2 Polynomial Dimensional Decomposition

Let $y(\Omega,\mathbf{X})$ be a multivariate stochastic response, representing any of y_k in Eq. (1) and depending on the random vector $\mathbf{X} = \{X_1,\cdots,X_N\}^T$, and $\mathcal{L}_2(\Omega_{\mathbf{X}}, \mathcal{F}, P)$ be a Hilbert space of square-integrable functions y with corresponding probability measure $f_{\mathbf{X}}(\mathbf{x})d\mathbf{x}$ supported on $\mathbb{R}^N$. Assuming independent coordinates, the PDD expansion of function y generates the following hierarchical decomposition [14,15]

$$y(\Omega,\mathbf{X}) = y_\emptyset(\Omega) + \sum_{\emptyset \ne u \subseteq \{1,\cdots,N\}} \sum_{\mathbf{j}_{|u|} \in \mathbb{N}^{|u|}} C_{u\mathbf{j}_{|u|}}(\Omega) \psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u;\Omega), \tag{2}$$

where $\psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u;\Omega) := \prod_{p=1}^{|u|} \psi_{i_p j_p}(X_i;\Omega)$ is a set of multivariate orthonormal polynomials and $\mathbf{j}_{|u|} = (j_1,\cdots,j_{|u|}) \in \mathbb{N}^{|u|}$ is a $|u|$-dimensional multi-index; $y_\phi(\Omega)$ represents the constant term; for $|u| = 1,2,\cdots,S$, $C_{u\mathbf{j}_{|u|}}(\Omega)\psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u;\Omega)$ are the univariate, bivariate, $\cdots$, and S-variate component functions representing the individual influence from a single input variable, the cooperative effect of two, $\cdots$, and S input variables, respectively. Retaining, at most, the interactive effects of $0 < S < N$ input variables and mth-order polynomials, Eq. (2) can be truncated as follows

$$\tilde{y}_{S,m}(\Omega,\mathbf{X}) = y_\emptyset(\Omega) + \sum_{\substack{\emptyset \ne u \subseteq \{1,\cdots,N\} \\ 1 \le |u| \le S}} \sum_{\substack{\mathbf{j}_{|u|} \in \mathbb{N}^{|u|} \\ \left\|\mathbf{j}_{|u|}\right\|_\infty \le m}} C_{u\mathbf{j}_{|u|}}(\Omega) \psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u;\Omega), \tag{3}$$

where

$$y_\emptyset(\Omega) = \int_{\mathbb{R}^N} y(\mathbf{x},\Omega) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{4}$$

and

$$C_{u\mathbf{j}_{|u|}}(\Omega) : \;\; = \int_{\mathbb{R}^N} y(\mathbf{x},\Omega) \psi_{u\mathbf{j}_{|u|}}(\mathbf{x}_u;\Omega) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}, \quad \emptyset \ne u \subseteq \{1,\cdots,N\},\; \mathbf{j}_{|u|} \in \mathbb{N}^{|u|}, \tag{5}$$

are various expansion coefficients. Eq. (3) generates the S-variate, mth-order PDD approximation by containing interactive effects of at most S input variables $X_{i_1},\cdots,X_{i_S}$, $1 \le i_1 < \cdots < i_S \le N$, and up to mth-order polynomial basis on y. It converges to y and engenders a sequence of hierarchical and convergent approximations of y when $S \to N$ and $m \to \infty$. Depending on the hierarchical structure and nonlinearity of an engineering problem, the truncation parameters S and m can be chosen accordingly. The higher the values of S and m permit the higher accuracy, but also demand the higher computational cost of an Sth-order polynomial computational complexity [14,15]. In the following sections of this paper, the S-variate, mth-order PDD approximation is simply referred to as *truncated PDD approximation*.

2.3 Stochastic Moments and Their Topology Derivatives

Given a random response y on certain topology design Ω, let $m^{(r)}(\Omega) := \mathbb{E}[y^r(\Omega,\mathbf{X})]$ denote the raw moment of y of order r. Let $\tilde{m}^{(r)}(\Omega) := \mathbb{E}[\tilde{y}^r_{S,m}(\Omega,\mathbf{X})]$ denote the raw moment of an S-variate, mth-order PDD approximation $\tilde{y}_{S,m}(\Omega,\mathbf{X})$ of $y(\Omega,\mathbf{X})$. The explicit formulae of the moments by PDD approximations are listed as follows. The first moment or mean [16]

$$\tilde{m}^{(1)}_{S,m}(\Omega) := \mathbb{E}\left[\tilde{y}_{S,m}(\Omega,\mathbf{X})\right] = y_{\emptyset}(\Omega) \tag{6}$$

is simply the constant term $y_{\emptyset}$, whereas the second moment [16]

$$\tilde{m}^{(2)}_{S,m}(\Omega) := \mathbb{E}\left[\tilde{y}^2_{S,m}(\Omega,\mathbf{X})\right] = y^2_{\emptyset}(\Omega) + \sum_{\substack{\emptyset\neq u\subseteq\{1,\cdots,N\}\\ 1\le|u|\le S}} \sum_{\substack{\mathbf{j}_{|u|}\in\mathbb{N}^{|u|}\\ \|\mathbf{j}_{|u|}\|_\infty\le m}} C^2_{u\mathbf{j}_{|u|}}(\Omega) \tag{7}$$

is just the sum of squares of the PDD expansion coefficients. It is straightforward that the estimation of the first two moments evaluated by above equations approaches their exact values when $S\to N$ and $m\to\infty$.

The proposed method for topology derivatives of those moments exploits the deterministic the topology derivative concept. When the total compliance of the structure is selected as the response function y, aided by the adjoint method, the deterministic topological derivative $D_T y(\Omega,\boldsymbol{\xi}_0)$ at a point $\boldsymbol{\xi}_0$ reads [17]

$$D_T y(\Omega,\boldsymbol{\xi}_0) = \tilde{\boldsymbol{\sigma}}(\boldsymbol{\xi}_0) : \mathbb{A} : \boldsymbol{\sigma}(\boldsymbol{\xi}_0). \tag{8}$$

For the case that the three-dimensional domain consist of isotropic linear elastic material, $\mathbb{A}$ is a fourth order tensor related to Young's modulus E and Poisson's ratio ν as follows

$$\mathbb{A} = \frac{2\pi\,(1-\nu)}{E(7-5\nu)}\left[10(1+\nu)\mathbb{I} - (5\nu+1)\,\mathbf{I}\otimes\mathbf{I}\right] \tag{9}$$

where $\mathbb{I} = \frac{1}{2}\left(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}\right)\mathbf{e}_i\otimes\mathbf{e}_j\otimes\mathbf{e}_k\otimes\mathbf{e}_l$ is the symmetric fourth order identity tensor and I $=\delta_{ij}\mathbf{e}_i\otimes\mathbf{e}_j$ is the second order identity tensor. In addition, $\boldsymbol{\sigma}$ is the stress solution of the original problem and $\tilde{\boldsymbol{\sigma}}$ is the stress solution of the associated adjoint problem.

For a point $\boldsymbol{\xi}_0\in\Omega$, taking topology derivative of rth moments of the response function $y(\Omega,\mathbf{X})$ and applying the Lebesgue dominated convergence theorem, which permits interchange of the differential and integral operators, yields

$$D_T m^{(r)}(\Omega,\boldsymbol{\xi}_0) := D_T\mathbb{E}\left[y^r(\Omega,\mathbf{X})\right]|_{\boldsymbol{\xi}_0} = \int_{\mathbb{R}^N} r y^{r-1}(\Omega,\mathbf{X}) D_T y(\Omega,\mathbf{X},\boldsymbol{\xi}_0) f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} = \mathbb{E}\left[r y^{r-1}(\Omega,\mathbf{X}) D_T y(\Omega,\mathbf{X},\boldsymbol{\xi}_0)\right], \tag{10}$$

that is, the stochastic topology derivative of a response function is obtained from the expectation on the product of the response function and its deterministic topology derivative.

For simplicity, we use a multi-variate function $z(\Omega,\mathbf{X},\boldsymbol{\xi}_0)$ to denote $D_T y(\Omega,\mathbf{X},\boldsymbol{\xi}_0)$, and it can be approximated by

$$\tilde{z}_{S,m}(\Omega,\mathbf{X},\boldsymbol{\xi}_0) := z_{\emptyset}(\Omega,\boldsymbol{\xi}_0) + \sum_{\substack{\emptyset\neq u\subseteq\{1,\cdots,N\}\\ 1\le|u|\le S}} \sum_{\substack{\mathbf{j}_{|u|}\in\mathbb{N}^{|u|}\\ \|\mathbf{j}_{|u|}\|_\infty\le m}} D_{u\mathbf{j}_{|u|}}(\Omega,\boldsymbol{\xi}_0)\psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u;\Omega). \tag{11}$$

Plugged in Eq. (10) and employing the zero mean property and orthonormal property of the PDD basis $\psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u;\Omega)$ yield the semi-analytical formulation for topology sensitivity of the first three moments

$$D_T\tilde{m}^{(1)}_{S,m}(\Omega,\boldsymbol{\xi}_0) = z_{\emptyset}(\Omega,\boldsymbol{\xi}_0), \tag{12}$$

$$D_T\tilde{m}^{(2)}_{S,m}(\Omega,\boldsymbol{\xi}_0) = 2\times\left[y_{\emptyset}(\Omega)z_{\emptyset}(\Omega,\boldsymbol{\xi}_0) + \sum_{\substack{\emptyset\neq u\subseteq\{1,\cdots,N\}\\ 1\le|u|\le S}} \sum_{\substack{\mathbf{j}_{|u|}\in\mathbb{N}^{|u|}\\ ||\mathbf{j}_{|u|}||_\infty\le m}} C_{u\mathbf{j}_{|u|}}(\Omega)D_{u\mathbf{j}_{|u|}}(\Omega,\boldsymbol{\xi}_0)\right], \tag{13}$$

$$D_T\tilde{m}^{(3)}_{S,m}(\Omega,\boldsymbol{\xi}_0) = 3\times\left[z_{\emptyset}(\Omega,\boldsymbol{\xi}_0)\tilde{m}^{(2)}_{S,m}(\Omega) + 2y_{\emptyset}(\Omega) \sum_{\substack{\emptyset\neq u\subseteq\{1,\cdots,N\}\\ 1\le|u|\le S}} \sum_{\substack{\mathbf{j}_{|u|}\in\mathbb{N}^{|u|}\\ ||\mathbf{j}_{|u|}||_\infty\le m}} C_{u\mathbf{j}_{|u|}}(\Omega)D_{u\mathbf{j}_{|u|}}(\Omega,\boldsymbol{\xi}_0) + T_k\right], \tag{14}$$

$$T_k = \sum_{\substack{\emptyset \neq u,v,w \subseteq \{1,\cdots,N\} \\ 1 \leq |u|,|v|,|w| \leq S}} \sum_{\substack{\mathbf{j}_{|u|},\mathbf{j}_{|v|},\mathbf{j}_{|w|} \in \mathbb{N}^{|u|} \\ ||\mathbf{j}_{|u|}||_\infty,||\mathbf{j}_{|v|}||_\infty,||\mathbf{j}_{|w|}||_\infty \leq m}} C_{u\mathbf{j}_{|u|}}(\Omega) C_{v\mathbf{j}_{|v|}}(\Omega) D_{w\mathbf{j}_{|w|}}(\Omega, \boldsymbol{\xi}_0) \times$$
$$\mathbb{E}_{\mathbf{d}} \left[\psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u;\Omega) \psi_{v\mathbf{j}_{|v|}}(\mathbf{X}_v;\Omega) \psi_{wj_{|w|}}(\mathbf{X}_w;\Omega) \right]. \tag{15}$$

which requires expectations of various products of three random orthonormal polynomials. However, if $\mathbf{X}$ follows classical distributions such as Gaussian, Exponential, and Uniform distribution, then the expectations are easily determined from the properties of univariate Hermite, Laguerre, and Legendre polynomials [18–20]. For general distributions, numerical integration methods will apply.

3 Level-set method for topology and shape changes

This paper employs level-set function for the topology representation. The structural domain is described by the positive values of the level-set function. The domain topology and boundaries are implicitly represented by its zero iso-surface. In addition, the evolution of level-set function thus the domain topology is updated by solving a reaction-diffusion equation [21], in which the reaction term is driven by the topology derivative of the stochastic moments. Therefore, it permits nucleation of new holes and new boundaries during the optimization iterations and does not need to preset an initial topology from guessing the proper number and configuration of initial holes.

In this research, the reaction-diffusion equation

$$\frac{\partial \phi}{\partial t} = \tau \nabla^2 \phi + D_T \tag{16}$$

with introducing a fictitious time parameter $t \in \mathbb{R}^+$ which corresponding to descent stepping in optimization iterations, will be used.

4 PDD expansion coefficients

The dimension-reduction integration (DRI) scheme [11], is employed to evaluate the PDD coefficients. Let $\mathbf{c}$ be a reference point, which is commonly taken as the mean of $\mathbf{X}$, and $y(\Omega, \mathbf{x}_v, \mathbf{c}_{-v})$ be an $|v|$-variate RDD component function [11] of $y(\Omega, \mathbf{x})$, where $v \subseteq \{1, \cdots, N\}$. Given a positive integer $S \leq R \leq N$, the coefficients $y_\emptyset(\Omega)$ and $C_{u\mathbf{j}_{|u|}}(\Omega)$ are estimated from [11]

$$y_\emptyset(\Omega) \cong \sum_{i=0}^{R} (-1)^i \binom{N-R+i-1}{i} \sum_{\substack{v \subseteq \{1,\cdots,N\} \\ |v| = R-i}} \int_{\mathbb{R}^{|v|}} y(\Omega, \mathbf{x}_v, \mathbf{c}_{-v}) f_{\mathbf{X}_v}(\mathbf{x}_v) d\mathbf{x}_v \tag{17}$$

and

$$C_{u\mathbf{j}_{|u|}}(\Omega) \cong \sum_{i=0}^{R} (-1)^i \binom{N-R+i-1}{i} \sum_{\substack{v \subseteq \{1,\cdots,N\} \\ |v| = R-i, u \subseteq v}} \int_{\mathbb{R}^{|v|}} y(\Omega, \mathbf{x}_v, \mathbf{c}_{-v}) \psi_{u\mathbf{j}_{|u|}}(\mathbf{x}_u, \Omega) f_{\mathbf{X}_v}(\mathbf{x}_v) d\mathbf{x}_v, \tag{18}$$

respectively, which need the evaluation of at most R-dimensional integrals. The DRI is significantly more efficient than performing one N-dimensional integration, particularly when $R \ll N$. For instance, when $R = 1$ or 2, Eqs. (17) and (18) involve one-, or at most, two-dimensional integration, respectively. Hence, the computational effort is significantly lowered.

5 Numerical Examples

To examine the efficiency of the PDD methods developed for RTO, two examples are solved in this section, which are a cantilever beam and a three-point bending beam, respectively. For both examples, the PDD expansion coefficients were estimated by DRI with the mean input as the reference point, $R = S$, and the number of Gauss points $n_g = m + 1$, where $S = 1$ and $m = 2$. In both examples, orthonormal polynomials and their consistent Gauss quadrature rules were employed for evaluating coefficients. No unit for length, force, and Young's modulus is specified in both examples for simplicity, while permitting any consistent unit system for the results. The response function y_0 in Eq. (1) is taken as the structure compliance $y_0(\Omega) = \int_\Omega \frac{1}{2} \boldsymbol{\sigma} : \boldsymbol{\epsilon} d\Omega$ of the structure for both examples. The two weights $w_1 = w_2 = 0.5$ in both examples.

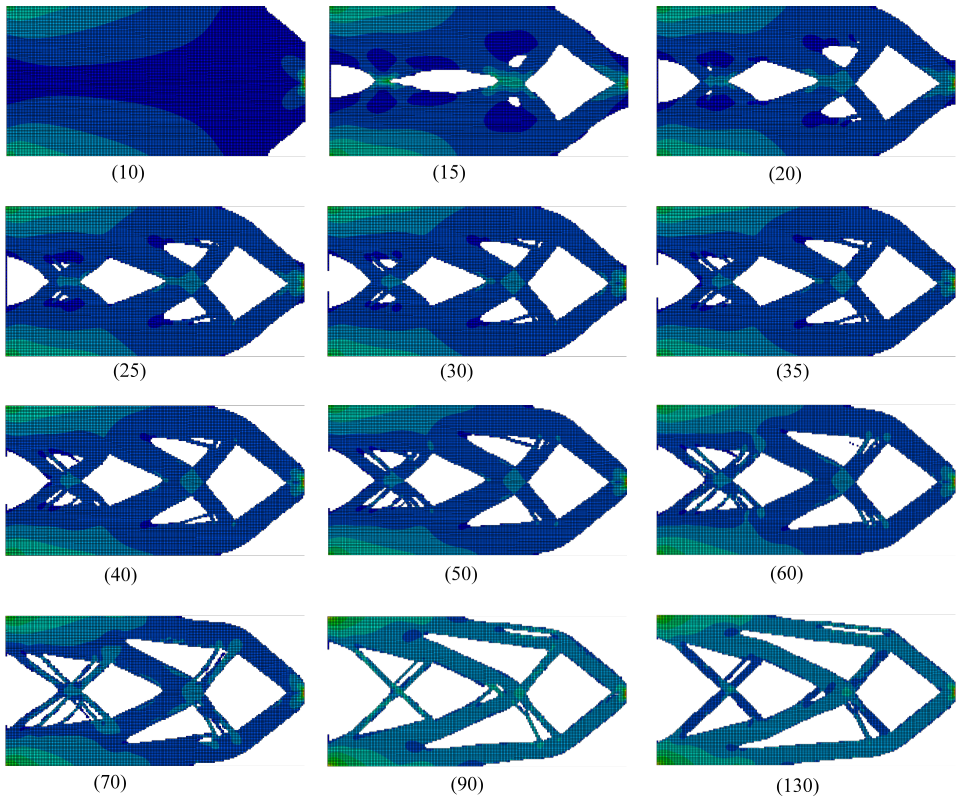

Figure 1: Topologies in selected iterations for the cantilever beam example (the number underneath each subfigure is the corresponding iteration number)

5.1 A Cantilever Beam

Consider a cantilever beam, of length $L = 200$ and height $H = 100$, subject to a distributed shear traction F along the downward direction on the segment of length $\frac{H}{16}$ located in the center of the right edge, where $F \sim \mathcal{N}(16, 0.16)$ is a Gaussian random variable with the mean value of 16 and standard deviation of 0.16. The beam consists of isotropic linear elastic material, of random Young's modulus E and random Poisson's ratio ν, where $E \sim \mathcal{N}(10^6, 10^5)$ and $\nu \sim \mathcal{N}(0.25, 0.0025)$. The RTO problem is also subject to a deterministic volume constraint which limits the maximal volume of the feasible design to be less than 35% of the initial one. The topology updates of selected design iterations are shown in Fig. 1. The total number of FEA for 130 iterations is only 910 attributing to the 2nd order univariate PDD approximation.

5.2 A Three-Point Bending Beam

The second example is an RTO of a three-point bending beam. Its length $L = 200$ and height $H = 100$, subject to a distributed normal traction F, pointing to the downward direction, on the segment of length $\frac{L}{32}$ located in the center of the bottom edge. The random normal traction F, random Young's modulus E, and random Poisson's ratio ν follow the same distribution of the corresponding variable in the cantilever beam example, respectively. In addition, the same deterministic volume constraint is applied in this example. The topology updates of selected design iterations are shown in Fig. 2. Only 980 FEA are required for 140 iterations by the 2nd order univariate PDD approximation.

6 Conclusions

The novel computational method proposed in this paper for robust topology optimization integrates truncated polynomial decomposition approximations, deterministic topology derivatives, and level-set functions, providing semi-analytical expressions of approximate topology sensitivities of the first three moments that are mean-square convergent. In addition, only a single stochastic analysis is required for both statistical moment analysis and their topology sensitivity analysis in each design iteration. The RTO driven by the proposed method requires no initial layout of holes, facilitating the design process. Two numerical examples indicate that the new method developed provides computationally efficient solutions. Although only three random variables are considered in both examples, the solution for RTO problems with 50-100 random variables can be envisioned according to authors' experience.

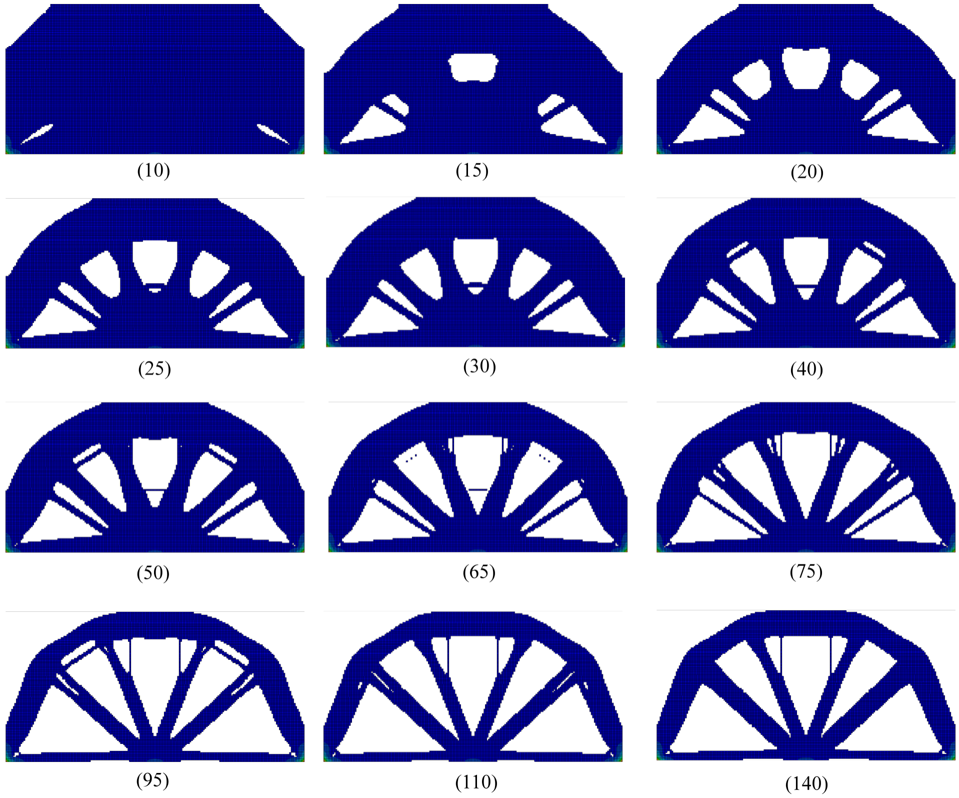

Figure 2: Topologies in selected iterations for the three-point bending example (the number underneath each subfigure is the corresponding iteration number)

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