Safe multi-cluster UAV continuum deformation coordination

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A R T I C L E   I N F O

Article history:
Received 29 March 2019
Received in revised form 2 May 2019
Accepted 2 May 2019
Available online 10 May 2019

A B S T R A C T

This paper proposes a paradigm for coordination of multiple unmanned aerial vehicle (UAV) clusters in a shared motion space. UAVs are arranged in a finite number of teams each bounded by a leading triangle. Collective motion of each UAV cluster is managed by a continuum deformation defined by three leaders at the vertices of a leading triangle and followers contained within this triangle. Each triangular cluster can deform substantially to support maneuverability in constrained spaces. This paper specifies necessary conditions to guarantee obstacle avoidance as well as collision avoidance within and across all clusters operating in a shared motion space. Given initial and target configurations, an existing planner (A*) identifies the shortest coordinated leader UAV paths from initial to final configuration in a manner that satisfies safety constraints. An illustrative simulation case study is presented. Continuum deformation containment offers scalability in collision-free UAV motion planning not previously realized in the detect-and-avoid literature. The proposed multi-cluster coordination protocol also extends previous cooperative control to address detect-and-avoid (DAA) multiple cooperative teams with different destinations.

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1. Introduction

Multi-agent system coordination is an active research area. Formation and cooperative control in a multi-agent system can enhance resilience to failure [1], improve efficiency, and reduce mission cost [2]. Applications include surveillance [3], air traffic management [4], formation flight [5–7], and connected vehicle control [8], and cooperative payload transport [9,10]. This paper studies optimal coordination of many UAVs, clustered into distinct teams, flying in a shared airspace. The paper treats agent optimal coordination as a multi-cluster continuum deformation problem and manages team path planning complexity by abstracting each team to a triangular geometry defined by three leaders.

1.1. Related work

Virtual structure, consensus, containment control, and continuum deformation are available methods for agent coordination in a 3D motion space. A virtual structure centrally coordinates agents; each agent’s desired position is defined by a reference position vector and a relative displacement vector with respect to this reference [11]. If the agents’ relative distances from the reference position remain constant, the multi-agent system can be treated as a rigid body [12]. A flexible virtual structure formation control has also been proposed [13].

Consensus [14–16], containment [17,18], and continuum deformation [19–23] are decentralized multi-agent system (MAS) coordination approaches. Consensus is the most common approach for MAS formation and cooperative control. Both leaderless [14] and leader-based consensus [16,24] approaches have been proposed. Consensus coordination under switching and fixed communication topologies is studied in Refs. [25–27]. Stability of consensus in the presence of communication delays is analyzed in Refs. [28,29]. Finite time consensus control of multi-agent systems is studied in Refs. [30,31].

Similar to consensus, containment control is a decentralized coordination approach. Under containment control, leaders move independently and guide overall team motion. Follower agents communicate with in-neighbor agents to coordinate motions through local communication [17,18]. Containment control under fixed and switching communication topologies is studied in [32]. MAS containment control, guided by stationary and moving leaders, is studied in [33]. Retarded containment control stability is analyzed in [34]. Finite-time MAS containment control [35] and containment control of heterogeneous MAS [36] have also been studied.

Continuum deformation is inspired by the principles of continuum mechanics. Under continuum deformation, inter-agent distance can change over time while inter-agent collision avoidance can be guaranteed [37,38]. Leader-follower formation control via continuum deformation was developed in [21]. Ref. [21] formulates an n-dimensional (n = 1, 2, 3) homogeneous transformation based
on trajectories of \( n + 1 \) leaders forming an \( n \)-dimensional polytope in a 3D motion space. Ref. [21] shows how follower agents can acquire desired trajectories through local communication. Decentralized continuum deformation coordination using area preservation and alignment strategy are demonstrated in [20] and [23]; Ref. [19] analyzes stability of continuum deformation coordination in the presence of communication delay. Sufficient conditions for inter-agent collision avoidance are defined in [19,20] while [22] formulates continuum deformation coordination under switching communication topologies.

Robot and vehicle motion planning has been widely studied. Optimal graph search planning methods include dynamic programming [39] which gives an admissible heuristic can improve search-space ordering in A∗ [40]. Rapidly-expanding Random Trees (RRT) [41] were proposed to offer a real-time graph search method applicable for path planning in known and unknown environments. Model predictive control (MPC) [42] is a well-known approach for trajectory (control vector) optimization that accounts for motion costs and vehicle dynamics constraints [43]. Researchers have proposed centralized [44–46] and decentralized [47–49] approaches for multi-agent path planning. Ref. [44] applies A∗ for first responder multi-agent team planning in a cluttered environment while Ref. [45] applies mixed integer programming. Multi-agent path planning using decentralized RRT is studied in Ref. [48]. Decentralized coordination of mini drones is investigated in Ref. [49]. Furthermore, Ref. [50] proposes a digital pheromone approach for autonomous coordination of a UAV team.

1.2. Contribution and outlines

This paper extends our previous contributions in single-cluster continuum deformation to support multi-cluster continuum deformation coordination over many UAVs. To manage coordination computational complexity, UAVs are clustered into multiple teams, where each team or cluster forms a bounded triangular geometry that evolves internally as particles of a deformable body or continuum. Given initial and target configurations for each UAV cluster, optimal team leader paths are planned to meet waypoint objectives and satisfy safety constraints. A feedback linearization controller performs optimal trajectory tracking. Compared to related work, this paper offers the following contributions:

- An innovative hierarchical coordination strategy is proposed to manage computational and coordination complexity. At the top level cluster leader UAV trajectories are optimized and de-conflicted. The remaining follower UAVs are contained within their cluster’s leading triangle thus pose no collision risk to other teams. Centralized (top-level) and decentralized (cluster-level) approaches are combined to optimize multi-cluster continuum deformation in a shared motion space. Collective motion of many UAVs, clustered into a finite number of groups, can be optimized with modest computation cost.

- Formal mathematical analysis assures UAV cluster containment and privacy. Cluster containment assures no UAV leaves the cluster over the entire collective motion period. Cluster privacy guarantees that a cluster is not trespassed (entered) by any agent from another cluster over the entire collective motion period. Continuum deformation is applied to assure inter-agent collision avoidance for any two agents within the same cluster.

- To the best of our knowledge, this is the first paper studying collective motion optimization of deformable clusters in a shared motion space. Cluster deformability enables flight through narrow passages and in constrained airspace volumes while allowing the cluster to resume an optimal geometric configuration for flight through open airspace. The paper formally verifies safety and mathematically formulates inter-agent collision avoidance conditions.

The paper is organized as follows. Preliminaries on graph theory, coordinate systems, and continuum deformation are presented in Section 2. Cluster containment and privacy are mathematically formulated in Section 3. The multi-cluster optimization problem is defined in Section 4, followed by formulation of a path-planning optimization strategy in Section 5. UAV dynamics and control are presented in Sections 6. Simulation case study results in Section 7 are followed by concluding remarks in Section 8.

2. Preliminaries

2.1. Coordinate systems

We define a ground coordinate system with bases \( \hat{e}_1, \hat{e}_2, \hat{e}_3 \). Bases of the ground coordinate system are fixed in an inertial reference. Each UAV has its own local or body coordinate system. Bases of the body coordinate system of UAV \( i \) are denoted by \( \hat{b}_i^1, \hat{b}_i^2, \hat{b}_i^3 \), and \( \hat{k}_i^b \), where subscript \( i \in \Omega^V \) denotes the quadcopter index number and superscript \( j \in \Omega^C \) assigns the cluster index number. Cluster identification numbers are defined by the set \( \Omega^C = \{1, \ldots, m\} \). Using a 3 − 2 − 1 Euler angle rotation, \( \hat{b}_i^j, \hat{k}_i^j, \) and \( \hat{k}_i^b \) are related to \( \hat{e}_1, \hat{e}_2, \hat{e}_3 \) by

\[
j \in \Omega^C, \; i \in \Omega^V, \; \begin{bmatrix} \hat{j}^i_{b,i} \\ \hat{j}^i_{k,i} \\ \hat{k}^i_{b,i} \end{bmatrix} = R^i_{\phi^j_{\psi^j_i}} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix} \tag{1a}
\]

\[
R^i_{\phi^j_{\psi^j_i}} := \begin{bmatrix}
 C^i_{\phi^j} C^i_{\phi^j} & C^i_{\theta^j} C^i_{\psi^j} & -S^i_{\phi^j} \\
 S^i_{\phi^j} C^i_{\phi^j} & C^i_{\theta^j} S^i_{\psi^j} & S^i_{\phi^j} \\
 S^i_{\phi^j} S^i_{\phi^j} & C^i_{\theta^j} S^i_{\psi^j} & S^i_{\phi^j} C^i_{\psi^j} \\
 C^i_{\phi^j} S^i_{\phi^j} + S^i_{\psi^j} S^i_{\phi^j} & C^i_{\theta^j} S^i_{\psi^j} & S^i_{\phi^j} C^i_{\psi^j} \\
 C^i_{\phi^j} S^i_{\phi^j} - S^i_{\psi^j} S^i_{\phi^j} & C^i_{\theta^j} S^i_{\psi^j} & S^i_{\phi^j} C^i_{\psi^j} \\
 C^i_{\phi^j} S^i_{\phi^j} & C^i_{\theta^j} S^i_{\psi^j} & C^i_{\phi^j} C^i_{\psi^j} \\
\end{bmatrix} \tag{1b}
\]

where \( \phi^j_i, \theta^j_i, \) and \( \psi^j_i \) are the roll, pitch, and yaw angles of UAV \( i \in \Omega^V \). Also, \( C^i_{\phi^j} \) and \( S^i_{\phi^j} \) stand for \( \cos(\phi^j) \) and \( \sin(\phi^j) \), respectively.

2.2. Position terminology

In this paper, the actual position of UAV \( i \in \Omega^V \) is given by

\[
j \in \Omega^C, \; i \in \Omega^V, \; \hat{r}^i_j = x^i_j \hat{e}_1 + y^i_j \hat{e}_2 + z^i_j \hat{e}_3. \tag{2}
\]

The global desired position of agent \( i \in \Omega^V \) is denoted by

\[
j \in \Omega^C, \; i \in \Omega^V, \; \hat{r}^i_{\Omega^V} = x^i_{\Omega^V} \hat{e}_1 + y^i_{\Omega^V} \hat{e}_2 + z^i_{\Omega^V} \hat{e}_3. \tag{3}
\]

In Section 2.4, global desired coordination, treated as continuum deformation, is formulated. This paper assumes that the \( z \) components of the global desired positions of all UAVs are the same:

\[
\forall j \in \Omega^C, \; \forall i \in \Omega^V, \; z^i_{\Omega^V} = z^j_{\Omega^V}. \tag{4}
\]

Local desired position, denoted by \( \hat{r}^i_{\Omega^V} (i \in \Omega^V) \), is defined:

\[
j \in \Omega^C, \; i \in \Omega^V, \; \hat{r}^i_{\Omega^V} = \sum_{i \in \Omega^V} w^i_{\Omega^V} \hat{r}^i_j \tag{4}
\]

where communication weight \( w^i_{\Omega^V} \) is positive and \( \sum_{i \in \Omega^V} w^i_{\Omega^V} = 1 \). In this paper, we assume communication weights are constant and consistent with UAV initial positions. Communication weight characteristic equations are obtained in Section 2.5.
Initial position of agent \(i \in \mathcal{V}_j\) is denoted by
\[
j \in \Omega_{CL}, \quad i \in \mathcal{V}_j, \quad \mathbf{r}_{i,0}^j = x_{i,0}^j \hat{\mathbf{e}}_1 + y_{i,0}^j \hat{\mathbf{e}}_2 + z_{i,0}^j \hat{\mathbf{e}}_3.
\] (5)

Target position of agent \(i \in \mathcal{V}_j\) is denoted by
\[
j \in \Omega_{CL}, \quad i \in \mathcal{V}_j, \quad \mathbf{r}_{i,F}^j = x_{i,F}^j \hat{\mathbf{e}}_1 + y_{i,F}^j \hat{\mathbf{e}}_2 + z_{i,F}^j \hat{\mathbf{e}}_3.
\] (6)

2.3. Definitions of shared airspace, no-flight zones (NFZ), and navigable (NAV) zones

Let \(\mathcal{M}\Omega \subset \mathbb{R}^2\) be a closed set defining a finite two-dimensional shared airspace. Let \(\Omega_{NFZ}\) be a set consisting of \(n_0\) convex polygons enclosing obstacles or No-Flight Zones. Mathematically speaking,
\[
\Omega_{NFZ} = \{O_1, \cdots, O_{n_0}\},
\] (7)
where \(O_i = \{o_{i,1}^j, \cdots, o_{i,n_0}^j\} \subset \mathcal{M}\Omega \quad (i = 1, \cdots, n_0)\) denotes the convex polygon \((i = 1, \cdots, n_0)\) defined as follows:
\[
i = 1, \cdots, n_0, \quad O_1 = \left\{ \sum_{k=1}^{p_i} \xi_k^j o_{k}^j \mid \xi_k^j \geq 0, \sum_{k=1}^{p_i} \xi_k^j = 1 \right\},
\] (8)
where
\[o_{k}^j = o_{k,x}^j \hat{\mathbf{e}}_1 + o_{k,y}^j \hat{\mathbf{e}}_2 \in \mathcal{M}\Omega\]
is the position of the vertex \(k\) of convex polygon \(O_i\) and \(\sum_{k=1}^{p_i} \xi_k^j \geq 0\) is an interior or boundary point of convex polygon \(O_i\). Navigable Zone
\[
\Omega_{NAV} = \mathcal{M}\Omega \setminus \Omega_{NFZ}
\] (9)
is an open set denoting an obstacle-free or navigable airspace. Notice that \(\Omega_{NFZ}\) defines the boundary of No-Flight Zone \(\Omega_{NFZ}\).

Shown in Fig. 1 is the schematic for a shared space containing three obstacles, denoted by \(O_1, O_2,\) and \(O_3\). Obstacle \(O_1\) is a quadrilateral \((p_1 = 4)\), obstacle \(O_2\) is a triangle \((p_2 = 3)\), and obstacle \(O_3\) is a pentagon \((p_3 = 5)\).

Motion space discretization: A uniform grid is distributed over the motion space \(\mathcal{M}\Omega\). Define distance increments \(\Delta x > 0\) and \(\Delta y > 0\), grid nodes are mathematically defined by
\[
\mathcal{M}\Omega = \{(x, y) \mid x = x_0 + k_x \Delta x, \quad y = y_0 + k_y \Delta y, \quad k_x, k_y = 0, 1, 2, \cdots \}.
\]

Key Assumption 1: This paper assumes that vertices of the polygons \(O_1\) through \(O_{n_0}\) are all positioned at the grid nodes defined by \(\mathcal{M}\Omega\). Mathematically speaking,
\[
\forall i \in \Omega_{NFZ}, \quad k = 1, \cdots, p_i, \quad o_{k}^j \in \mathcal{M}\Omega.
\]

2.4. Continuum deformation coordinate definition

Consider a group of \(N\) UAVs divided into \(m\) clusters. Let
\[
j = \Omega_{CL}, \quad \mathcal{V}_j = \{V_{j,1}, \cdots, V_{j,m}\}
\]
define local UAV index numbers in cluster \(j \in \Omega_{CL}\), where \(V_{j,1} = \{1, 2, 3\}\) define three leaders and \(V_{j,2} = \{4, \cdots, N_j\}\) define all followers in cluster \(j\). It is assumed that cluster \(j\) contains \(N_j\) UAVs \((\sum_{j=1}^{m} N_j = N)\). Note that each UAV is identified by a number \(j \in \Omega_{CL}\) and a local index number \(i \in \mathcal{V}_j\). The paper treats UAVs in cluster \(j\) as a finite number of particles of a deformable body or continuum.1 The desired coordination of cluster \(j\) is defined by a homogeneous transformation:
\[
t \geq t_0, \quad j \in \Omega_{CL}, \quad i \in \mathcal{V}_j, \quad \mathbf{r}_{i,HT}^j = \mathbf{Q}_j^i(t, t_0)\mathbf{r}_{i,0}^j + \mathbf{d}_j^i(t, t_0),
\] (10)
where \(t_0\) is the initial time and \(\mathbf{Q}_j^i \in \mathbb{R}^{3 \times 3}\) and \(\mathbf{d}_j^i \in \mathbb{R}^{3 \times 1}\) are the continuum deformation Jacobian matrix and rigid body displacement vector, respectively. Without loss of generality, we consider 2-D continuum deformation, where
\[
j \in \Omega_{CL}, \quad \mathbf{Q}^j = \begin{bmatrix} \mathbf{Q}_{ij}^j & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} Q_{1,1}^j & Q_{1,2}^j & 0 \\ Q_{2,1}^j & Q_{2,2}^j & 0 \\ 0 & 0 & 1 \end{bmatrix},
\] (11a)
\[
j \in \Omega_{CL}, \quad \mathbf{d}^j = \begin{bmatrix} d_1^j \\ d_2^j \\ 0 \end{bmatrix}.
\] (11b)

Elements of \(\mathbf{Q}_j^i \) (\(Q_{1,1}^j, Q_{1,2}^j, Q_{2,1}^j, Q_{2,2}^j\)) can be defined based on the global desired positions of the leaders [19]:
\[
\begin{bmatrix} Q_{1,1}^j(t) \\ Q_{1,2}^j(t) \\ Q_{2,1}^j(t) \\ Q_{2,2}^j(t) \end{bmatrix} = \begin{bmatrix} x_{1,0}^j & y_{1,0}^j & 0 & 0 & 1 & 0 \\ x_{2,0}^j & y_{2,0}^j & 0 & 0 & 1 & 0 \\ x_{3,0}^j & y_{3,0}^j & 0 & 0 & 1 & 0 \\ 0 & 0 & x_{1,0}^j & y_{1,0}^j & 0 & 1 \\ 0 & 0 & x_{2,0}^j & y_{2,0}^j & 0 & 1 \\ 0 & 0 & x_{3,0}^j & y_{3,0}^j & 0 & 1 \end{bmatrix},
\] (12)

Polar Decomposition of the Continuum Deformation Jacobian Matrix: Using polar decomposition, \(\mathbf{Q}_j^i\) can be expressed as
\[
j \in \Omega_{CL}, \quad \mathbf{Q}_j^i = \mathbf{R}_j^i \mathbf{U}_j^i
\] (13)
where \(\mathbf{U}_j^i\) is a positive definite (and symmetric) matrix and \(\mathbf{R}_j^i\) is an orthogonal matrix, i.e., \((\mathbf{R}_j^i)^T \mathbf{R}_j^i = \mathbf{I}_2\) where \(\mathbf{I}_2\) is the identity matrix. Eigenvalues of the matrix \(\mathbf{U}_j^i\) are positive and real and denoted by \(\lambda_1^j\) and \(\lambda_2^j\) \((0 < \lambda_1^j \leq \lambda_2^j)\).

Key Property of a Homogeneous Deformation: Let the three leaders of cluster \(j\) form a triangle at all times \(t\). Therefore,
\[
\forall t \geq t_0, \quad j \in \Omega_{CL}, \quad \text{Rank} \left[ \begin{bmatrix} r_{1,HT}^j - r_{1,HT}^i \\ r_{2,HT}^j - r_{2,HT}^i \\ r_{3,HT}^j - r_{3,HT}^i \end{bmatrix} \right] = 2.
\]
The triangle formed by the leaders of cluster \(j \in \Omega_{CL}\) is called leading triangle \(j\). Under a homogeneous deformation, \(x\) and \(y\) components of the global desired position of UAV \(i \in \mathcal{V}_j\) can be expressed as [19]
\footnote{1 A continuum is a continuous domain consisting of infinite number of particles with infinitesimal size [51].}
\[ j \in \Omega_{CL}, \ i \in V_f^j, \quad y_{i,HT}^j = \left[ \begin{array}{c} x_{i,HT}^j(t) \\ y_{i,HT}^j(t) \end{array} \right] = \sum_{l=1}^{3} \alpha_{i,l}^{j} \left[ \begin{array}{c} x_{l,HT}^j(t) \\ y_{l,HT}^j(t) \end{array} \right], \]

(14)

where \( \alpha_{i,1}^{j}, \ alpha_{i,2}^{j}, \) and \( \alpha_{i,3}^{j} \) are \textit{time-invariant} parameters and

\[ \alpha_{i,1}^{j} + \alpha_{i,2}^{j} + \alpha_{i,3}^{j} = 1. \]

(15)

Parameters \( \alpha_{i,1}^{j}, \ alpha_{i,2}^{j}, \) and \( \alpha_{i,3}^{j} \) are computed from the initial position of UAV \( i \) and the three leaders as follows [19]:

\[ j \in \Omega_{CL}, \ i \in V_f^j, \quad \left[ \begin{array}{ccc} x_{i,0}^j & x_{i,0}^j & x_{i,0}^j \\ y_{i,0} & y_{i,0} & y_{i,0} \end{array} \right] \left[ \begin{array}{ccc} \alpha_{i,1}^{j} & \alpha_{i,2}^{j} & \alpha_{i,3}^{j} \end{array} \right] = \left[ \begin{array}{c} x_{i,0}^j \\ y_{i,0} \end{array} \right]. \]

(16)

### 2.5. Continuum deformation acquisition through local communication

We assume that cluster \( j \)'s collective motion is guided by three UAV leaders moving independently. Leaders all communicate with each other and evolve in a centralized fashion. However, follower UAVs communicate with local UAVs to update their positions. Inter-agent communication among UAVs is defined by graph \( G^j = (V^j, E^j, W^j) \), where \( E^j \subset V^j \times V^j \) defines edges of the graph \( E^j \). We define

\[ j \in \Omega_{CL}, \ i \in V_f^j, \quad N^j_i = \{ (l, i) \in E^j | l \in V_f^j \} \]

as the in-neighbor set of follower UAV \( i \in V_f^j \). Communication of follower \( i \in V_f^j \) with in-neighbor UAV \( l \in N^j_i \) is assigned weights \( w_{i,l} \), where \( \sum_{l \in N^j_i} w_{i,l} = 1 \). Communication graph \( G^j \) can be expressed as \( G^j = \partial \Psi \cup \Psi \), where \( \partial \Psi \cup \Psi = \emptyset \). \( \partial \Psi \) defines the boundary of graph \( G^j \) and sub-graph \( \Psi = (V_f^j, E_f^j) \) is strongly connected.\(^2\) \( \Psi \) defines inter-agent communication for followers in cluster \( j \in \Omega_{CL} \), i.e., \( E_f^j \subset V_f^j \times V_f^j \) defines \( \Psi \) graph edges. Note that nodes of the boundary graph \( \partial \Psi \) represents cluster \( j \)'s leaders and nodes of sub-graph \( \Psi \) represent follower UAVs in cluster \( j \in \Omega_{CL} \).

**FL and FF Communication Matrices:** We define weight matrix \( W^j = [W_{i,l}^j] \in \mathbb{R}^{(N_f-3) \times N_f} \) with the \( i,l \) entry specified as follows:

\[ W_{i,l}^j = \begin{cases} w_{i,l}^j, & \text{if } (i+3,l) \in V_f^j \land l \in N^j_{i+3}, \\ -1, & \text{if } l = i+3, \\ 0, & \text{otherwise.} \end{cases} \]

(17)

Partitioning \( W^j \)

\[ W^j = \left[ \begin{array}{c} B^j \\ A^j \end{array} \right] \in \mathbb{R}^{(N_f-3) \times N_f}, \]

(18)

where matrices \( B^j \in \mathbb{R}^{(N_f-3) \times (N_f-3)} \) and \( A^j \in \mathbb{R}^{(N_f-3) \times (N_f-3)} \) are called Follower-Leader (or FL) communication matrix and Follower-Follower (or FF) communication matrix, respectively. If communication weight \( w_{i,j}^j \) is positive, matrix \( A^j \) is Hurwitz [19].

**Follower’s Communication Weights in a Continuum Deformation Coordination:** Followers’ communication weights are assigned based on UAV positions at initial time \( t_0 \). Let UAV \( i \in V_f^j \), initially positioned at \( r_{i,0}^j = x_{i,0}^j \tilde{e}_1 + y_{i,0}^j \tilde{e}_2 + z_{i,0}^j \tilde{e}_3 \), communicate with in-neighbor \( i_1, i_2, \) and \( i_3 \) \((i_1, i_2, i_3 \in N^j_i, \ i \in \Omega_{CL})\), initially positioned at \( r_{i_1,0}^j = x_{i_1,0}^j \tilde{e}_1 + y_{i_1,0}^j \tilde{e}_2 + z_{i_1,0}^j \tilde{e}_3, \) and \( r_{i_2,0}^j = x_{i_2,0}^j \tilde{e}_1 + y_{i_2,0}^j \tilde{e}_2 + z_{i_2,0}^j \tilde{e}_3 \). It is further assumed that in-neighbor UAVs \( i_1, i_2, \) and \( i_3 \) form a triangle at the initial time \( t_0 \), therefore:

\[ \text{Rank} \left[ r_{i_2,0}^j - r_{i_1,0}^j \quad r_{i_3,0}^j - r_{i_1,0}^j \right] = 2. \]

(19)

and \( r_{i,0}^j \) is uniquely expressed as follows:

\[ r_{i,0}^j = w_{i_2,i_1} (r_{i_2,0}^j - r_{i_1,0}^j) + w_{i_3,i_1} (r_{i_3,0}^j - r_{i_1,0}^j) \]

(20)

Defining \( w_{i_1,i}, \ w_{i_2,i}, \) and \( w_{i_3,i} \) are uniquely obtained by

\[ \left[ \begin{array}{c} x_{i_1,0}^j \\ y_{i_1,0}^j \\ z_{i_1,0}^j \\ w_{i_1,i} \\ x_{i_2,0}^j \\ y_{i_2,0}^j \\ z_{i_2,0}^j \\ w_{i_2,i} \\ x_{i_3,0}^j \\ y_{i_3,0}^j \\ z_{i_3,0}^j \\ w_{i_3,i} \end{array} \right] = \left[ \begin{array}{c} x_{i,0}^j \\ y_{i,0}^j \end{array} \right]. \]

(21)

If follower \( i \in V_f^j \) is inside the communication triangle defined by \( i_1, i_2, i_3 \in N^j_i \), then communication weights are all positive [19].

**Proposition 1.** Define

\[ Z_{X,L,0}^j = \left[ \begin{array}{c} x_{1,0}^j \\ x_{2,0}^j \\ x_{3,0}^j \end{array} \right] \]

(22a)

\[ Z_{Y,L,0}^j = \left[ \begin{array}{c} y_{1,0}^j \\ y_{2,0}^j \\ y_{3,0}^j \end{array} \right] \]

(22b)

\[ Z_{X,F,0}^j = \left[ \begin{array}{c} x_{4,0}^j \\ \ldots \\ x_{M_{IJ},0}^j \end{array} \right] \]

(22c)

\[ Z_{Y,F,0}^j = \left[ \begin{array}{c} y_{4,0}^j \\ \ldots \\ y_{M_{IJ},0}^j \end{array} \right] \]

(22d)

If communication weights satisfy Eq. (21), then the \( x \) and \( y \) components of cluster \( j \)'s UAVs satisfy the following relations [19]:

\[ B^j Z_{X,L,0}^j + A^j Z_{X,F,0}^j = 0, \]

(23a)

\[ B^j Z_{Y,L,0}^j + A^j Z_{Y,F,0}^j = 0, \]

(23b)

where \( A^j \) and \( B^j \) are determined from Eqs. (17) and (18).

**Proposition 2.** If followers’ communication weights are consistent with agents’ initial positions and satisfy Eq. (21), then the FL communication matrix \( B^j \in \mathbb{R}^{(N_f-3) \times (N_f-3)} \) and the FF communication matrix \( A^j \in \mathbb{R}^{(N_f-3) \times (N_f-3)} \), obtained by Eqs. (17) and (18), satisfy the following relation [19]:

\[ W^j = A^{-1} B^j \]

(24)

where \( \alpha_{i,j} \) is uniquely assigned using Eq. (16) given initial position of follower \( i \in V_f^j \) and leaders of cluster \( j \).

**UAV Global Desired Trajectories:** Defining \( Z_{q,L,HT}^j(t) = [q_{1,L,HT}(t) \ q_{2,L,HT}(t) \ q_{3,L,HT}(t)]^T \) and \( Z_{q,F,HT}^j(t) = [q_{4,L,HT}(t) \ q_{5,L,HT}(t) \ q_{6,L,HT}(t)]^T \), components of the global desired trajectories are defined by

\[ j \in \Omega_{CL}, \ q = x, y, \forall t, \quad Z_{q,F,HT}^j(t) = W^j Z_{q,L,HT}^j(t). \]

(25)
3. Safety requirements: obstacle collision avoidance cluster containment and cluster privacy

Before proceeding to specify continuum deformation safety requirements, we define

\[
\begin{bmatrix}
  a_1^j \\
  a_2^j \\
  a_3^j
\end{bmatrix} = \begin{bmatrix}
  x_{1,HT}^j \\
  y_{1,HT}^j \\
  x_{2,HT}^j \\
  y_{2,HT}^j \\
  x_{3,HT}^j \\
  y_{3,HT}^j
\end{bmatrix}^{-1} \begin{bmatrix}
  x \\
  y
\end{bmatrix},
\]

where \( a_h^i = a_h^j (x_{HT}^j, \ldots, y_{3,HT}^j) \) (\( h \in \mathcal{V}_3 \)), \( r = \hat{x}_t + y_t \hat{y}_t \) is the position of a point in the \( x - y \) plane and \( x_{HT}^j \) and \( y_{HT}^j \) are position components of leader \( i \in \mathcal{V}_L \). The solution of Eq. (16) can be expressed as

\[
a_i^j = \frac{(x_{HT}^j - x_{p,HT}^j)(y - y_{p,HT}^j) - (y_{HT}^j - y_{p,HT}^j)(x - x_{p,HT}^j)}{(x_{HT}^j - x_{p,HT}^j)(y_{HT}^j - y_{p,HT}^j) - (y_{HT}^j - y_{p,HT}^j)(y_{HT}^j - y_{p,HT}^j)}.
\]

(26)

where

\((p, q, r) \in \{(1, 2, 3), (2, 3, 1), (3, 1, 2)\}\). (27)

Both cluster containment and privacy must be satisfied and obstacle collision avoidance must be ensured in a safe multi-cluster continuum deformation. Propositions 3, 4, and 5 formulate these three necessary conditions. By ensuring cluster containment, follower quadruplet of cluster \( j \in \Omega_{CL} \) does not leave the cluster at any time \( t \). By ensuring cluster privacy, a cluster is not trespassed by a UAV from a different cluster. Proposition 5 provides guarantee condition for obstacle collision avoidance.

**Key Assumption 2.** In this paper, we assume that the desired waypoint of leader \( i \), denoted by \((x_{HT}^j, y_{HT}^j)_i \), is positioned on the motion space grid nodes. Mathematically speaking,

\( j \in \Omega_{CL}, i \in \mathcal{V}_L \), \( \left( x_{HT}^j, y_{HT}^j \right)_i \in \mathcal{M}_O \).

**Proposition 3 (Containment condition).** Followers are all inside the leading triangle \( j \in \Omega_{CL} \) at any time \( t \), if

\[
\forall i \in \mathcal{V}_F, \quad \left( a_1^i \geq 0 \right) \land \left( a_2^i \geq 0 \right) \land \left( a_3^i \geq 0 \right),
\]

(28)

where \( a_h^i = a_h^j (x_{HT}^j, x_{2,HT}^j, x_{3,HT}^j, x_{1,HT}^j, y_{HT}^j, y_{2,HT}^j, y_{3,HT}^j, y_{1,HT}^j) \) \( (r = 1, 2, 3, \forall i \in \mathcal{V}_F) \).

**Proof.** \( a_1^i \) constant \( (r = 1, 2, 3) \) is an equation of the line parallel to the line segments connecting leaders \( p \) and \( q \) (\( p \neq q \), \( q \neq r \), \( r \neq p \), \( p, q, r \in \{1, 2, 3\} \)). If follower \( i \in \mathcal{V}_F \) is located along the line segment connecting leaders \( p \) and \( q \), the denominator of Eq. (16) vanishes and \( a_h^i \) \( (r = 1, 2, 3) \) is an equation of the line parallel to the line segments connecting leaders \( p \) and \( q \) (\( p \neq q \), \( q \neq r \), \( r \neq p \), \( p, q, r \in \{1, 2, 3\} \)). The horizontal plane is divided into the following 10 zones:

- Zone 1: \( a_{1,1}^i > 0, a_{1,2}^i > 0, a_{1,3}^i > 0 \).
- Zone 2: \( a_{1,1}^i > 0, a_{1,2}^i < 0, a_{1,3}^i < 0 \).
- Zone 3: \( a_{1,1}^i > 0, a_{1,2}^i < 0, a_{1,3}^i > 0 \).
- Zone 4: \( a_{1,1}^i < 0, a_{1,2}^i > 0, a_{1,3}^i > 0 \).
- Zone 5: \( a_{1,1}^i < 0, a_{1,2}^i < 0, a_{1,3}^i > 0 \).
- Zone 6: \( a_{1,1}^i > 0, a_{1,2}^i < 0, a_{1,3}^i < 0 \).
- Zone 7: \( a_{1,1}^i < 0, a_{1,2}^i > 0, a_{1,3}^i < 0 \).
- Zone 8: \( a_{1,1}^i = 0 \).
- Zone 9: \( a_{1,2}^i = 0 \).
- Zone 10: \( a_{1,3}^i = 0 \).

As shown in Fig. 2, \( a_{1,2}^i, a_{1,3}^i \), and \( a_{1,1}^i \) are all positive, only if follower \( i \in \mathcal{V}_F \) is inside leading triangle \( j \in \Omega_{CL} \).

**Proposition 4 (Privacy condition).** It is ensured that cluster \( j_1 \in \Omega_{CL} \) is not trespassed by a UAV \( i_2 \in \Omega_{F} \) at any time \( t \), if

\[
\forall i_1 \in \mathcal{V}_F, j_1 \in \Omega_{CL}, j_1 \neq j_2, \forall i_2 \in \mathcal{V}_F, j_1 \in \Omega_{CL}, (a_1^j (x_{HT}^j, y_{HT}^j, x_{2,HT}^j, y_{2,HT}^j, y_{3,HT}^j) < 0) \lor \left( a_2^j (x_{HT}^j, y_{HT}^j, x_{2,HT}^j, y_{2,HT}^j, y_{3,HT}^j) < 0 \right) \lor \left( a_3^j (x_{HT}^j, y_{HT}^j, x_{2,HT}^j, y_{2,HT}^j, y_{3,HT}^j) < 0 \right).
\]

(29)

**Proof.** If UAV \( i_2 \in \mathcal{V}_F \) is outside cluster \( j_1 \), then, \( a_h^j (x_{HT}^j, y_{HT}^j, x_{2,HT}^j, y_{2,HT}^j, y_{3,HT}^j) = 0 \), \( a_h^j (x_{HT}^j, y_{HT}^j, x_{2,HT}^j, y_{2,HT}^j, y_{3,HT}^j) = 0 \), \( a_h^j (x_{HT}^j, y_{HT}^j, x_{2,HT}^j, y_{2,HT}^j, y_{3,HT}^j) = 0 \), and \( a_h^j (x_{HT}^j, y_{HT}^j, x_{2,HT}^j, y_{2,HT}^j, y_{3,HT}^j) = 0 \) \( (j_1 \in \Omega_{CL}, i_2 \in \mathcal{V}_F) \) cannot be all non-negative (See Fig. 2). Therefore, privacy of cluster \( j_1 \) is assured if condition (29) is satisfied for all UAVs in cluster \( j_2 \).

**Proposition 5 (Obstacle collision avoidance).** Cluster \( j \in \Omega_{CL} \) is assured to not collide with obstacles \( O_i = O_i (o_1^i, \ldots, o_p^i) \), i.e., cluster \( j \) does not enter No-Flight Zone \( O_i \) if

\[
\begin{align*}
&i = 1, \ldots, n_o, O_i \in \Omega_{NFZ}, \forall j \in \Omega_{CL}, k = 1, \ldots, p_i,
&\forall \{(x_{HT}^j, y_{HT}^j, x_{2,HT}^j, y_{2,HT}^j, y_{3,HT}^j, o_k^i) \} \lor \quad \text{(30)}
&\forall \{(x_{HT}^j, y_{HT}^j, x_{2,HT}^j, y_{2,HT}^j, y_{3,HT}^j, o_k^i) \} \lor \quad \text{(30)}
&\forall \{(x_{HT}^j, y_{HT}^j, x_{2,HT}^j, y_{2,HT}^j, y_{3,HT}^j, o_k^i) \}.
\end{align*}
\]

**Proof.** Because obstacles (No-Flight Zones) are convex regions in the shared airspace, the position of any interior point of \( O_{F} \) can be expressed as a convex combination of vertices \( o_1^j, \ldots, o_p^j \).
Therefore, obstacle \( O_{j_3} \) can be represented by its vertices and cluster \( j_1 \in \Omega_{CL} \) does not collide with obstacle \( O_{j_2} \) if condition (30) is satisfied.

4. Problem statement and formulation

Consider a collection of \( m \) UAV clusters in a shared motion space. Each cluster forms a triangular domain and consists of a large number of small UAVs. Three leaders, located at the vertices of a triangular cluster, guide the collective motion. It is assumed that initial and target configurations of each triangular cluster are known. Given initial and target configurations, the objective is to minimize travel distance (path length) of every UAV from its initial destination to its target location ensuring inter-agent and obstacle collision avoidance, i.e., “No-Flight Zone” avoidance.

This problem is mathematically defined as follows:

\[
\min \sum_{j \in \Omega_{CL}} \sum_{i \in V_j^l} \int_0^{s_{i,j}} dS_i.
\]

subject to containment, privacy, and obstacle avoidance guarantee conditions (28), (29), and (30) as well as the following constraints:

\[
\forall j, j_1, j_2 \in \Omega_{CL}, \forall l \in V_i^j, \forall t \in V_i^j, i \neq l,
\]

\[
\|r_{i,j}^t - r_{i,j_2}^t\| > 2\epsilon,
\]

\[
\|r_{i,j}^t - r_{i,j_2}^t\| > 2\epsilon,
\]

\[
\|r_{i,j}^t - r_{i,j_2}^t\| > 2\epsilon.
\]

Constraint equation (32a) ensures that no two separate UAVs collide. Constraint equation (32b) ensures that UAV \( i \in V_j^l \) ultimately reaches the target destination given the initial position assigned in constraint equation (32b). Constraint equation (32d) guarantees leaders form a triangle at any time \( t \). Eq. (32d) must be satisfied to ensure that the Jacobian matrix \( Q_i^j \) is nonsingular at any time \( t \), i.e., if rank condition (32d) is not satisfied, the leading triangle \( j \) is either mapped to a line or a single point.

The above optimization problem is highly nonlinear and computationally expensive. To deal with complexity, we assume that each UAV \( i \in V_j^l \) moves on a straight path over time \( t \in [t_{k-1}, t_k] \) (\( k = 1, 2, \ldots \)). Therefore, path planning can be treated as waypoint planning where leaders’ paths can be simply planned by connecting consecutive optimal waypoints. The paper applies a well-established \( A^* \) method to find optimal leader waypoints and optimize continuum deformation of UAV clusters in a shared motion space such that geometric motion constraints are all satisfied. While \( A^* \) has substantially overhead, its application to only leaders can provide manage complexity sufficiently given a limited number of interacting clusters. Once waypoints are defined, each UAV desired speed profile is then planned along the optimal waypoint sequence as described in Section 5.3.

5. Multi-cluster continuum deformation planning

In this paper we assume that the \( x - y \) plane is uniformly discretized. Let

\[
j \in \Omega_{CL}, l = 1, 2, 3,
\]

\[
P_{l,c}^j = P_{x,i,c}^j \hat{e}_1 + P_{y,i,c}^j \hat{e}_2 \in M_{lO}
\]

define the current desired waypoint of leader \( l \in V_i^j \). The next waypoint of each leader \( l \) is denoted by

\[
j \in \Omega_{CL}, l = 1, 2, 3,
\]

\[
P_{l,n}^j = P_{x,i,n}^j \hat{e}_1 + P_{y,i,n}^j \hat{e}_2 \in M_{lO},
\]

where

\[
q = x, y,
\]

\[
p_{q,l,n}^j = P_{q,l,c}^j + h_q \Delta q
\]

Note that \( \Delta q > 0 \) (\( q = x, y \)) is constant. Let \( \bar{T}_l^j = \{P_{l,c}^j, P_{l,n}^j, P_{l,g}^j\} \) denote the desired configuration of leading triangle \( j \in \Omega_{CL} \) \( \{j \in [1, \ldots, m] \} \) in the \( x - y \) plane at the current time. The next desired configuration of leading triangle \( j \) is denoted by \( \bar{T}_l^j = \{P_{l,c}^j, P_{l,n}^j, P_{l,g}^j\} \). In addition, \( \bar{T}_l^j = \{P_{l,c}^j, P_{l,n}^j, P_{l,g}^j\} \) and \( \bar{T}_0 = \{P_{l,c}^j, P_{l,n}^j, P_{l,g}^j\} \) are the goal and initial configurations of leading triangle \( j \in \Omega_{CL} \), where

\[
l = 1, 2, 3,
\]

\[
P_{l,c}^j = P_{x,i,c}^j \hat{e}_1 + P_{y,i,c}^j \hat{e}_2 \in M_{lO}
\]

We assume leader \( l \in V_i^j \) moves along the line segment connecting \( P_{l,c}^j \) and \( P_{l,n}^j \) (see Fig. 3). The desired path of leader \( l \in V_i^j \) is defined by

\[
l \in V_i^j,
\]

\[
r_{l,H}^j = (1 - \beta)P_{l,c}^j + \beta P_{l,n}^j + z_{HT} \hat{e}_3
\]

where multi-UAV system (MUS) elevation \( z_{HT} \) is constant, \( l \in V_i^j \), \( j \in \Omega_{CL} \), and \( \beta \in [0, 1] \).

5.1. Collision avoidance

It is computationally expensive to directly check for inter-agent collision avoidance with constraint Eq. (32a) for every UAV pair in the shared motion space. Because each cluster evolution is treated as continuum deformation, inter-agent collision avoidance can be assured with reduced computation cost. For this purpose, we first specify inter-agent collision avoidance and containment guarantee conditions in Theorem 1. Assuming containment guarantee and collision avoidance at every cluster, Theorem 2 provides sufficient conditions for inter-agent collision avoidance between any two agents in the shared motion space.

Theorem 1. Let \( d_{i}^j \) be the minimum separation distance between two UAVs in cluster \( j \in \Omega_{CL} \) at initial time \( t_0 \), \( d_{i}^j \) be the minimum distance of a UAV from the edges of the leading triangle \( j \in [1, \ldots, m] \) at time \( t_0 \) (see Fig. 4), and each UAV be enclosed by a ball with radius \( \epsilon \). Define

\[
\theta_{\max} = \min \left\{ \frac{1}{2} \left( d_{i}^j - 2\epsilon \right), \left( d_{i}^j - \epsilon \right) \right\}
\]

Let \( r_{i,H}^j \) be the global desired position of a UAV \( i \in V_i^j \), given by a continuum deformation (see Eq. (10)), \( r_{i}^j \) be the actual position of UAV \( i \in V_i^j \),
and δ be an upper limit for deviation of a cluster j UAV from continuum deformation desired position:
\[ \forall t \in [t_k, t_{k+1}], \forall i \in V^J, \quad \| r_i^J(t) - r_i^J(t_k) \| \leq \delta. \]  \hspace{1cm} (36)

Define
\[ \lambda_{\text{CD, min}}^j = \frac{\delta^j + \epsilon}{\delta_{\text{max}}}, \]  \hspace{1cm} (37)

If
\[ \forall t \in [t_k, t_{k+1}], \quad C_{\text{COL}, 1}^j = \lambda_{\text{CD, min}}^j - \lambda_{\text{CD, min}}^J \left( U_{\text{CD}}^J(t) \right) \leq 0, \]  \hspace{1cm} (38)

then,
1. Inter-agent collision avoidance in cluster j (constraint Eq. \((32d)\)) is guaranteed and
2. All followers remain inside leading triangle j at all times t, i.e., containment condition \((28)\) is satisfied.

**Proof.** See the proof in [19,20]. \(\Box\)

**Theorem 2.** Assume \(C_{\text{COL}, 1}^j\) and \(C_{\text{COL}, 2}^j\) \((j_1, j_2 \in \Omega_{CL}, j_1 \neq j_2)\) are both satisfied at any time \(t \in [t_k, t_{k+1}]\) and
\[ \bigcup_{j \in \Omega_{CL}} \bigcup_{l \in V^L(j_1), j_1 \neq j_2} \| r_{l1}^{J_l} - r_{l2}^{J_l} \| \geq \left( \delta_{J_1} + \delta_{J_2} + 2\epsilon \right). \]  \hspace{1cm} (39)

Furthermore, assume privacy condition \((29)\) is satisfied at all times t. Then, no two UAVs in clusters \(j_1\) and \(j_2\) collide.

**Proof.** If \(C_{\text{COL}, 1}^j \leq 0\) and \(C_{\text{COL}, 2}^j \leq 0\), then,
\[ \forall l \in V^L, \quad \| r_{l1}^{J_l} - r_{l2}^{J_l} \| \leq \delta_{J_1}, \]  \hspace{1cm} (40a)
\[ \forall l \in V^L, \quad \| r_{l2}^{J_l} - r_{l2}^{J_l} \| \leq \delta_{J_2}, \]  \hspace{1cm} (40b)

and the following statements are true:
1. Inter-agent collision avoidance is guaranteed in cluster \(j_u\) \((u = 1, 2)\).
2. No follower UAV leaves leading triangle \(j_u\) \((u = 1, 2)\).

Furthermore, cluster \(j_1 \in \Omega_{CL}\) is not trespassed by a UAV from cluster \(j_2\), if privacy condition \((29)\) is satisfied. UAV \(l \in V^L(j_1)\) never collides with UAV \(l \in V^L(j_2)\) if leaders of clusters \(j_1\) and \(j_2\) do not collide. By considering Eq. \((40)\),
\[ \| r - r_{l1}^{J_l} \| \leq \delta_{J_1} + \epsilon, \]  \hspace{1cm} (41)
\[ \| r - r_{l2}^{J_l} \| \leq \delta_{J_2} + \epsilon, \]  \hspace{1cm} (42)

are the safe zones of leaders \(l_1 \in V^L_1(j_1)\) and \(l_2 \in V^L_2(j_2)\) and must not be trespassed by any UAV. Collision avoidance between leaders \(l_1 \in V^L_1(j_1)\) and \(l_2 \in V^L_2(j_2)\) is guaranteed if inequality \((39)\) is satisfied. \(\Box\)

**Definition 1** (Valid deformation). Define current configurations \(T_1^J = (P_{1,1}^{J_1}, P_{1,2}^{J_1}, ..., P_{1,n}^{J_1})\) and \(T_2^J = (P_{2,1}^{J_2}, P_{2,2}^{J_2}, ..., P_{2,n}^{J_2})\) and next configurations \(T_1^J = (P_{1,1}^{J_1}, P_{2,2}^{J_2}, ..., P_{1,n}^{J_1})\) and \(T_2^J = (P_{2,1}^{J_2}, P_{2,2}^{J_2}, ..., P_{2,n}^{J_2})\). Assume leader \(l_u \in V_{l_u}(u = 1)\) moves along a straight path connecting \(P_{l_u,1}^{J_1}\) and \(P_{l_u,n}^{J_1}\) as defined by Eq. \((34)\):
\[ u = 1, 2, \quad l_u \in V_{l_u}^J, \quad r_{l_u,HT}^J = (1 - \beta) P_{l_u,1}^{J_1} + \beta P_{l_u,2}^{J_1} + z_{HT} \]  \hspace{1cm} (43)

Evolution of leading triangle \(j_u\) is called a valid deformation if
1. \(P_{l_u}^J\) is defined by Eq. \((33)\) and
2. Containment and collision avoidance condition \((38)\), privacy condition \((29)\), obstacle avoidance condition \((30)\) and inequality constraint \((39)\) are satisfied at any \(\beta \in [0, 1]\).

**5.2. Cluster deformation optimization**

We apply A* search to optimally plan the multi-cluster continuum deformation leader waypoints given clusters’ initial and target configurations. To set up the path-planning problem, we define the following symbols:
\[ s_0 = (\bar{T}_1, ..., \bar{T}_m) := \text{Initial node} \]  \hspace{1cm} (44)
\[ s_g = (\bar{T}_1, ..., \bar{T}_m) := \text{Goal node} \]  \hspace{1cm} (45)
\[ s_c = (\bar{T}_1, ..., \bar{T}_m) := \text{Current node} \]  \hspace{1cm} (46)
\[ s_n = (\bar{T}_1, ..., \bar{T}_m) := \text{Next node} \]  \hspace{1cm} (47)

Note that \(s_n\) is a valid continuum deformation. Superscripts 1, 2, , , m denote cluster index numbers. Cluster leaders’ optimal paths are assigned by minimizing deformation cost
\[ f(s_n) = g(s_n) + h(s_n), \]  \hspace{1cm} (48)

where \(h(s_n)\) is the admissible heuristic cost assigned as follows:
\[ h(s_n) = \sqrt{\sum_{j \in V^J} \sum_{l \in V^L} \| P_{l,n}^{J_l} - P_{l,g}^{J_l} \|^2}. \]  \hspace{1cm} (49)

Note that \(h(s_n)\) is the sum of leaders’ straight line distances from their target destinations. Furthermore, \(g(s_n)\) is the minimum estimated cost from \(s_0\) to \(s_n\)
\[ g(s_n) = \min \{ g(s_c) + C_{c,n} \}, \]  \hspace{1cm} (50)

where
\[ C_{c,n} = \sqrt{\sum_{j \in V^J} \sum_{l \in V^L} \| P_{l,n}^{J_l} - P_{l,c}^{J_l} \|^2}. \]  \hspace{1cm} (51)

**5.3. Leader trajectory planning**

Leaders’ paths are all piecewise linear. To ensure that leaders’ trajectories are \(C^2\) continuous, \(\beta\) in Eq. \((34)\) is given by a fifth order polynomial:
\[ t \in [t_{k-1}, t_k], \quad \beta(t) = \sum_{i=0}^{5} a_{i,k} t^{5-i}, \]  \hspace{1cm} (52)

subject to
\[ \beta(t_{k-1}) = 0, \]
\[ \beta(t_k) = 1, \]
\[ \dot{\beta}(t_{k-1}) = \dot{\beta}(t_k) = 0. \]

Assuming \( \Delta t = t_k - t_{k-1} \), \( a_{0,k} \) through \( a_{5,k} \) are determined by solving the following linear equality constraints:

\[
\begin{bmatrix}
\Delta t^5 & \Delta t^4 & \Delta t^3 & \Delta t^2 & \Delta t & 1 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
a_{0,k} \\
a_{1,k} \\
a_{2,k} \\
a_{3,k} \\
a_{4,k} \\
a_{5,k}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
1 \\
0 \\
0 \\
0
\end{bmatrix}.
\]

(47)

5.4. Follower coordination planning

Leaders’ global and local desired positions are the same and defined by Eq. (34) given current and new waypoints assigned by \( A' \). Knowing leaders’ desired trajectories, UAV desired trajectories are assigned using Eq. (4).

6. Quadcopter dynamics and control

**Dynamics**: Define UAV \( i \in V^j \) centroid position \( r_i^j = [x_i^j, y_i^j, z_i^j]^T \). Euler angles \( \phi_i^j, \theta_i^j, \psi_i^j \), mass \( m_i \), and thrust force \( F_{T,i} \). The dynamics of UAV \( i \in V^j \) is given by

\[
\begin{align*}
\dot{r}_i^j &= v_i^j, \\
v_i^j &= [0, 0, -g]^T + \vec{F}_{T,i} \vec{k}_{d,i}^j, \\
\dot{\vec{F}}_{T,i} &= \begin{bmatrix}
\vec{F}_{T,i}^T \\
\vec{K}_{d,i}^j
\end{bmatrix}
\end{align*}
\]

(48)

Note that gravity \( g = 9.81 \frac{m}{s^2} \) and \( \vec{F}_{T,i} = \frac{f_{T,i}}{m_i} \) is thrust force per mass \( m_i \), and \( \vec{k}_{d,i}^j \) is the thrust direction unit vector \( \vec{F}_{T,i} \). Dynamics (48) can be rewritten in the following form:

\[
\begin{align*}
\dot{x}_i^j &= F_i(\mathbf{x}_i^j) + G_i v_i^j, \\
r_i^j &= h_i(\mathbf{x}_i^j) = [x_i^j, y_i^j, z_i^j]^T, \\
\mathbf{x}_i^j &= [x_i^j, y_i^j, z_i^j, v_{x,i}^j, v_{y,i}^j, v_{z,i}^j, \dot{v}_{x,i}^j, \dot{v}_{y,i}^j, \dot{v}_{z,i}^j]^T
\end{align*}
\]

(49)

where

\[
\begin{align*}
\mathbf{r}_i^j &= [r_i^j, v_i^j]^T,
\mathbf{V}_i^j &= [u_i^j, v_i^j, u_{\psi,i}^j, v_{\psi,i}^j]
\end{align*}
\]

is the control state, \( \mathbf{r}_i^j \) is the control output, \( \mathbf{V}_i^j = [u_i^j, v_i^j, u_{\psi,i}^j, v_{\psi,i}^j] \) is the control input vector.

\[
\begin{align*}
\mathbf{F}_i^j &= \begin{bmatrix}
f_{\dot{x,i}^j} & f_{\dot{y},i^j} & f_{\dot{z},i^j} & f_{\dot{\phi},i} & f_{\dot{\theta},i} & f_{\dot{\psi},i} & f_{\dot{\phi},i} & f_{\dot{\theta},i} & f_{\dot{\psi},i}
\end{bmatrix}^T,
\mathbf{G} &= \begin{bmatrix}
0_{3 \times 3} \\
I_3 \\
0_{2 \times 3}
\end{bmatrix}
\end{align*}
\]

(50a)

Internal Dynamics (Yaw Control): This paper chooses \( u_{\psi,i}^j = f_{\dot{\psi},i} \), as follows:

\[
\begin{align*}
\dot{\mathbf{x}}_i^j &= \mathbf{F}_i(\mathbf{x}_i^j) + \mathbf{G} \mathbf{V}_i^j, \\
\mathbf{r}_i^j &= [x_i^j, y_i^j, z_i^j]^T
\end{align*}
\]

Fig. 5. UAV controller block diagram.

\[
\begin{align*}
u_{\psi,i} &= f_{\dot{\psi},i} = \dot{\bar{\psi}}_i + k_{\psi,i}(\dot{\psi}_i - \bar{\psi}_i) + k_{\dot{\psi}_i}(\dot{\psi}_i - \bar{\psi}_i),
\end{align*}
\]

where \( k_{\psi,i} \) and \( k_{\dot{\psi},i} \) are positive constants and \( \dot{\psi}_i \) are known. Therefore, \( \dot{\bar{\psi}}_i \) is updated by the following stable second order dynamics (Fig. 5):

\[
\dot{\bar{\psi}}_i = \dot{\psi}_i - \ddot{\psi}_i, \quad \dot{\psi}_i = \dot{\psi}_i - \ddot{\psi}_i = 0.
\]

Control: Defining the state transition \( \mathbf{x}_i^j \rightarrow (\mathbf{r}_i^j, \mathbf{r}_i^j, \mathbf{V}_i^j, \psi_i^j) \), UAV dynamics (49) is feedback linearizable and can be expressed as follows:

\[
\begin{align*}
\frac{d^4 \mathbf{r}_i^j}{dt^4} &= \mathbf{U}_i^j, \\
\frac{d^2 \psi_i^j}{dt^2} &= \dot{\psi}_i, \\
\mathbf{U}_i^j &= \mathbf{M}_{\phi,i} \mathbf{V}_i^j + \mathbf{N}_{\phi,i} \mathbf{V}_i^j,
\end{align*}
\]

(52)

where

\[
\begin{align*}
\mathbf{M}_{\phi,i} &= \begin{bmatrix}
\Lambda_{1,i}^j & \Lambda_{2,i}^j & \Lambda_{3,i}^j
\end{bmatrix}, \\
\mathbf{N}_{\phi,i} &= \begin{bmatrix}
\Lambda_{0,i}^j + \Lambda_{1,i}^j \hat{F}_{T,i} + \Lambda_{2,i}^j \dot{\phi}_i + \Lambda_{3,i}^j \dot{\theta}_i
\end{bmatrix}, \\
\Lambda_{1,i}^j &= \begin{bmatrix}
0 & C_{\phi_i} S_{\phi_i} C_{\psi_i} + S_{\phi_i} C_{\psi_i}
\end{bmatrix}, \\
\Lambda_{2,i}^j &= \begin{bmatrix}
0 & C_{\phi_i} C_{\psi_i} + S_{\phi_i} C_{\psi_i}
\end{bmatrix}, \\
\Lambda_{3,i}^j &= \begin{bmatrix}
0 & C_{\psi_i} S_{\phi_i} C_{\psi_i} + S_{\phi_i} C_{\psi_i}
\end{bmatrix}, \\
\Lambda_{0,i}^j &= \begin{bmatrix}
0 & C_{\phi_i} C_{\psi_i} + S_{\phi_i} C_{\psi_i}
\end{bmatrix}.
\end{align*}
\]

(53)

(54a)

(54b)

(54c)

(54d)

(54e)

(54f)
Equating right-hand sides of Eq. (53) and the first row of Eq. (52), $U_i^j$ and $V_i^j$ are related by

$$V_i^j = M_{i_{\theta(i)},i}^{-1}(U_i^j - N_{i_{\theta(i)},i}).$$  

(55)

Let $U_i^j$ be chosen as follows:

$$U_i^j = \begin{cases} \mu_i, & i \in \mathcal{V}_i^j, \\ 0, & \text{otherwise} \end{cases},$$

(56)

where

$$P_i^j = \sum_{l=1}^{4} \gamma_{i}^{l} \frac{d^4 r_{i}^{j}}{dr^4},$$

(57a)

$$P_{d,i}^j = \sum_{l=0}^{4} \gamma_{i}^{l} \frac{d^4 r_{d,i}^{j}}{dr^4},$$

(57b)

$$\forall j \in \Omega_{CL}, \quad \mu_i = \begin{cases} 4 & i \in \mathcal{V}_i^j, \\ 3 & \text{otherwise} \end{cases}.$$  

(57c)

where $\gamma_{i}^{1}, \gamma_{i}^{2}, \gamma_{i}^{3}, \gamma_{i}^{4} > 0$ are appropriately chosen such that the MUS collective dynamics is stable. Stability of MUS collective dynamics is discussed in Theorem 3.

Define

$q = x, y, z, 
\begin{align*}
Z_q^j &= [q_1^T \cdots q_{N_j}^T], \\
Z_{q,L}^j &= [q_1^T, q_2^T]^T, \\
Z_{q,F}^j &= [q_1^T, q_2^T, \cdots, q_{N_j}^T], \\
Z_{MUS}^j &= \left[ Z_q^j, Z_{q,L}^j, Z_{q,F}^j \right]^T.
\end{align*}$

(58)

If position of follower $i \in \mathcal{V}_j^i$ is updated according to Eq. (56), then, collective dynamics of cluster $j$ is expressed by

$$\frac{d^4 Z_{q,L}^j}{dr^4} = \frac{d^4 Z_{Q,F}^j}{dr^4} = \sum_{l=1}^{4} \gamma_l^j \frac{d^4 Z_q^j}{dr^4} - \sum_{l=1}^{4} \gamma_l^j \frac{d^4 Z_q^j_{L,H,T}}{dr^4} - \frac{d^4 Z_q^j_{L,H,T}}{dr^4},$$

(59a)

$$\frac{d^4 Z_{q,F}^j}{dr^4} = \sum_{l=1}^{4} \gamma_l^j \frac{d^4 Z_q^j}{dr^4} + \sum_{l=1}^{4} \gamma_l^j \frac{d^4 Z_q^j_{L,H,T}}{dr^4} + \sum_{l=1}^{4} \gamma_l^j \frac{d^4 Z_q^j_{L,H,T}}{dr^4}.$$  

(59b)

Collective dynamics (59) can be rewritten in the following state-space form:

$$q = x, y, z, 
\begin{align*}
\dot{Z}_{MUS}^j &= A_{MUS} Z_{MUS}^j + B_{MUS} U_{MUS}^j,
\end{align*}$$

(60)

where

$$A_{MUS} = \begin{bmatrix} 0_{N_j} & 1_{N_j} & 0_{N_j} & 0_{N_j} \\
0_{N_j} & 0_{N_j} & 1_{N_j} & 0_{N_j} \\
0_{N_j} & 0_{N_j} & 0_{N_j} & 1_{N_j} \\
\gamma_4 A_{MUS} & \gamma_2 A_{MUS} & \gamma_1 A_{MUS} \end{bmatrix},$$

(62a)

$$A_{MUS}^j = \begin{bmatrix} -I_3 & 0_3 \times (N_j-3) \\
0 & A \\
0 & 0 \end{bmatrix}.$$  

(62b)

$$A_{MUS}^j = \begin{bmatrix} 0_{3 \times N_j} & I_3 & 0_{3 \times (N_j-3)} \end{bmatrix}^T.$$  

(62c)

Note that $0_{N_j} \in \mathbb{R}^{N_j \times N_j}$ and $0_{3 \times (N_j-3)} \in \mathbb{R}^{3 \times (N_j-3)}$ are zero-entry matrices and $I_3$ and $I_{N_j}$ are identity matrices.

**Theorem 3.** Define $E_{q,L}^j = Z_{q,L}^j - Z_{q,L,H,T}$ and $E_{q,F}^j = Z_{q,F}^j - Z_{q,F,H,T}$ as the error signals specifying deviation of cluster $j$ leader and follower UAVs from global desired positions given by (10). If the position of UAV $i \in \mathcal{V}_i^j$ is updated according to dynamics in (52) and (56), then the following statements hold:

- The error signals $E_{q,L}^j$ and $E_{q,F}^j (j \in \Omega_{CL}, q = x, y, z)$ are updated by the following fourth order dynamics:

$$\frac{d^4 E_{q,L}^j}{dr^4} = \sum_{l=1}^{4} \gamma_l^j \frac{d^4 E_{q,L}^j}{dr^4},$$

(63a)

$$\frac{d^4 E_{q,F}^j}{dr^4} = \sum_{l=1}^{4} \gamma_l^j \left( A \frac{d^4 E_{q,F}^j}{dr^4} + B \frac{d^4 E_{q,L}^j}{dr^4} \right) - W \frac{d^4 Z_{q,F}^j}{dr^4},$$

(63b)

- Error dynamics (63a) and (63b) are asymptotically stable, if $\sum_{l=0}^{3} s^l a_{l,j} = 0$, roots are all located in the left-hand side of the s-plane.

**Proof.** By subtracting $\frac{d^4 Z_{Q,F}^j}{dr^4}$ from both sides of Eq. (59a), Eq. (59a) can be rewritten as Eq. (63a) and (60)

$$\frac{d^4 Z_{q,F}^j}{dr^4} = \sum_{l=1}^{4} \gamma_l^j \left( A \frac{d^4 E_{q,F}^j}{dr^4} + B \frac{d^4 E_{q,L}^j}{dr^4} \right) + \sum_{l=1}^{4} \gamma_l^j \frac{d^4 E_{q,F}^j}{dr^4}.$$  

(64)

Substituting $B = -AW_l$, Eq. (64) can be rewritten as follows:

$$\frac{d^4 Z_{q,F}^j}{dr^4} = \sum_{l=1}^{4} \gamma_l^j \left( A \frac{d^4 E_{q,F}^j}{dr^4} + B \frac{d^4 E_{q,L}^j}{dr^4} \right) + \sum_{l=1}^{4} \gamma_l^j \frac{d^4 E_{q,F}^j}{dr^4}.$$  

(65)

By subtracting $\frac{d^4 E_{q,F}^j}{dr^4}$ from both sides of Eq. (65), $E_{q,F}^j$ is updated by Eq. (63b). Note that the error dynamics (63b) is asymptotically stable if the roots of the error characteristic equation,

$$\begin{align*}
j \in \Omega_{CL}, \quad \sum_{l=0}^{3} s^l a_{l,j} = 0, \\
\text{are all located in the left-half s-plane.} \quad \Box
\end{align*}$$

7. Simulation results

Consider an MUS consisting of two clusters ($\Omega_{CL} = \{1, 2\}$, $m = 2$ with initial configurations shown in Fig. 6. Each cluster consists of 18 UAVs ($N_1 = N_2 = 18$). Leader UAVs are identified by local index numbers 1, 2, and 3, and follower UAVs are locally indexed by 4, $\cdots$, 18. As shown in Fig. 7, leaders of clusters 1 and 2 are initially placed at $r_{1,0}^1 = 10e_1 + 10e_2 + 10e_3$, $r_{1,0}^2 = 30e_1 + 10e_2 + 30e_3$, $r_{1,0}^3 = 10e_1 + 10e_2 + 10e_3$, $r_{1,0}^4 = 10e_1 + 10e_2 + 10e_3$, $r_{1,0}^5 = 10e_1 + 10e_2 + 10e_3$, $r_{1,0}^6 = 10e_1 + 10e_2 + 10e_3$, and $r_{2,0}^7 = 10e_1 + 10e_2 + 10e_3$.

**Followers’ Evolution:** Communication graphs shown in Fig. 6 define inter-agent communication among UAVs in clusters 1 and 2. Given initial positions (shown in Fig. 6), communication weights are computed using Eq. (21). For instance, UAV 17 communicates with in-neighbor UAVs 11, 13, and 16 in cluster 1, where $r_{17,0}^1 = \cdots$
The remaining communication weights are all 0.225. UAVs all choose $\gamma^j_1 = 30.9375$, $\gamma^j_2 = 52.8438$, $\gamma^j_3 = 33.6875$, and $\gamma^j_4 = 9.5000$ ($j \in \Omega_{CL} = \{1, 2\}$).

**Inter-agent Collision Avoidance Verification:** We choose $\lambda^j_{CD, \text{min}} = 0.5$ ($j \in \Omega_{CL} = \{1, 2\}$). In cluster 1 $\in \Omega_{CL}$, UAVs 9 $\in \mathcal{V}^j_1$ and 15 $\in \mathcal{V}^j_1$ have the minimum separation distance $d^1_{ij} = 2.989m$ at the initial time. Furthermore, UAV 6 $\in \mathcal{V}^j_1$ is at the closest distance from the boundary of the leading triangle 1 $\in \Omega_{CL}$. In cluster 2 $\in \Omega_{CL}$, UAVs 10 $\in \mathcal{V}^j_2$ and 16 $\in \mathcal{V}^j_2$ have minimum separation distance $d^2_{ij} = 2.4789m$ at the initial time. Moreover, UAV 6 $\in \mathcal{V}^j_2$ is at the closest distance $d^2_{ij} = 2.0206m$ from sides 1 $\sim 3$ of leading triangle 2 $\in \Omega_{CL}$. We assume each UAV can be enclosed by a ball with radius $\epsilon = 0.5m$. Using Eq. (35),

$$
\delta^1_{\text{max}} = \min \left\{ \frac{1}{2} (d^1_s - 2\epsilon), (d^1_b - \epsilon) \right\} = 0.9944m
$$

$$
\delta^2_{\text{max}} = \min \left\{ \frac{1}{2} (d^2_s - 2\epsilon), (d^2_b - \epsilon) \right\} = 0.7395m
$$

Therefore, $\delta^1 = 0.2472m$ and $\delta^2 = 0.1198m$ assign upper limits for UAV deviations in clusters 1 and 2, respectively, such that $\| \mathbf{r}_j(t) - \mathbf{r}_{i,HT}(t) \| \leq \delta^j_i$, $j \in \Omega_{CL} = \{1, 2\}$, $i \in \mathcal{V}^j = \{1, \cdots, 18\}$.

**7.1. Case-study 1: limited airspace without no-flying zone**

It is aimed that cluster leaders ultimately reach $\mathbf{r}^j_{1,F} = 100\mathbf{e}_1 + 40\mathbf{e}_2 + 10\mathbf{e}_3$, $\mathbf{r}^j_{2,F} = 100\mathbf{e}_1 + 70\mathbf{e}_2 + 10\mathbf{e}_3$, $\mathbf{r}^j_{3,F} = 70\mathbf{e}_1 + 60\mathbf{e}_2 + 10\mathbf{e}_3$, $\mathbf{r}^j_{4,F} = 40\mathbf{e}_1 + 60\mathbf{e}_2 + 10\mathbf{e}_3$, $\mathbf{r}^j_{5,F} = 40\mathbf{e}_1 + 40\mathbf{e}_2 + 10\mathbf{e}_3$, and $\mathbf{r}^j_{6,F} = 10\mathbf{e}_1 + 30\mathbf{e}_2 + 10\mathbf{e}_3$. The path of leader $i \in \mathcal{V}^j_1$ ($j \in \Omega_{CL} = \{1, 2\}$) must satisfy motion constraints, defined in Eq. (32), to ensure collective motion safety. Optimal leaders' waypoints are obtained using $A^*$ and listed in Table 1. Leaders' straight paths

---

**Fig. 6.** Initial configurations of clusters 1 and 2. This figure also shows graphs defining inter-agent communication for clusters 1 and 2.

$$
18.9548\mathbf{e}_1 + 37.6103\mathbf{e}_2, \mathbf{r}^j_{1,0} = 17.6720\mathbf{e}_1 + 40.7236\mathbf{e}_2, \mathbf{r}^j_{1,7} = 14.8149\mathbf{e}_1 + 25.1496\mathbf{e}_2, \mathbf{r}^j_{1,7} = 26.2303\mathbf{e}_1 + 32.7599\mathbf{e}_2.
$$

Therefore, communication weights $w^j_{17,11}$, $w^j_{17,13}$, and $w^j_{17,16}$ are obtained as follows:

$$
\begin{bmatrix}
17.6720 & 14.8149 & 26.2303 \\
40.7236 & 25.1496 & 32.7599 \\
1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
18.9548 \\
14.8149 \\
1
\end{bmatrix}
= 0.225
$$

**Fig. 7.** Initial and target configurations of UAV clusters 1 and 2 in Case Study 1. (a-c) Optimal path for leaders guiding cluster 1. (d) Optimal path for leaders guiding cluster 2.
Fig. 8. Case-study 1: (a) $x$ components of UAVs in cluster 1 versus time. (b) $y$ components of UAVs in cluster 1 versus time. (c) $x$ components of UAVs in cluster 2 versus time. (d) $y$ components of UAVs in cluster 2 versus time.
Table 1
Optimal waypoints assigned by $A^*$ in case study 1.

<table>
<thead>
<tr>
<th>Time</th>
<th>$P_{1,1}^t$ (m)</th>
<th>$P_{1,2}^t$ (m)</th>
<th>$P_{1,3}^t$ (m)</th>
<th>$P_{2,1}^t$ (m)</th>
<th>$P_{2,2}^t$ (m)</th>
<th>$P_{2,3}^t$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_0$ = 0s</td>
<td>(10, 10)</td>
<td>(40, 30)</td>
<td>(10, 50)</td>
<td>(100, 10)</td>
<td>(100, 40)</td>
<td>(70, 30)</td>
</tr>
<tr>
<td>$t_1$ = 20s</td>
<td>(20, 10)</td>
<td>(50, 30)</td>
<td>(20, 50)</td>
<td>(90, 10)</td>
<td>(90, 40)</td>
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<tr>
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<td>(2050)</td>
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<td>(90, 40)</td>
<td>(60, 30)</td>
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<tr>
<td>$t_3$ = 60s</td>
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<td>(20, 50)</td>
<td>(90, 10)</td>
<td>(90, 40)</td>
<td>(60, 30)</td>
</tr>
<tr>
<td>$t_4$ = 80s</td>
<td>(50, 10)</td>
<td>(50, 40)</td>
<td>(20, 50)</td>
<td>(90, 10)</td>
<td>(90, 40)</td>
<td>(60, 30)</td>
</tr>
<tr>
<td>$t_5$ = 100s</td>
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<td>(20, 50)</td>
<td>(90, 10)</td>
<td>(90, 40)</td>
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<tr>
<td>$t_6$ = 120s</td>
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<td>(50, 50)</td>
<td>(20, 50)</td>
<td>(90, 10)</td>
<td>(90, 40)</td>
<td>(60, 30)</td>
</tr>
<tr>
<td>$t_7$ = 140s</td>
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<td>(50, 60)</td>
<td>(30, 50)</td>
<td>(90, 10)</td>
<td>(90, 40)</td>
<td>(60, 30)</td>
</tr>
<tr>
<td>$t_8$ = 160s</td>
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<td>(60, 60)</td>
<td>(40, 60)</td>
<td>(80, 10)</td>
<td>(80, 40)</td>
<td>(50, 30)</td>
</tr>
<tr>
<td>$t_9$ = 180s</td>
<td>(60, 50)</td>
<td>(60, 70)</td>
<td>(40, 60)</td>
<td>(70, 10)</td>
<td>(70, 40)</td>
<td>(40, 30)</td>
</tr>
<tr>
<td>$t_{10}$ = 200s</td>
<td>(70, 50)</td>
<td>(70, 70)</td>
<td>(50, 60)</td>
<td>(60, 10)</td>
<td>(60, 40)</td>
<td>(30, 30)</td>
</tr>
<tr>
<td>$t_{11}$ = 220s</td>
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<td>(80, 70)</td>
<td>(50, 60)</td>
<td>(60, 10)</td>
<td>(60, 40)</td>
<td>(20, 30)</td>
</tr>
<tr>
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<td>(90, 70)</td>
<td>(60, 60)</td>
<td>(50, 10)</td>
<td>(50, 40)</td>
<td>(20, 30)</td>
</tr>
<tr>
<td>$t_{13}$ = 260s</td>
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<td>(90, 80)</td>
<td>(70, 60)</td>
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<tr>
<td>$t_{14}$ = 280s</td>
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<td>(70, 60)</td>
<td>(60, 10)</td>
<td>(60, 40)</td>
<td>(10, 30)</td>
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Fig. 9. Case-study 1: (a) Deviation of cluster 1 UAVs from global desired coordination. (b) Deviation of cluster 2 UAVs from global desired coordination.

Table 2
Optimal waypoints assigned by $A^*$ in case study 2.

<table>
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<tr>
<th>Time</th>
<th>$P_{1,1}^t$ (m)</th>
<th>$P_{1,2}^t$ (m)</th>
<th>$P_{1,3}^t$ (m)</th>
<th>$P_{2,1}^t$ (m)</th>
<th>$P_{2,2}^t$ (m)</th>
<th>$P_{2,3}^t$ (m)</th>
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<td>(100, 10)</td>
<td>(100, 40)</td>
<td>(70, 30)</td>
</tr>
<tr>
<td>$t_1$ = 20s</td>
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<td>(90, 40)</td>
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<td>(90, 10)</td>
<td>(90, 40)</td>
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<tr>
<td>$t_4$ = 80s</td>
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<td>(90, 40)</td>
<td>(60, 30)</td>
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<tr>
<td>$t_5$ = 100s</td>
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<td>(20, 50)</td>
<td>(90, 10)</td>
<td>(90, 40)</td>
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<tr>
<td>$t_6$ = 120s</td>
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<td>(20, 50)</td>
<td>(90, 10)</td>
<td>(90, 40)</td>
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<tr>
<td>$t_7$ = 140s</td>
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<td>(50, 60)</td>
<td>(30, 50)</td>
<td>(90, 10)</td>
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<tr>
<td>$t_8$ = 160s</td>
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<td>(40, 60)</td>
<td>(80, 10)</td>
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<tr>
<td>$t_9$ = 180s</td>
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<td>(60, 70)</td>
<td>(40, 60)</td>
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<td>$t_{10}$ = 200s</td>
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<td>(50, 60)</td>
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<tr>
<td>$t_{11}$ = 220s</td>
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<tr>
<td>$t_{12}$ = 240s</td>
<td>(90, 50)</td>
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<tr>
<td>$t_{13}$ = 260s</td>
<td>(90, 60)</td>
<td>(90, 80)</td>
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<td>$t_{14}$ = 280s</td>
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<td>(60, 40)</td>
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connecting two consecutive waypoints are shown in Fig. 7. We choose $\Delta t = t_{k+1} - t_k = 20s \ (k = 0, 1, \ldots, 13)$ such that $P^j_{i,n}(t_k) = P^j_{l,c}(t_{k+1}), \ i \in V^j_1, j \in \Omega_{CL} = \{1,2\}$. The global desired trajectory of leader $l \in V^j_1, \Omega_{CL} = \{1,2\}$ is assigned using Eq. (34), where $\beta(t)$ is specified by Eqs. (45) and (46).

The $x$ and $y$ components of UAV positions in the $x-y$ plane are shown in Fig. 8. It is seen that UAVs all reach their target destinations while no two UAVs collide. Fig. 9 shows deviations of all UAVs from the global desired coordination references in clusters 1 and 2. As shown the deviations of UAVs in cluster 1 are less than $\delta^1 = 0.2472m$. Also, UAV deviations in cluster 2 are less than $\delta^2 = 0.1198m$.

7.2. Case-study 2: limited airspace with no-flying zone

Clusters must avoid flying over No-Flight Zones $O_1$ and $O_2$. $O_1$ is a rectangular shape with vertices placed at $o_1^1 = 20\hat{e}_1 + 80\hat{e}_2, o_1^2 = 35\hat{e}_1 + 100\hat{e}_2, o_1^3 = 20\hat{e}_1 + 100\hat{e}_2$, and $o_1^4 = 35\hat{e}_1 + 100\hat{e}_2$. No Flight Zone $O_2$ is a triangle with vertices placed at $o_2^1 = \ldots$
90\hat{e}_1 + 50\hat{e}_2, \quad o_2^2 = 100\hat{e}_1 + 60\hat{e}_2, \quad \text{and} \quad o_3^2 = 85\hat{e}_1 + 60\hat{e}_2. \quad \text{Target destinations of cluster 1 leaders are the same as in Case Study 1 but target destinations of cluster 2 leaders are different and given by:} \quad r_{1,F}^1 = 100\hat{e}_1 + 70\hat{e}_2 + 10\hat{e}_3, \quad r_{2,F}^1 = 100\hat{e}_1 + 100\hat{e}_2 + 10\hat{e}_3, \quad r_{1,F}^2 = 70\hat{e}_1 + 90\hat{e}_2 + 10\hat{e}_3, \quad r_{1,F}^3 = 40\hat{e}_1 + 10\hat{e}_2 + 10\hat{e}_3, \quad r_{2,F}^2 = 40\hat{e}_1 + 40\hat{e}_2 + 10\hat{e}_3, \quad \text{and} \quad r_{3,F}^2 = 10\hat{e}_1 + 30\hat{e}_2 + 10\hat{e}_3. \quad \text{In Figs. 10 (a-d), leaders' optimal paths connecting initial and target destinations are shown. Figs. 10 (e-h) show UAV clusters at } t = 60s, \quad t = 120s, \quad t = 180s, \quad \text{and} \quad t = 240s. \quad \text{Leaders' optimal paths are also listed in Table 2. Eigenvalues of matrices } U_j^{CD}, \text{denoted by } \lambda_j^1 \text{ and } \lambda_j^2, \text{are plotted versus time in Fig. 11 (The superscript } j = 1, 2 \text{ is the cluster index number). Note that cluster 1 aggressively deforms}
in order to reach the desired final formation by passing through a narrow channel. However, cluster 2 moves as a rigid body; therefore, $\lambda_2^1(t) = \lambda_2^2(t) = 1$ at any time $t$. Furthermore, $\lambda_{ij}(t) > \lambda_{\text{CD,min}}^1$ at any time $t$ ($i = 1, 2$ and $j = 1, 2$). Therefore, followers all remain inside the triangular domain defined by the cluster leaders and no two quadcopters collide while cluster 1 aggressively deforms to avoid obstacles and reach the target formation. $x$ and $y$ components of actual positions of quadcopters are plotted versus time in Fig. 12. Furthermore, Fig. 13 plots deviations of agents of cluster 1 and 2 from the desired continuum deformation.

8. Conclusion and future work

This paper studies the problem of multi-cluster continuum deformation optimization in which multiple UAV teams form clusters that are safely coordinated for flight through a shared motion space. By treating MUS evolution with continuum deformation for contained UAV clusters, collision-free collective motion with minimal communication and manageable planning overhead are achieved despite changes in each cluster’s shape over the trajectory. The paper contributes a novel hierarchical strategy for leader planning above continuum deformation collision-free coordination. Future work is necessary to extend the triangular clusters and constraint to constant altitude to a full three-dimensional cluster shape and trajectory space.

This work is based on the assumption that the total number of agents is fixed within every cluster in a continuum deformation coordination. In future work, we aim to enhance the resilience and scalability by forming a hybrid multi-cluster deformation coordination framework with two modes: (i) “Merge-Split” and (ii) “Continuum Deformation”. In the “Merge-Split” mode, our recently-proposed deployment method [52] is used to safely merge and split clusters, and reduce or increase the number of clusters. In the “Continuum Deformation” mode, the multi-cluster continuum deformation coordination approach, proposed in this paper, can be applied to safely manage coordination of many agents in an obstacle-laden environment.

Declaration of Competing Interest

None declared.

Acknowledgements

This work is dedicated to the late Professor Suhada Jayasuriya. This work has been supported by the National Science Foundation under Award Nos. 1134669, 1250280, and 1739525.

References
