

# A level-set based approach for the path planning of human evacuations in contaminated indoor environments

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**Abstract**—This work considers a level-set based method for path planning of human evacuation in a hazardous indoor environment. The algorithm developed examines the accumulated inhalation of carbon monoxide over a generated path and determines the evacuees survivability. The accumulated inhalation of a hazardous substance is calculated via the line integral of the substance concentration over the selected path. The candidate paths are generated by calculating the constant angle paths towards the exit if the critical level-set of the gas concentration has not been reached. Once the path overlaps with the selected level set, a new set of angles is calculated so that the path follows the level-set. Simulations of both spatially and spatiotemporally varying fields of the hazardous gas are examined.

## I. INTRODUCTION

One of the most common reasons for fatalities during a fire is Carbon Monoxide (CO) poisoning. When inhaled, CO molecules bind with hemoglobin forming carboxyhemoglobin which has a lower oxygen-carrying capacity, [1]. Long term exposure is also dangerous even at levels as low as 100ppm. During a fire, however, concentrations can reach as much as 4000ppm which leads to acute poisoning and unconsciousness in a few minutes, [2]. This is even more acute when a person is breathing faster which occurs during an evacuation because of the running and the carbon dioxide ( $CO_2$ ) present from the fire, [3].

Many complex physiological aspects such as the ability to escape are important in these situations. For a human, top speed and acceleration depend on many factors such as impulse and energy applied at each step, [4], [5]. Higher peak force and impulse mean higher acceleration, [6], [7]. Although these factors are acknowledged as important for a full dynamic model, we can use the kinematic model instead since the top speed is reached in negligible time compared to the total evacuation time. Additional models may examine the interactions between multiple evacuees, and either a particle model with collision or a macroscopic model can be considered, [8], [9].

During evacuations in indoor environments such as commercial, educational, civic and government buildings, casualty increases can be attributed to chaotic evacuation procedures. This is due to evacuees being unaware of the presence and location of harmful substances and being unable to locate the exits. According to extensive research, see [10] and references therein, exit signs are not easily located during

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a fire; in fact only 8% of fire survivors remember seeing the exit signs. Additionally evacuees usually exhibit herd behavior and follow the mass even if the mass doesn't know the correct or safest path. Furthermore serious injuries might be caused due to a stampede or people getting pressed against walls due to flight instincts, [11], [12]. Disorientation and the effects of the harmful substances only serve to amplify the problem as CO poisoning can lead to dizziness and unconsciousness.

It has been shown from research and modelling that humans during evacuations are not efficient, [13]. Because of flight instincts to run away from fire, humans will run faster than the optimum evacuation speed. This will cause confusion and a blockade at narrower exits when there is a significant amount of people trying to exit. This can lead to injuries that are not directly related to the fire because of pushing and panicking, [12]. As people push, more panic is created that leads to more pushing creating a vicious circle. The effect can not be attributed to lack of escape routes but rather it is due to people not using them right as they tend to follow the herd (mass behavior). Dynamic crowd emergency exit selection research has been conducted for fire models but do not take into account the physical models of harmful chemicals like CO, [14], [15].

Another significant physiological factor is the breathing rate. CO poisoning is accelerated by faster breathing and exercise. Because of this and the panic created, *hyperventilation* can occur, [16]. Hyperventilation can cause a person's breathing rate to rise as high as 40 times per minute. It can occur when the amount of CO in the body rises and in an attempt to decrease it, the body begins hyperventilation, [17]. This is very serious and dangerous as it can cause *hypocapnia* and *respiratory alkalosis*, that can lead to dizziness, fainting and seizures, [18]. During a fire the breathing rate of evacuees can increase because of the high  $CO_2$  levels. This faster breathing rate can accelerate CO poisoning.

Lastly we look at various ways for path planning. Rapidly-Exploring Random Trees (RRT) is considered. RRT uses a branch system to create a path from a known origin to a known goal, [19], [20]. The algorithm uses a known origin and creates a path in a random direction and known distance. The process is repeated until an obstacle interrupts the path. The algorithm repeats itself until a path can be created to the goal. Certain variations have been suggested and implemented such as speeding up the process by running multiple trees simultaneously with one starting from the origin and another starting from the goal, [21]. This works in an evacuation procedure with a known environment as there

is no time for an algorithm that has to learn the environment first. To ensure the survival of the evacuee another path planning method is the level-set approach in which instead of being constrained only by the environment we also take into consideration the hazardous field in order for the total amount of CO inhaled to decrease. We therefore generate a path in order to follow a certain level-set of the CO field. The value chosen for the level-set has to be so that the path length is not increased significantly in comparison with a straight line, thus the evacuee will exit the building as quickly as possible, [22] [23].

The goal is to increase the survivability of the evacuee by ensuring that the impact of the above factors that can affect this negatively are reduced or removed entirely. Such an example would be the psychological process involved. Although panic cannot be removed from a person, we can reduce the number of decisions to be made in order to reduce the effect of panic. Additionally any environmental effects can be limited by ensuring a quicker evacuation along a safer path so that both the symptoms from CO as well as low visibility due to fires do not come into play. To achieve this, this paper will examine the relation between the available paths and CO and its effects, both instantaneous and accumulated, and attempt to limit them as much as possible. More specifically a level set method will be applied in order to ensure that the instantaneous concentration does not exceed certain dangerous levels. We will consider both known stationary and time-varying spatial fields of CO inside large buildings. The algorithm will evaluate the paths and determine the safest one. To reach that decision, physiological data will be used in order to calculate the CO inhaled. At this stage a single evacuee will be evaluated but the algorithm will be expanded to a multi-agent problem with collisions and interaction.

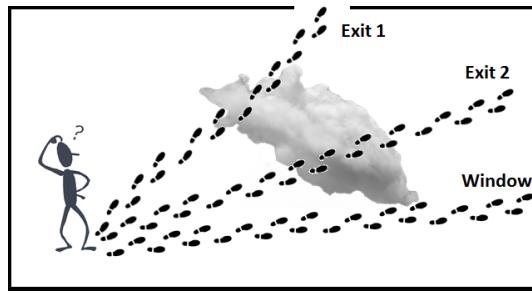


Fig. 1. Deciding which exit to use under the presence of hazardous plume.

Therefore, the contributions of this paper are the

- 1) establishment of a distributed parameter system framework for the mathematical modeling of the indoor environment CO concentration,
- 2) design of a time-varying angle path planning that is dependent on *both* the current (instantaneous) value of the CO field *and* the accumulated amount inhaled by each evacuee,
- 3) presentation of an optimization for the admissible evacuation routes that guarantee the accumulated CO

inhaled is below harmful levels, ensuring the evacuee's survival.

Figure 1 captures in a simple way the contribution of this work whereby the evacuation dilemma is which exit to take. While under normal conditions one would always choose the obstacle-free and minimum-distance path to escape exit, the incorporation of the accumulated CO effects adds a new dimension to the path planning problem, as one now must choose the path that results in the projected amount of CO inhaled be below the allowable threshold needed for survival.

A brief description of the mathematical framework for the spatial evolution of the CO concentration field in an indoor environment is presented in Section II along with the kinematic equations of motion of a human during an evacuation. The proposed constant-angle and modified (level set based) varying-angle path evacuation policies that are dictated by both the accumulated amount of CO inhaled and the instantaneous level of CO are presented in Section III. The proposed level-set based path planning is demonstrated via numerical studies of an evacuee in a typical office building floor in Section IV. Section V provides concluding remarks along with future work by the authors.

## II. MATHEMATICAL FRAMEWORK AND MODELING

The spatial variability of the concentration of a hazardous substance, such as CO, is described by a field which is modeled by a spatially or a spatiotemporally varying process. Such processes are mathematically modelled by Poisson-type elliptic partial differential equations (PDEs) or advection-diffusion PDEs. Often one has to consider multispecies transport models with reactions [24] and this is especially the case for indoor environments with fire and/or smoke. However, as a starting point we consider a single-species.

For a spatial domain  $\Omega = [0, L_x] \times [0, L_y] \times [0, L_z]$  representing an enclosed indoor environment, the former is

$$\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2} = f \quad (1)$$

where  $c(x, y, z)$  is the concentration at coordinates  $(x, y, z) \in \Omega$  and  $f(x, y, z)$  is the source, and furnished with the appropriate boundary conditions. Mixed boundary conditions can be assumed for (1): denoting the part of the boundary where Dirichlet conditions are imposed by  $\Gamma_D$  and the part of the boundary where Neumann conditions are imposed by  $\Gamma_N$ , then we have  $\partial\Omega = \Gamma_D \cup \Gamma_N$ . For the latter, we have

$$\frac{\partial c}{\partial t} = \nabla \cdot (D \nabla c) - \nabla \cdot (uc) + f, \quad (2)$$

where  $c(t, x, y, z)$  is the time-varying concentration,  $D$  is the diffusivity,  $u$  is the velocity and  $f(t, x, y, z)$  is the source, and with inhomogeneous boundary conditions prescribed in different parts of  $\partial\Omega$  as in the Poisson PDE case.

For the specific application of an indoor environment, the region of interest is given by a slice of the rectangular domain described by the  $xy$  plane and the  $z$  axis in the interval  $[z_l, z_h]$ , thus giving the effective spatial domain to be  $\Omega_e = [0, L_x] \times [0, L_y] \times [z_l, z_h]$ , see Figure 2.

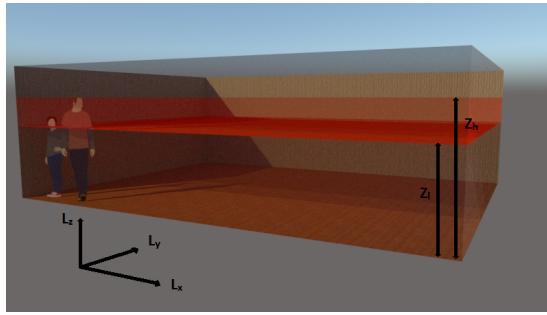


Fig. 2. Typical indoor environment and effective domain  $\Omega_e$  dictated by human height variability.

The reason is that we are interested in the height of the spatial domain that corresponds to the height of a person's face with  $z_l$  representing the average height of a shorter person and  $z_h$  representing the average height of a taller person. Within this effective spatial domain  $\Omega_e$ , we can further assume that the vertical variation of the concentration  $c$  is smaller than the variation in the  $x$  and  $y$  directions and thus assume an axisymmetry in the vertical direction. In this case, the concentration function will be given by  $c(t, x, y)$  or  $c(x, y)$ . Towards this, we make the following assumptions.

*Assumption 1 (effective domain):* The effective domain is

$$\Omega_e = [0, L_x] \times [0, L_y] \times [z_l, z_h],$$

where  $z_l$  denotes the average height of a short person taken to be one standard deviation below the average adult height and  $z_h$  represents the average height of a tall person, taken to be one standard deviation above the average adult height.

*Assumption 2 (axisymmetry):* The variability of the concentration in the vertical direction within  $\Omega_e$  is negligible compared to the variability in the  $x$  and  $y$  directions and hence the concentration can be given by the 2D solution to either (1) or (2). The concentration is then

$$\forall z \in [z_l, z_h], \quad \begin{cases} c(t, x, y, z) \approx c(t, x, y) \\ c(x, y, z) \approx c(x, y) \end{cases}$$

Additionally, we will be making the following assumption on the full knowledge of the state of the process as given by either (1) or (2) for now. In a future study, we will be considering the real-time estimation of the spatial field using the techniques developed by the first author, [25], [26], [27].

*Assumption 3 (full process information):* Full state information of either (1) or (2) is available to the user.

We begin by assuming that there is a single evacuee in an indoor environment and that the equations below and which are based on the simple kinematic model [28] describe the motion within the horizontal plane of  $\Omega_e$

$$\begin{aligned} \dot{x}(t) &= v(t) \cos(\theta), & x(0) &= x_0, \\ \dot{y}(t) &= v(t) \sin(\theta), & y(0) &= y_0, \end{aligned} \quad (3)$$

Since no interactions with other evacuees are considered, then (3) does not need to include the interactions due to collisions, [8]. In the above, the constant angle  $\theta$  is the angle between the direction of motion and the  $x$ -axis of the reference frame. The above describe the case where there are no obstacles and in this case, the path planning reduces

to simply choosing the angle  $\theta$ . The speed of the evacuee is denoted by  $v(t)$  and is assumed constant in time.

*Assumption 4:* The evacuee speed in (3) is constant.

For a given path described by a constant angle  $\theta$ , the solution to (3) provides the location (coordinates)  $(x(t), y(t))$  of the evacuee with respect to the origin. To compute the total amount of CO inhaled for a particular path, one must compute the line integral of the concentration along the chosen path. Denote by  $\mathbf{r}(t) = (x(t), y(t))$  the path, then the line integral along the path defined by  $\theta$

$$J(\theta) = \int_{\theta} c(\mathbf{r}) ds, \quad (4)$$

provides the accumulated amount of the hazardous substance inhaled by the evacuee. However, this does not take into account the breathing pattern of an evacuee. Additional modelling assumptions include the breathing rate of a human under duress. This is estimated to be around 20 breaths per minute with equal length of inhaling and exhaling. Furthermore, we need to model CO poisoning. This is something that is not standard from person to person. It is also affected by a persons breathing rate and whether they are performing a certain activity that increases heart rate and blood flow. Empirical data given in Table I can be used to calculate a safe limit in total CO inhalation over a given path. From

Carbon Monoxide Poisoning	
Concentration	Effects
3200 ppm	Headache, dizziness and nausea in five to ten minutes. Death in 30 minutes.
6400 ppm	Headache and dizziness in one to two minutes. Convulsions, respiratory arrest and death in less than 20 minutes.
12800 ppm	Unconsciousness after 2-3 breaths. Death in less than 3 minutes.

TABLE I  
QUANTITATIVE EFFECTS OF CARBON MONOXIDE POISONING.

these data, it can be concluded that the safe limit in total CO inhalation,  $C_{\text{thresh}}$  is 25,000 ppm for peak concentrations over 2,000 ppm and 64,000 for peak concentrations under 2,000 ppm. This limit includes a safety factor of 5 as we wanted to lower the probability of a person experiencing dizziness or nausea. Using these realistic physiological data for the total CO inhaled by the evacuee, the line integral over the path travelled as given by (3) can be used with a modification. To model the breathing of a person, (4) is divided by two as a person would only inhale on an average half of the time. Thus the total CO inhaled is now given by

$$J(\theta) = \frac{1}{2} \int_{\theta} c(\mathbf{r}) ds. \quad (5)$$

### III. LEVEL-SET BASED EVACUATION IN PRESENCE OF HAZARDOUS FIELDS IN INDOOR ENVIRONMENTS

The problem at hand is to ensure that each evacuee with initial coordinates  $(x_i(0), y_i(0))$ ,  $i = 1, \dots, n$ , follows an escape path that obeys (3) and which ensures that the accumulated CO inhaled, as predicted by (5), is below the

allowable limit. This ensures that each evacuee reaches any of the escape exits conscious.

We define  $\Theta$  as the set of admissible paths that satisfy the motion constraints. Then the optimization problem can be stated as the selection of the path  $\theta(t) \in \Theta$  that ensures that the CO inhaled, as predicted by (5), is less than the  $C_{\text{thresh}}$ . **Problem statement:** Using the level sets to generate a time varying angle  $\theta \in \Theta$  and (5) to calculate the inhalation of CO, design a path that both minimizes the CO inhaled and keeps the total amount less than  $C_{\text{thresh}}$ .

$$\text{optimization : } \begin{cases} \underset{\theta(t) \in \Theta}{\text{maximize}} & C_{\text{thresh}} - J(\theta) \\ \text{subject to} & \begin{cases} \dot{x}(t) = v \cos(\theta) \\ \dot{y}(t) = v \sin(\theta) \end{cases} \end{cases}$$

The *constant-angle paths* for each evacuee are defined by

$$\Theta_i = \left\{ \theta_{ij} : \tan(\theta_{ij}) = \frac{Y_j - y_i(0)}{X_j - x_i(0)}, \forall j = 1, \dots, N \right\}. \quad (6)$$

where the initial position is  $(x_i(0), y_i(0))$ ,  $i = 1, \dots, n$  and  $(X_j, Y_j)$  are the coordinates for each emergency exit.

### A. Algorithms

The algorithms presented here assume an empty floor with no obstacles such as cubicles, divisions, furniture, incapacitated humans, etc. While a path to an exit is unobstructed, and a straight line, the effects of CO inhalation will act as obstacles. The input to the proposed algorithm is the initial location of each evacuee  $(x_i(0), y_i(0))$ ,  $i = 1, \dots, n$ , the coordinates of each of the exits and the CO concentration. The value of  $v = 7\text{m/s}$  is used as the average top speed of a human. The algorithm will predict the total accumulated CO inhaled for each admissible path and will subsequently select the smallest one that results in total accumulated inhalation that is below the allowable threshold necessary for survival. The sampling rate corresponding to the amount of CO inhaled is calculated every second since the breathing rate of humans is on a timescale of seconds.

#### Algorithm 1 Evacuation based on a *constant-angle path* in indoor environment

- 1: For each evacuee estimate the maximum velocity
- 2: Using (6) generate the admissible path set for each evacuee with coordinates  $(x_i(0), y_i(0))$ ,  $i = 1, \dots, n$
- 3: Using (5), calculate the inhalation of CO for each  $i$ ,  $i = 1, \dots, n$  over all admissible paths
- 4: Generate the subset of  $\Theta_{ij}$  for which  $J_i(\theta_{ij}) < C_{\text{thresh}}$ ,  $j = 1, \dots, N$ , i.e.

$$\Theta_i^{\text{opt}} = \left\{ \theta_{ij} \in \Theta_i : J_i(\theta_{ij}) < C_{\text{thresh}} \right\} \quad (7)$$

- 5: The smallest  $\Theta_i^{\text{opt}}$  below  $C_{\text{thresh}}$  is the optimal path, i.e.

$$\theta_i^{\text{opt}} = \arg \min_{\theta \in \Theta_i^{\text{opt}}} J_i(\theta) \quad (8)$$

The above constant-angle path may not be feasible if  $\Theta_i^{\text{opt}} = \emptyset$ , prompting for a varying-angle path.

To follow the level-set needed to ensure the highest chance of survival, in this case 2,000 ppm, new angles need to be

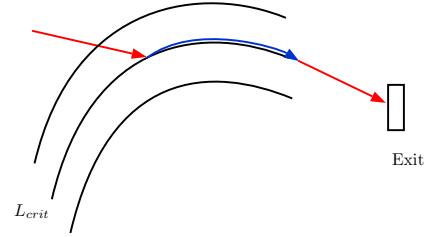


Fig. 3. Level set-based varying-angle path.

generated at every time step instead of a constant angle. To do so, we follow Algorithm 2 as depicted in Figure 3. The idea behind this, is to initially follow a constant-angle path and when the critical level set  $L_m = \{(x, y) : c(x, y) = m\}$  is encountered, march along this level set and when the line of sight pointing to an exit is tangent to the level set, get off and follow the new constant-angle path towards the exit.

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#### Algorithm 2 Evacuation based on a *varying-angle path* in indoor environment: level-set tracking

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- 1: Use knowledge of CO to calculate the instantaneous peak concentration.
- 2: If peak concentration is less than 2,000 ppm then the angle  $\theta$  is the constant angle path from the current location to the exit as calculated in (6).
- 3: If peak concentration is over 2,000 ppm, calculate the gradient of the CO field  $\nabla c$  at current location.
- 4: Using  $\nabla c$ , a new  $\theta_{\text{level}}$  is generated at every  $k$  iteration so that the path follows the critical level set  $L_{2,000}$ ,

$$\theta_k = \arctan \left( \frac{\nabla_x c}{\nabla_y c} \right) \quad (9)$$

- 5: At every iteration  $k$  of the algorithm the constant angle path is generated. If the instantaneous peak concentration at the adjacent point is less than 2,000 ppm then the constant angle path is followed.

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## IV. NUMERICAL RESULTS

The two algorithms are implemented for three different case studies. Over the rectangular domain  $\Omega = [0, 100] \times [0, 30]\text{m}$ , we investigate the cases of (1) a steady field, (2) an unsteady field and finally (3) a steady field where both constant-angle paths and the level-set based varying-angle paths fail to save the evacuee who instead could take a longer path to the exit that guarantee their survival. Comparison of the level-set based varying-angle approach and the constant-angle approach is provided.

### A. Case 1: Steady CO field

For a simulation where the source of the CO field is located at  $(\mu_x, \mu_y)$ , solving (1) gives us

$$c(x, y) = 4100e^{-\left( \frac{(x-\mu_x)^2}{2\sigma_x^2} + \frac{(y-\mu_y)^2}{2\sigma_y^2} \right)} \quad (10)$$

where the standard deviations are  $\sigma_x = 10$ ,  $\sigma_y = \sqrt{30}$ . For our simulation the source is located at  $(60, 10)$ .

Three escape exits were selected at (100,10), (100,30) and (60,30). For an evacuee initially at  $(x_1(0), y_1(0)) = (35, 5)$ , an initial constant-angle path is generated. If the level-set condition is met, then the angle  $\theta$  changes accordingly. Figure 4 depicts the constant-angle path for all three exits. It is observed that the first two exits are not viable with an evacuee starting at (35,5) and using a constant-angle path. Specifically the constant-angle paths for the first two exits predict a total CO inhalation of over 35,000 ppm which is above the safe limit. The evacuation path that employs a constant-angle path for the third exit proves to be a safe one for the evacuee. To ensure survival for any of the three exits, the level-set of 2,000 ppm is selected and followed as seen in Figure 5. The optimal path of the route is shown in green and yellow lines for the first and second exit paths, respectively. The total CO inhaled for the two constant-angle paths was estimated at 43,511 ppm and 35,339 ppm respectively. The varying-angle path to be followed reduces those two numbers to 32,095 ppm and 25,899 ppm, respectively. However, since the peak value is below the threshold of 2,000 ppm these are not dangerous levels.

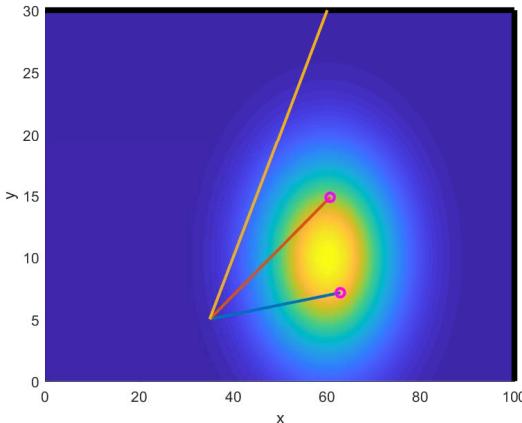


Fig. 4. Case1: Stationary field with constant-angle paths.

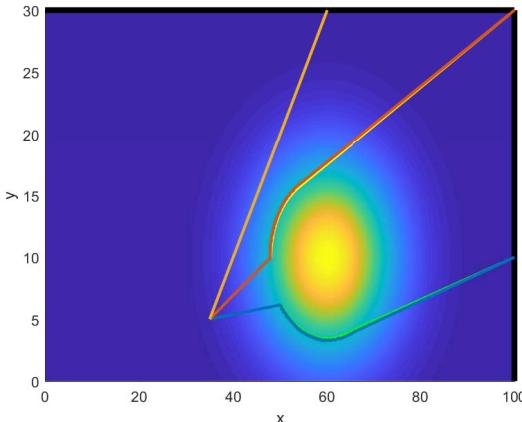


Fig. 5. Case 1: Stationary field with level-set based variable-angle path.

### B. Case 2: Unsteady CO field

For the time varying (unsteady) CO field in (3), we assume a variation in the location of the source or the peak of the CO field. This can be due to a gust of wind or a change in the morphology of the fire. The solution (10) remains the same with the only modification being that the means are described by  $\mu_x(t) = 5 \cos(9t - \frac{\pi}{2}) + \mu_{x0}$ ,  $\mu_y(t) = 5 \sin(9t - \frac{\pi}{2}) + \mu_{y0}$ . The exits of the evacuees are the same as in the first case. The CO source is initially centered at  $(\mu_{x0}, \mu_{y0}) = (60, 15)$ . The motion of the source is described by the parametric equations of a circle for the means  $(\mu_x(t), \mu_y(t))$ . The motion of the mean  $(\mu_x(t), \mu_y(t))$  is depicted in Figure 6 by the black line.

The evacuee starts at  $(x_1(0), y_1(0)) = (20, 0)$ . The second path does not ensure survivability if the constant-angle path is selected. Applying the level-set based approach though as seen in Figure 6 the evacuee survives for all three exits in the same way they did in the first case thus proving the algorithm works even for a time varying field.

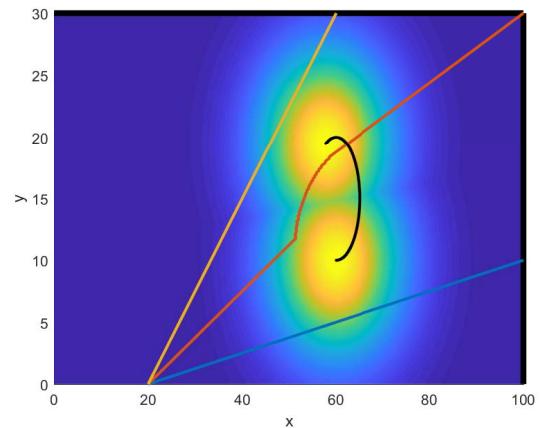


Fig. 6. Case 2: Unsteady field with level-set based variable-angle path.

### C. Case 3: Extreme Scenario

In the third case we examine the extreme situation where both the constant-angle paths in Figure 7, and the level-set based varying-angle paths in Figure 8, fail to provide a survivable path. A different proposed path is then examined which involves the evacuee following the wall furthest away from the source of the hazardous field. Although this means that the time for evacuation is significantly increased, the CO inhaled is at 17,733 ppm, a value significantly less than the limit. The flight time along the wall required 19s whereas a constant-angle path, unaffected by the CO concentration, would have taken 12s.

## V. CONCLUSIONS

An abstract framework that brought forth the interdependence of an escape path and the survivability of an evacuee in a contaminated indoor environment was presented. A given path affects the accumulated amount of CO and subsequently defines the survivability of an evacuee. The amount of accumulated CO inhaled by an evacuee was calculated via

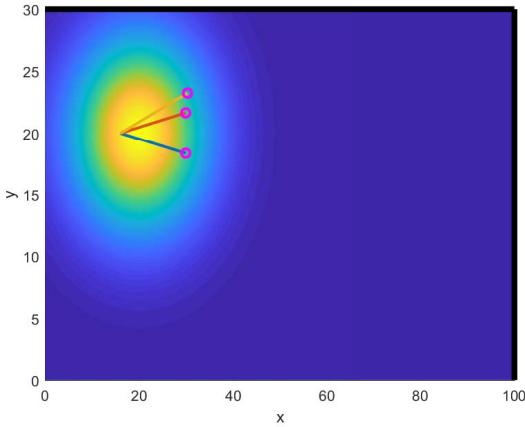


Fig. 7. Case 3: Extreme case with constant-angle paths.

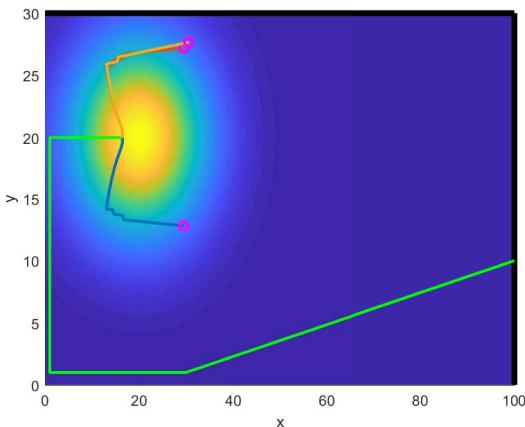


Fig. 8. Case 3: Extreme case with level-set based variable-angle path and additional path following the wall.

the line integral of the CO concentration over the selected path. The proposed scheme examined the modification of constant angle escape paths and if the projected amount of the accumulated CO exceeded the threshold for survivability, then it altered the escape path by marching along level sets of the CO distribution. Residence along a single level-set was considered whereby the constant-angle path was initially followed and then a curved path along a critical level set of concentration was followed. When the line of sight toward an exit was tangent to a level set, then the path switched to a constant-angle path terminating at the exit. Numerical simulations provided an insight on the level-set path planning and which provided evacuation paths that ensure the survivability of evacuees. Direct extension involve a CO-dependent evacuee speed and a multi level set modification in the varying-angle path planning.

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