Joint Energy Procurement and Demand Response Towards Optimal Deployment of Renewables

Xuanyu Cao, Junshan Zhang, and H. Vincent Poor

***Abstract*—In this paper, joint energy procurement and demand response is studied from the perspective of the operator of a power system. The operator procures energy from both renewable energy sources (RESs) and the spot market. We observe the fact that the RESs may incur considerable infrastructure cost. This cost is taken into account and the optimal planning of renewables is examined by controlling the investment in RES infrastructures. Due to the uncertainty of renewables, the operator can also purchase energy directly from the spot market to compensate for the possible deficit incurred by the realization of the random renewable energy. By setting appropriate prices, the operator sells the collected energy to heterogeneous end users with different demand response characteristics. We model the decision making process of the operator as a two-stage optimization problem. The optimal decisions on the renewable deployment, energy purchase from the spot market, and pricing schemes are derived. Several solution structures are observed and a computationally efficient algorithm, requiring only closed-form calculation and simple bisection search, is proposed to compute the optimal decisions. Finally, numerical experiments are conducted to verify the optimality of the proposed algorithm and the solution structures observed theoretically. In particular, the impact of renewable penetration and the importance of its optimal design are highlighted.**

***Index Terms*—Renewable energy sources, demand response, pricing, smart grid, optimization, resource allocation.**

I. Introduction

DUE to their low generation costs and low production of pollution, renewable energy sources (RESs), e.g., wind and solar energy, are envisioned as indispensable elements of future power grids [1]–[3]. RESs are usually uncertain (random) and intermittent (time-varying), which makes them nondispatchable and cannot be readily incorporated into existing power systems. Therefore, tremendous research efforts have been devoted to the integration of RESs into smart grid operations in the recent decade.

In [4], motivated by the fact that the forecast of renewables may be accurate within a certain time window, Ilic *et al.* advocated a model predictive control approach to dynamically adjust the economic dispatch (ED) in response to the updated forecast of renewables. In [5], modeling the renewables as random variables with known distribution, e.g., Weibull distribution for wind speed, Liu proposed chance constrained methods for ED so that the supply demand balance is satisfied with high probability under the known distribution of the renewables. In practice, energy trade is performed at different timescales, e.g., day ahead planning, real time energy procurement, before/after the realization of random renewables. Thus, several multi-stage dynamic optimization approaches were proposed in [6], [7] where the energy is generated or purchased at different time periods. An analogous multi-stage stochastic programming approach, named risk limit dispatch (RLD), was proposed by Varaiya *et al.* in [8] to combat the uncertainty of renewable generation and user demands. Later, Zhang *et al.* extended it to network RLS by taking into account the power network topology and gave explicit expression for the price of uncertainty [9]. Besides, to match the time-varying user demands, integration of renewables in a given power network topology was investigated in [10]. Recently, multi-stage optimization methods were proposed to integrate renewables for energy efficient wireless communications [11]. A more robust (and also more conservative) approach to incorporate renewables was the robust optimization framework pursued in [12] and [13], where the outputs of RESs were known to be located in some uncertainty set and the objective was to optimize the worst-case performance of the power system. Furthermore, in [14], inspired by the dual gradient method widely used in communication networks [15], Enyioha *et al.* presented an online power allocation algorithm to adapt to the time-varying energy supply and user demands. In addition, a network calculus approach was proposed in [16] to incorporate renewables by accounting for the presence of energy storage devices.

In most existing works on RESs, the renewables are considered stochastically given (e.g., with a fixed distribution) or deterministically given (e.g., known within a time window or known to be located in a uncertainty set). In other words, the renewables are not subject to design and optimization. Moreover, the renewables are assumed to be completely cost free. In practice, though the operator of RESs incurs little cost, their infrastructures necessitate substantial cost such as the construction cost and the maintenance cost. For example, the typical unit

Manuscript received September 30, 2017; revised March 8, 2018; accepted June 6, 2018. Date of publication June 11, 2018; date of current version July 27, 2018. This work was supported in part by the U.S. Army Research Office under Grant W911NF-16-1-0448 and in part by the Defense Threat Reduction Agency under Grant HDTRA1-13-1-0029. The guest editor coordinating the review of this manuscript and approving it for publication was Prof. Nikolaos Gatsis. *(Corresponding author: Xuanyu Cao.)*

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Digital Object Identifier 10.1109/JSTSP.2018.2846043

infrastructure cost of solar panels is around \$0.1218 per kW·h (calculation of this number is based on information from the website "Solar Power Authority" and is detailed in Section II) while the residential electricity rate in the US is between \$0.0837-0.3734 per kW·h [17]. These numbers suggest that the infrastructure cost of RESs is comparable with the cost of traditional energy sources and thus should not be neglected. Therefore, in this paper, we are motivated to take into account the cost of renewables and design *optimal* deployment of RESs in a power system. We can "control" the distribution, particularly expected generation, of the RESs by investing on RES infrastructures appropriately, e.g., constructing appropriate amount of solar panels or wind turbines. Further, we note that trading and management of renewable energy have been studied through the lens of stochastic optimization in [18], where the energy management system (EMS) can purchase the surplus of distributed renewable generation with certain prices. This cost of real-time purchase of renewable energy is fundamentally different from the infrastructure cost of renewable generation (usually incurred in the planning phase of the power system) considered in this paper.

After the construction of RES infrastructures, in each following time frame, e.g., each hour or each day, the generation of the RESs is still uncertain since RESs are random and intermittent. Thereby, after the renewables at a time frame are realized, if the realization level is low, the operator (e.g., the utility company) of the power system may need to purchase extra energy from some spot market (usually with relatively high unit price) to compensate the energy deficit. Then, the operator sells the collected energy to end users through judious pricing schemes so that appropriate demands are elicited. This energy sale method is called demand response (DR), a widely used technique in the demand side management (DSM) in smart grid.

In the literature, extensive DR and DSM schemes have been proposed [19]–[21]. In [22], the device scheduling problems at a single user's side was studied to maximize the net benefit. In [23], Kim and Giannakis examined the demand management problem with multiple subscribers and dynamic prices and proposed parallel algorithms for the corresponding mixed integer programming. Additionally, DR with real-time pricing was also investigated in [24] and [25], in which the effects of price uncertainty and prediction were incorporated. Furthermore, the competition between multiple end users was studied under game-theoretic frameworks in [26]. Later, Maharjan *et al.* took the competition among utility companies into consideration and proposed a Stackelberg game approach for DR in [27]. Besides, a VCG auction mechanism was proposed in [28] to incentivize users to reveal their private information truthfully. A review of game-theoretic approaches in DSM was presented in [29]. One important principle in DR is to shift the loads away from the peak hours so that the generation cost is reduced. This matches well with the charging needs of plug-in electric vehicles (EVs), which only require to be charged in some given time intervals and are flexible in charging rates across time. Thus, many works were devoted to schedule EV charging in the context of DR and DSM [30]–[34]. Moreover, DR in the presence of RESs was examined in [35]–[39].

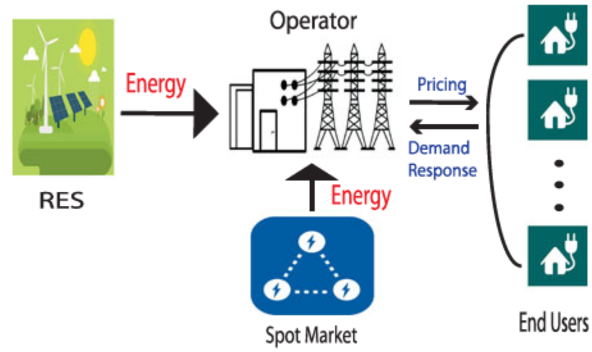

Fig. 1. System model.

The main focus of this paper is on the impact of optimal design of RES deployment. Thus, we use a simplified DR model for users. Specifically, we assume that the DR functions of the heterogeneous users are known to the operator through past usage history and these DR functions remain unchanged across time (if they do change, the operator needs to remodel the power system and compute new optimal decisions). We summarize the main contributions of this paper as follows.

- A profit maximization problem is formulated from the perspective of the operator of a power system. The operator procures energy from the RESs and the spot market and then sells the energy to heterogeneous end users with different demand response functions. The optimization problem consists of two stages: (1) optimal deployment design of RESs (e.g., investment in RES infrastructures) in the planning phase; (2) optimal energy procurement from the spot market and optimal pricing to users in each time frame. In the latter stage, optimal decisions are made based upon the realization of the random renewables in that time frame.
- The solution of the formulated profit maximization problem is derived and several structures of the solution are observed. Accordingly, a computationally efficient algorithm, involving only closed-form calculations and simple bisection search, is proposed to sequentially find the optimal decisions on RES deployment, purchase from the spot market and pricing. Numerical experiments are implemented to corroborate the optimality of the algorithm and the observed solution structures. In particular, the importance of the optimal design of RES penetration is highlighted.

The remaining part of this paper is organized as follows. In Section II, we formally introduce the system model and problem formulation. In Section III, we solve the formulated optimization problems and propose an efficient algorithm to compute the solution. Simulation results are presented in Section IV and we conclude this paper in Section V.

II. Problem Formulation

Consider a power system comprised of an operator, an RES, a spot market selling electrical power and some end users of power, as illustrated in Fig. 1. The operator can be a utility company who serves users in a region. The operator procures energy from two sources: the RES and the spot market. The RES may consist of solar farms and wind farms. The energy sold in the spot market mostly originates from traditional energy sources, e.g., coal and gas, and is thus more costly. Though the energy of RES is potentially cheaper and cleaner than that of the spot market, the former is subject to uncertainty and intermittence owing to the stochastic temporal variations of wind and solar energy. Thus, when the realization of the renewable energy is not enough, the operator may need to purchase extra energy from the spot market to compensate the energy deficit. Aftering procuring the energy from the RES and the spot market, the operator sells it to the end users by setting appropriate prices. The users' demand response (DR) depends on the announced prices: high prices suppress demands while low prices enhance demands. In this paper, we consider from the operator's perspective and our goal is to maximize its (expected) profit from procuring and selling the energy. In the following, we present the system model and formulate the optimization problems in detail.

A. Renewable Energy Source

In existing works [4]–[7], [9]–[14], [16], renewable energy is usually modeled as random variables with fixed distribution or deterministic quantities with partial knowledge, e.g., predictable within some time window or known to be located in some uncertainty set. In other words, the renewable energy is either stochastically or deterministically given and it is not subject to design and optimization. Additionally, in most existing models, renewable energy does not incur any cost. In practice, though the operation of RES facilities, e.g., solar panel and wind turbine, incurs little cost, their construction and maintenance necessitate remarkable infrastructure cost. For example, according to the website "Solar Power Authority", the infrastructure cost of solar panels is between \$7–\$9 per watt, which is approximated as \$8000 per kW. The typical lifespan of solar panels is around 20 years. In each year, we assume that 75% of days are clear days with sufficient sunshine and roughly one half of each day is daytime. Thus, a solar panel can work for $20 \times 365 \times \frac{3}{4} \times 12 = 65700$ hours. Thus, the typical cost of a solar panel is \$8000/65700 = \$0.1218 per kW·h. As comparison, in the United States, Hawaii residents have the highest electricity rate of \$0.3734 per kW·h while Louisiana residents have the lowest of \$0.0837 per kW·h [17]. These numbers suggest that the infrastructure cost of RES is comparable with the cost of traditional energy. Thus, the cost of RES should not be neglected when incorporating renewables into power systems. Since renewable energy is random and intermittent, we cannot control its exact realization at each time. However, we can control the *expected* amount of harvested renewables by constructing appropriate amount of RES facilities. Therefore, different from most existing works, in this paper, we are motivated to *design* the expected amount of renewables optimally by taking the (infrastructure) cost of RES into account.

Consider a certain time frame, e.g., one hour or one day. The amount of renewable energy harvested during this time frame is a random variable R. For simplicity, we assume that R is uniformly distributed over some interval $[0, r]$, where $r \geq 0$ is

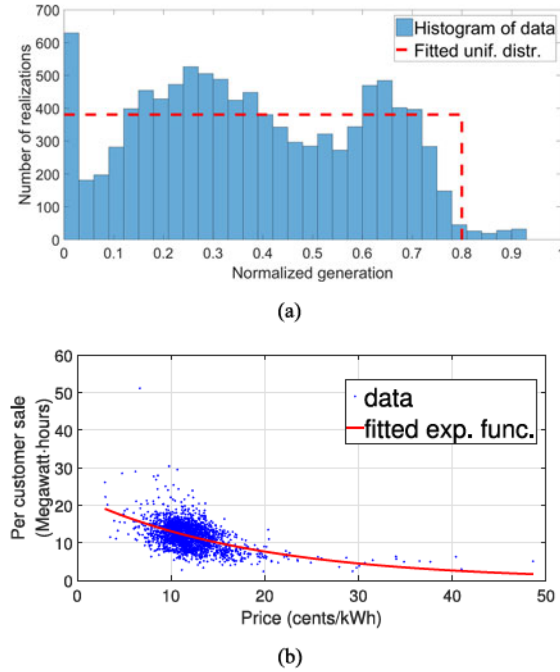

Fig. 2. Justification of the uniform distribution of renewable generation (left) and the exponential form of the demand response function (right). (a) Histogram of wind generation [40]. (b) Per customer electricity sale versus prices [41].

the maximum possible realization of renewables. We note that this uniform distribution can serve as an approximation of the distribution of real-world renewable generation. To verify this claim, we make use of the wind power data (made publicly available by the Australian Electricity Market Operator) of several wind farms in south-east Australia from 2012 to 2013 [40]. In Fig. 2(a), we plot the histogram of the normalized wind power generation data and it fits well with a uniform distribution. We observe that, though the distribution of the real data is complicated with several peaks and valleys, it can be approximated by the uniform distribution reasonably well. In practice, if more accurate approximation is needed, one can use piecewise uniform distribution (i.e., the support is split into several intervals and the probability density function (PDF) is constant within each interval) in lieu of uniform distribution. The analysis in this paper can be extended to the scenario with piecewise uniformly distributed renewable generation at the cost of cluttered notations. Actually, in such a scenario, the integrals in the proofs in Section III-C need to be split into multiple integrals over individual intervals of the piecewise uniform distribution.

We denote the unit cost of the expected renewables as $\alpha > 0$. Thus, the cost of the RES is $\alpha\mathbb{E}[R] = \frac{\alpha r}{2}$. Here, r is a design variable to be chosen by a private party such as an investor or a system planner, who owns the RES infrastructures. This can be achieved by constructing a certain amount of RES facilities. Note that r is determined in the planning phase of the power system as it is related to the construction of RES infrastructures. Once determined, its value will be used for the following multiple time frames, e.g., one year. Define $\theta = \frac{R}{r}$ to be the realization factor, which is a random variable uniformly located on $[0, 1]$. The realization factor θ changes across time frames since renewables are intermittent and time-varying depending on the weather conditions. When choosing r in the planning phase, the private party does not know the realizations of θ in later time frames. In the planning phase, the private party is only aware of the statistical distribution (uniform distribution in this case) of θ.

B. Spot Market

After r is chosen and the planning phase is over, the power system starts operating in the following multiple time frames. In each time frame, a new θ is realized, i.e., the renewable energy $R = \theta r$ is realized. After the realization of θ, the operator may purchase extra electrical energy from the spot market to compensate the energy deficit if the realization of θ is low. The unit price of the energy from the spot market is $\beta > 0$. Denote the amount of energy purchased from the spot market as $s \geq 0$, which is chosen by the operator. Thus, the operator pays βs to the spot market. Note that, different from the amount of renewables R, s is a deterministic quantity without any randomness. Usually, the unit price β in the spot market is higher than that of the RES α as the energy in the spot market mainly originates from traditional energy sources. Notice that r and s are decided at different phases: r is determined in the planning phase before θ is realized while s is determined in each time frame after θ is realized for this frame. This information gap is the advantage of spot market over the RES. The operator should weigh this information gap and the price gap between α, β judiously to make the optimal decisions.

C. Pricing

In each time frame, after s is chosen, the operator has $\theta r + s$ amount of energy supply. Then, it sells the energy to the end users through pricing and demand response. Suppose there are n users in total. If the operator announces a unit energy price of p_i to user i, the demand response of user i is $d_i(p_i)$, which is some monotonically decreasing function of p_i. In this paper, for mathematical tractability, we assume an exponential form of demand response function $d_i(p_i) = \xi_i e^{-\phi_i p_i}$, where ξ_i and ϕ_i are two positive constants describing the price-demand characteristic of user i. This exponential demand response function can be used as an approximation of the real-world price-demand relations. To validate this approximation, we make use of the annual retail sales data (by state and utility company) of electricity in the U.S. in 2016 [41]. The relations between the price and the per customer sale are plotted in Fig. 2(b), and an exponential function ae^{bx} is fitted, where $a = 22.35$ and $b = -0.05333$. We remark that the exponential function can fit the real data reasonably well, considering that the exponential function is very simple with only two parameters. This justifies the choice of exponential demand response function. Given the demand response function $d_i(p_i)$, the operator obtains $p_i \xi_i e^{-\phi_i p_i}$ amount of revenue from user i. In general, different users have different price-demand parameters, i.e., $(\xi_i, \phi_i) \neq (\xi_j, \phi_j), \forall i \neq j$. This captures the heterogeneity of real-world users. Facing with the same price, different users often have different demands as they may have different living standards and economic conditions. We assume that the operator is aware of the price-demand parameters (ξ_i, ϕ_i) of each node i. This assumption is reasonble as these parameters can be learned from the past power usage history of the users. Since users are heterogeneous, the operator uses differentiated pricing $\boldsymbol{p} = [p_1, \ldots, p_n]^\mathsf{T}$, i.e., it sets different prices for different users, to obtain more revenue. Intuitively, if a user i has high demand, i.e., ξ_i is large and/or ϕ_i is small, the operator will set high price for her to extract more profit from her. Thus, the differentiated pricing scheme $\boldsymbol{p}$ can potentially enhance the fairness of energy allocation among the heterogeneous users: it suppresses high demands of individual users and encourages low demand users to consume more energy. We note that various setups/methods of differentiated pricing have been used worldwide and have been studied in the research literature. For instance, in 2004, the National Development and Reform Commission of China established a policy permitting differential electricity pricing for high energy-consuming industries in which electricity prices can be set based on the energy efficiency level of each enterprise. The differentiated energy tarrifs are designed to phase-out energy inefficient enterprises by imposing high electricity prices on them. Various differentiated pricing schemes have been proposed and analyzed in the research literature to penalize enterprises with high energy consumptions [42], [43]. Further, differentiated rate plans have been proposed in [44] for residential electricity based on individual customers' consumption behaviors. Spatially varying differential pricing schemes have also been applied to electric vehicle charging in [45]. Another example of differentiated pricing is the locational marginal pricing (LMP), which has been used in the U.S. and investigated in the literature extensively [46], [47]. In light of the above, differentiated pricing considered in this paper can be applied to industrial enterprises, electric vehicles, and even residential electricity in the future.

Additionally, we remark that electricity prices are consequences of many complicated factors in real-world markets. The U.S. Energy Information Administration (EIA) has identified several key factors that influence the electricity prices, including costs of fuels, costs of power plants, weather conditions, and costs of the transmission/distribution systems [48]. The operators of the power systems, e.g., the utilities, adjust electricity prices based on these factors. In the model of this paper, the costs of fuels can be incorporated into the energy price β of the spot market. The costs of power plants are embodied by the unit cost of renewable infrastructure α and can be controlled by the design of renewable deployment r. The time-varying random weather conditions are captured by the random renewable realization factor θ, which varies across time frames. The costs of transmission/distribution systems can be partially reflected by the spot market price β. More comprehensively speaking, the costs of transmission systems should depend on the topologies of the underlying power networks and the electricity prices need to be set to satisfy the operation constraints of the power networks [49]. In this paper, for simplicity, we do not take into account the effects of power network constraints and only focus on the optimal design of renewable deployment. The impact of power networks is a promising direction for future work. The notations of the system model are summarized in Table I.

TABLE I
NOTATIONS OF THE MODEL

Notations	Definitions
r	The maximum possible amount of renewable energy
R	The realized amount of renewable energy
θ	The realization factor of the RES
α	The unit cost of the expected renewable energy
s	The amount of energy purchased from the spot market
β	The unit price of energy from the spot market
ξ_i, ϕ_i	The price-demand parameters of user i
$\boldsymbol{p}$	The price vector comprised of prices of all users

D. Optimization Problems

Based on the above system model, the private party (e.g., an investor owning the RES) aims at solving the following optimization problem to maximize the expected profit:

$$\text{Maximize}_{r\geq 0}\ \mathbb{E}_\theta[w(\theta,r)], \tag{1}$$

where $w(\theta,r)$ is the profit with fixed r and θ and the corresponding optimal s and $\boldsymbol{p}$. The computation of the function $w(\theta,r)$ is through the following optimization problem:

$$w(\theta,r) = \sup_{s\in\mathbb{R},\boldsymbol{p}\in\mathbb{R}^n}\left\{\sum_{i=1}^n p_i\xi_i e^{-\phi_i p_i} - \beta s - \frac{\alpha r}{2}\ \middle|\ \theta r + s \geq \sum_{i=1}^n \xi_i e^{-\phi_i p_i},\ s\geq 0,\ \boldsymbol{p}\succeq \boldsymbol{0}\right\}, \tag{2}$$

which is used by the operator to determine the optimal s and $\boldsymbol{p}$. The objective function in (2) is the total revenue from all users substracted by the costs from the RES and the spot market while the constraint points to the fact that the total procured energy should be no less than the total sold energy. We note that real-world power systems must balance the supply and the demand, i.e., the total procured energy must be equal to the total sold energy, so that the first constraint in (2) must hold with equality in practice. Nevertheless, in most practical scenarios, surplus energy or oversupply can usually be disposed of at negligible cost. For example, the operator can cut some solar panels off from the grid for supply curtailment [50]. Better solutions include sharing/selling the excessive energy to other utilities or storing surplus energy if local storages are available. As such, in problem (2), we allow the total procured energy to be larger than the total sold energy. Moreover, in (2), we restrict the prices to be nonnegative since excessive energy can be handled at negligible expense and the operator never needs to pay (i.e., sell at negative prices) to get rid of surplus energy. Additionally, we note that the cost of renewable energy generation mainly comes from the infrastructure cost, which does not depend on the specific renewable realization factor θ in a certain time frame. Instead, this infrastructure cost is proportional to the *expected* renewable generation $\frac{r}{2}$ (with proportion factor α), which is directly related to the quantity/quality of infrastructure construction. The renewable realization factor θ depends on the weather conditions and is not related to the infrastructure directly. As such, in the objective function of (2), the cost of renewable energy is $\frac{\alpha r}{2}$, which does not depend on θ.

Further, the computation of $w(\theta,r)$ in (2) can be decomposed into two optimization problems:

$$w(\theta,r) = \sup_{s\geq 0}\{h(\theta r+s) - \beta s\} - \frac{\alpha r}{2}, \tag{3}$$

where the function $h(\cdot)$ is defined as: $\forall t>0$,

$$h(t) = \sup_{\boldsymbol{p}\in\mathbb{R}^n}\left\{\sum_{i=1}^n p_i\xi_i e^{-\phi_i p_i}\ \middle|\ \sum_{i=1}^n \xi_i e^{-\phi_i p_i}\leq t,\ \boldsymbol{p}\succeq\boldsymbol{0}\right\}. \tag{4}$$

The derivation of this optimization decomposition is given as follows. Starting from (2), we have:

$$w(\theta,r)$$

$$\overset{(a)}{=} \sup_{s\in\mathbb{R},\boldsymbol{p}\in\mathbb{R}^n}\left\{\sum_{i=1}^n p_i\xi_i e^{-\phi_i p_i} - \beta s\ \middle|\ \theta r + s \geq \sum_{i=1}^n \xi_i e^{-\phi_i p_i},\ s\geq 0,\ \boldsymbol{p}\succeq\boldsymbol{0}\right\} - \frac{\alpha r}{2} \tag{5}$$

$$\overset{(b)}{=} \sup_{s\geq 0}\left(\sup_{\boldsymbol{p}\in\mathbb{R}^n}\left\{\sum_{i=1}^n p_i\xi_i e^{-\phi_i p_i}\ \middle|\ \theta r+s\geq\sum_{i=1}^n \xi_i e^{-\phi_i p_i}, \boldsymbol{p}\succeq\boldsymbol{0}\right\} - \beta s\right) - \frac{\alpha r}{2} \tag{6}$$

$$\overset{(c)}{=} \sup_{s\geq 0}(h(\theta r+s) - \beta s) - \frac{\alpha r}{2}, \tag{7}$$

where in (a) we move the term $\frac{\alpha r}{2}$ out of the sup since it does not depend on s and $\boldsymbol{p}$; in (b) we first hold s fixed and optimize over $\boldsymbol{p}$ only, and then optimize over $s\geq 0$ (the term βs is taken out of the inner sup because it does not depend on $\boldsymbol{p}$); in (c) we simply make use of the definition of the function $h(\cdot)$ in (4).

Our goal is to make optimal decisions on r, s and $\boldsymbol{p}$ by solving the two-stage optimization problem (1) and (2), where the former is before the realization of θ and the latter is after it. To this end, we need to sequentially solve the optimal pricing problem in (4), the optimal energy procurement from spot market in (3) and the optimal design of RES penetration in (1), which will be accomplished in Section III. Note that the optimal s and $\boldsymbol{p}$ depends on the realization of θ because the purchase from the spot market and pricing happen after the random renewable energy is realized.

Problem (1) and problem (2) are solved at different timescales. An instance of problem (2) with a particular realization of θ is solved in each time frame, e.g., each hour or each day. Its solution $\{s^*,\boldsymbol{p}^*\}$ is used in this time frame only. In contrast, problem (1) is solved at the planning phase of the power system (because r corresponds to the infrastructure construction) and its solution r^* will be used for multiple time frames, e.g., one year. For instance, a practical timescale of problems (1) and (2) can be as follows. The optimal r^* is decided for one whole year by constructing an appropriate amount of RES infrastructures. Afterwards, in each day of the following year, given the previously determined r for the whole year and the realization of θ in this day, the operator solves (2) and makes optimal decisions

s^* and $\boldsymbol{p}^*$, which are used in this day only. Notice that problems (3) and (4) are nothing but equivalent reformulation of problem (2) and they are solved at the same timescale, i.e., each time frame.

We note that, in practice, model parameters ξ_i, ϕ_i and β can be time-varying and/or random, which necessitates some modifications to the problem formulation in (1) and (2). Suppose these parameters are time-varying and denote their realizations at time frame t as $\xi_{i,t}$, $\phi_{i,t}$ and β_t, respectively. Define their time-averages as $\bar{\xi}_i = \frac{1}{T}\sum_{t=1}^T \xi_{i,t}$, $\bar{\phi}_i = \frac{1}{T}\sum_{t=1}^T \phi_{i,t}$, and $\bar{\beta} = \frac{1}{T}\sum_{t=1}^T \beta_t$, where T is the total number of time frames. Though the parameters $\xi_{i,t}$, $\phi_{i,t}$ and β_t may be random, we assume that reasonably accurate estimates of their time-averages $\bar{\xi}_i$, $\bar{\phi}_i$ and $\bar{\beta}$ are available in the planning phase based on past histories and patterns. Then, in the planning phase, these time-average quantities are used to compute the optimal r^* by solving a modified version of problem (1), i.e.,

$$r^* = \arg\max_{r\geq 0} \mathbb{E}_\theta[\bar{w}(\theta, r)], \tag{8}$$

where

$$\bar{w}(\theta, r) = \sup_{s\in\mathbb{R},\boldsymbol{p}\in\mathbb{R}^n} \left\{ \sum_{i=1}^n p_i \bar{\xi}_i e^{-\bar{\phi}_i p_i} - \bar{\beta} s - \frac{\alpha r}{2} \;\middle|\; \theta r + s \geq \sum_{i=1}^n \bar{\xi}_i e^{-\bar{\phi}_i p_i},\, s\geq 0,\, \boldsymbol{p}\succeq \boldsymbol{0} \right\}. \tag{9}$$

After r^* is determined, in each time frame t, the operator observes the current values of $\xi_{i,t}$, $\phi_{i,t}$, β_t and the current renewable realization factor θ_t. Then, she computes the optimal s_t^* and $\boldsymbol{p}_t^*$ by solving a modified version of problem (2), i.e.,

$$(s_t^*, \boldsymbol{p}_t^*) = \arg\max_{s\in\mathbb{R},\boldsymbol{p}\in\mathbb{R}^n} \left\{ \sum_{i=1}^n p_i \xi_{i,t} e^{-\phi_{i,t} p_i} - \beta_t s - \frac{\alpha r^*}{2} \;\middle|\; \theta_t r^* + s \geq \sum_{i=1}^n \xi_{i,t} e^{-\phi_{i,t} p_i},\, s\geq 0,\, \boldsymbol{p}\succeq \boldsymbol{0} \right\}. \tag{10}$$

We note that the optimization methods for problems (1) and (2) can be readily transformed to those of problems (8) and (10) with minor notation adaptations. In the analytical part of this paper, for simplicity, we still stick to the assumption that ξ_i, ϕ_i and β are time-invariant deterministic quantities because the main goal of this paper is to examin the optimal deployment of renewables instead of studying the dynamics of user demands and spot market prices. Nevertheless, numerical experiments based on real-world time-varying data will be conducted in Section IV, confirming the applicability of the proposed algorithm to time-varying parameters.

Additionally, in practice, multiple types of RESs (e.g., solar power, wind power and hydropower) may be used simultaneously to reduce the volatility of renewable generation. Different RESs can have different costs per expected renewable generation, which can be accommodated by some adaptation of the problem formulation (1) and (2) as follows. Suppose m types of RESs are used and the cost per expected renewable generation of RES j is α_j. Denote the maximum possible renewable generation and the renewable realization factor of RES j as r_j and θ_j, respectively, where θ_j is uniformly distributed over $[0, 1]$. Components r_j and θ_j of all RES types are stacked as vectors $\boldsymbol{r}\in\mathbb{R}^m$ and $\boldsymbol{\theta}\in\mathbb{R}^m$, respectively. In such a case, the two-stage optimization problems (1) and (2) become:

$$\text{Maximize}_{\boldsymbol{r}\succeq\boldsymbol{0}}\ \mathbb{E}_{\boldsymbol{\theta}}[\widetilde{w}(\boldsymbol{\theta}, \boldsymbol{r})], \tag{11}$$

where the profit $\widetilde{w}(\boldsymbol{\theta}, \boldsymbol{r})$ is given as:

$$\widetilde{w}(\boldsymbol{\theta}, \boldsymbol{r}) = \sup_{s\in\mathbb{R},\boldsymbol{p}\in\mathbb{R}^n} \left\{ \sum_{i=1}^n p_i \xi_i e^{-\phi_i p_i} - \beta s - \sum_{j=1}^m \frac{\alpha_j r_j}{2} \;\middle|\; \sum_{j=1}^m \theta_j r_j + s \geq \sum_{i=1}^n \xi_i e^{-\phi_i p_i},\, s\geq 0,\, \boldsymbol{p}\succeq \boldsymbol{0} \right\}. \tag{12}$$

Analogous to (3) and (4), optimization problem (12) can also be decomposed into two problems with respect to s and $\boldsymbol{p}$, respectively:

$$\widetilde{w}(\boldsymbol{\theta}, \boldsymbol{r}) = \sup_{s\geq 0} \left\{ h\left(s + \sum_{j=1}^m \theta_j r_j \right) - \beta s \right\} - \sum_{j=1}^m \frac{\alpha_j r_j}{2}, \tag{13}$$

where the function $h(\cdot)$ is defined in (4). In principle, the analysis and solution methods of problems (1) and (2) can be extended to those of problems (11) and (12) for multiple types of RESs, though the notations will be more cluttered.

Lastly, if the assumptions of uniformly distributed renewable generation and exponential demand response functions do not hold, the detailed/quantitative results in this paper, e.g., Algorithm 1 to be presented, may need to be modified. Nevertheless, most of the structural/qualitative results, e.g., the threshold structures to be stated in Section IV, are still true, i.e., they are robust to these assumptions.

III. Optimal Decisions

In this section, we derive the optimal decisions of the renewable energy r, the purchase from spot market s and the pricing scheme $\boldsymbol{p}$ by solving the optimal pricing problem in (4), the optimal purchase from spot market in (3) and the optimal design of RES penetration in (1) sequentially. Accordingly, a computationally efficient algorithm is proposed to find the optimal decisions. The proposed algorithm only involves closed-form computation and simple bisection search.

A. Optimal Pricing

In this section, we solve the optimal pricing problem in (4), which can be rewritten as:

$$\begin{aligned} &\text{Minimize}_{\boldsymbol{p}\in\mathbb{R}^n}\ f(\boldsymbol{p}) := -\sum_{i=1}^n p_i \xi_i e^{-\phi_i p_i} \\ &\text{subject to}\ \sum_{i=1}^n \xi_i e^{-\phi_i p_i} \leq t, \\ &\qquad \boldsymbol{p}\succeq \boldsymbol{0}, \end{aligned} \tag{14}$$

where $t > 0$ is the problem parameter representing the total available energy supply from both the RES and the spot market. We note that the objective function $f(\boldsymbol{p})$ is not a convex function. Hence, problem (14) is a non-convex optimization problem, for which strong duality does not hold in general [51], [52]. However, in the sequel (Proposition 1), we show that strong duality actually holds for problem (14) and its solution can be obtained in pseudo-closed-form by using duality theory. Denote the optimal point and the optimal objective function value of problem (14) as $\boldsymbol{p}^*$ and $f^* = f(\boldsymbol{p}^*)$, respectively. The solution of problem (14) is given by the following result.

Proposition 1: The solution to problem (14) is as follows.

1) If $t \leq e^{-1}\sum_{i=1}^{n}\xi_i$, then the optimal point is given by $p_i^* = \lambda^* + \frac{1}{\phi_i}, i = 1, \ldots, n$, where $\lambda^* \geq 0$ is the unique solution to the following equation:

$$\sum_{i=1}^{n} \xi_i e^{-\phi_i \lambda^* - 1} = t, \tag{15}$$

and the corresponding optimal value is:

$$f^* = -\sum_{i=1}^{n} \frac{\xi_i}{\phi_i} e^{-\phi_i \lambda^* - 1} - \lambda^* t. \tag{16}$$

2) If $t > e^{-1}\sum_{i=1}^{n}\xi_i$, then the optimal point is given by $p_i^* = \frac{1}{\phi_i}, i = 1, \ldots, n$, and the corresponding optimal value is:

$$f^* = -e^{-1}\sum_{i=1}^{n} \frac{\xi_i}{\phi_i}. \tag{17}$$

Proof: The proof is presented in Appendix A. ■

In problem (14), t is the total available energy supply at the operator. According to the system model or equation (3), we know that $t = \theta r + s$. When the total supply t is large enough so that case (2) of Proposition 1 takes place, we observe that the total energy will not be sold out. In other words, in case (2), with optimal pricing, the total supply t is strictly larger than the total demands from all end users. The reason is that, to sell all energy, i.e., to elicit large demands from users, the prices need to be very low, which strongly hurt the revenue of the operator and are thus not optimal. Further, we note that the optimal prices $\boldsymbol{p}^*$ exhibit interesting structures. When the total supply t is very large, i.e., in case (2) of Proposition 1, $\boldsymbol{p}^*$ is a constant vector and does not decrease with t. The reason is that further lowering $\boldsymbol{p}^*$ will hurt the revenue. On the other hand, when the total supply t is small, i.e., in case (1) of Proposition 1, since λ^* decreases with t (c.f. (15)), so do the optimal prices $\boldsymbol{p}^*$. This is reasonable because lowering prices can boost user demands, which can help sell the increasing supply t.

To refer to the solution of equation (15) more compactly, we make the following definition.

Definition 1: Define $\delta : (0, +\infty) \mapsto \mathbb{R}$ to be the inverse function of $\sum_{i=1}^{n}\xi_i e^{-\phi_i x - 1}$, i.e., for any $y > 0$, $\delta(y)$ is the unique solution x of the following equation:

$$\sum_{i=1}^{n} \xi_i e^{-\phi_i x - 1} = y. \tag{18}$$

Note that $\sum_{i=1}^{n}\xi_i e^{-\phi_i x - 1}$ is a strictly decreasing function of x. Thus, its inverse function $\delta(\cdot)$ is well-defined and is also strictly decreasing. By definition, we immediately know:

$$\lim_{y\to 0^+} \delta(y) = +\infty, \ \lim_{y\to +\infty} \delta(y) = -\infty, \tag{19}$$

$$\delta\left(e^{-1}\sum_{i=1}^{n}\xi_i\right) = 0. \tag{20}$$

According to Proposition 1 and Definition 1, the function $h(\cdot)$ defined in (4) can be written in the following compact form. For any $t > 0$:

$$h(t) = \begin{cases} \sum_{i=1}^{n} \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(t) - 1} + t\delta(t), & \text{if } t \leq e^{-1}\sum_{i=1}^{n}\xi_i, \\ e^{-1}\sum_{i=1}^{n}\frac{\xi_i}{\phi_i}, & \text{if } t > e^{-1}\sum_{i=1}^{n}\xi_i. \end{cases} \tag{21}$$

The physical meaning of $h(t)$ is the maximal revenue the operator can obtain from the end users through differentiated pricing when t unit of energy supply is available.

B. Optimal Procurement From the Spot Market

Next, given θ and r, we compute the optimal decision on purchasing energy from the spot market by solving the problem in (3), i.e.,

$$\text{Maximize}_{s\geq 0} \ q(s), \tag{22}$$

where $q(s) := h(\theta r + s) - \beta s$. Note that the dependence of $q(s)$ on θ and r is implicit. The physical meaning of $q(s)$ is the partial profit, i.e., the revenue collected from the users substracted by the energy purchasing cost from the spot market. This partial profit does not contain the cost of the RES, which is independent of s. Define the optimal point and the optimal value of (22) as s^* and $q^* = q(s^*)$, respectively. The solution to problem (22) is given in the following result.

Proposition 2: The solution to problem (22) is given as follows.

1) If $\theta r > e^{-1}\sum_{i=1}^{n}\xi_i$, then the optimal point is $s^* = 0$ and the corresponding optimal value is $q^* = e^{-1}\sum_{i=1}^{n}\frac{\xi_i}{\phi_i}$.

2) If $\sum_{i=1}^{n}\xi_i e^{-\phi_i\beta - 1} \leq \theta r \leq e^{-1}\sum_{i=1}^{n}\xi_i$, then the optimal point is $s^* = 0$ and the corresponding optimal value is:

$$q^* = \sum_{i=1}^{n}\frac{\xi_i}{\phi_i} e^{-\phi_i\delta(\theta r) - 1} + \theta r\delta(\theta r). \tag{23}$$

3) If $\theta r < \sum_{i=1}^{n}\xi_i e^{-\phi_i\beta - 1}$, then the optimal point and the corresponding optimal value are given as:

$$s^* = \sum_{i=1}^{n}\xi_i e^{-\phi_i\beta - 1} - \theta r, \tag{24}$$

$$q^* = \sum_{i=1}^{n}\frac{\xi_i}{\phi_i} e^{-\phi_i\beta - 1} + \beta\theta r. \tag{25}$$

Proof: The proof is presented in Appendix B. ■

From Proposition 2, we can write the optimal purchase from the spot market compactly as $s^* = \left[\sum_{i=1}^{n}\xi_i e^{-\phi_i\beta - 1} - \theta r\right]^+$, where $[x]^+ = \max\{x, 0\}$. This suggests a threshold structure of the optimal s^*: the operator should purchase energy from the spot market if and only if the realized renewable energy θr is less than a constant threshold of $\sum_{i=1}^n \xi_i e^{-\phi_i\beta-1}$. Further, the optimal partial profit q^* also possesses interesting structures. When the renewable realization θr is very large, i.e., in case (1) of Proposition 2, q^* is a constant independent of θ and r. The reason is that, in such a case, the user demands are saturated even if no energy is purchased from the spot market (c.f. case (2) of Proposition 1). Thus, extra renewable supply will be wasted and cannot boost the partial profit q^*. When the renewable realization θr is medium, i.e., in case (2) of Proposition 2, q^* depends on θr nonlinearly through (23). In addition, when the renewable realization θr is small, i.e., in case (3) of Proposition 2, q^* increases linearly with θr.

C. Optimal Design of RES Penetration

According to (3), we know $w(\theta, r) = q^* - \frac{\alpha r}{2}$, where q^* is given in Proposition 2 and depends on θ, r implicitly. To simplify notations, we define the following four positive constants $\mu_1, \mu_2, \pi_1, \pi_2$:

$$\mu_1 = \sum_{i=1}^n \xi_i e^{-\phi_i\beta-1},\ \mu_2 = e^{-1}\sum_{i=1}^n \xi_i, \tag{26}$$

$$\pi_1 = \sum_{i=1}^n \frac{\xi_i}{\phi_i} e^{-\phi_i\beta-1},\ \pi_2 = e^{-1}\sum_{i=1}^n \frac{\xi_i}{\phi_i}. \tag{27}$$

With these definitions, we immediately know:

$$\delta(\mu_1) = \beta,\ \delta(\mu_2) = 0, \tag{28}$$

which will be used frequently in later analysis. Combining the definitions in (26), (27) and Proposition 2, we can write $w(\theta, r)$ compactly as:

$$w(\theta, r) = \begin{cases} \pi_1 + \beta\theta r - \frac{\alpha r}{2}, & \text{if } \theta r < \mu_1, \\ \sum_{i=1}^n \frac{\xi_i}{\phi_i} e^{-\phi_i\delta(\theta r)-1} + \theta r\delta(\theta r) - \frac{\alpha r}{2}, & \\ & \text{if } \mu_1 \le \theta r \le \mu_2, \\ \pi_2 - \frac{\alpha r}{2}, & \text{if } \theta r > \mu_2. \end{cases} \tag{29}$$

We further define another positive constant ϵ, which will be used later:

$$\epsilon = e^{-2}\sum_{i,j=1}^n \frac{\xi_i\xi_j\phi_j}{\phi_i+\phi_j}\left[\frac{1}{\phi_i+\phi_j} - \frac{1}{\phi_i+\phi_j}e^{-(\phi_i+\phi_j)\beta} - \beta e^{-(\phi_i+\phi_j)\beta}\right]. \tag{30}$$

In this section, we compute the optimal RES penetration r by solving the optimization problem (1). To this end, we first evalute the objective function of (1), i.e., $\mathbb{E}_\theta[w(\theta, r)]$, which is accomplished in the following lemma.

Lemma 1: As a function of r, $\mathbb{E}_\theta[w(\theta, r)]$ satisfies the following statements.

1) When $r > \mu_2$, we have:

$$\mathbb{E}[w(\theta, r)] = \pi_2 - \frac{\alpha r}{2} - \frac{\beta\mu_1^2}{2r} - \frac{\epsilon}{r}, \tag{31}$$

which is a concave function over the interval $r \in [\mu_2, +\infty)$.

2) When $\mu_1 \le r \le \mu_2$, we have:

$$\mathbb{E}[w(\theta, r)] = -\frac{\alpha r}{2} - \frac{\beta\mu_1^2}{2r} + \beta\mu_1 + \pi_1 + \int_{\mu_1}^r \delta(\mu)d\mu - \frac{1}{r}\int_{\mu_1}^r \delta(\mu)\mu d\mu, \tag{32}$$

which is a concave function over the interval $r \in [\mu_1, \mu_2]$.

3) When $0 \le r < \mu_1$, we have:

$$\mathbb{E}[w(\theta, r)] = \pi_1 + \frac{(\beta-\alpha)r}{2}. \tag{33}$$

Proof: The proof is presented in Appendix C. ■

Based on Lemma 1, we can solve (1) in the following proposition, in which we identify three regimes for the cost of RES.

Proposition 3: The optimal point r^* of the optimal RES penetration problem (1) is given as follows.

1) *(Low RES Cost Regime)* When $0 < \alpha \le \frac{\beta\mu_1^2+2\epsilon}{\mu_2^2}$, the optimal point is:

$$r^* = \sqrt{\frac{\beta\mu_1^2+2\epsilon}{\alpha}}. \tag{34}$$

2) *(Medium RES Cost Regime)* When $\frac{\beta\mu_1^2+2\epsilon}{\mu_2^2} < \alpha \le \beta$, the optimal point is:

$$r^* = \sum_{i=1}^n \xi_i e^{-\phi_i x^*-1}, \tag{35}$$

where x^* is the unique solution of the following equation over the interval $x \in [0, \beta]$:

$$e^{-2}\sum_{i,j=1}^n \frac{\xi_i\xi_j\phi_j}{\phi_i+\phi_j}\left[\left(x+\frac{1}{\phi_i+\phi_j}\right)e^{-(\phi_i+\phi_j)x} - \left(\beta+\frac{1}{\phi_i+\phi_j}\right)e^{-(\phi_i+\phi_j)\beta}\right] - \frac{\alpha}{2}\left(\sum_{i=1}^n \xi_i e^{-\phi_i x-1}\right)^2 + \frac{\beta\mu_1^2}{2} = 0. \tag{36}$$

(3) *(High RES Cost Regime)* When $\alpha > \beta$, the optimal point is: $r^* = 0$.

Proof: The proof is presented in Appendix D. ■

From Proposition 3, the optimal point r^* can be computed in closed-form in the low and high RES cost regimes. In the medium RES cost regime, define $\psi(x)$ as the L.H.S. of (36). We note $\psi(0) = \mu_2^2(-\frac{\alpha}{2} + \frac{\epsilon}{\mu_2^2} + \frac{\beta\mu_1^2}{2\mu_2^2}) \le 0$ and $\psi(\beta) = \frac{\mu_1^2(\beta-\alpha)}{2} \ge 0$. Therefore, equation (36) can be solved very efficiently by using simple bisection method. Besides, in accordance with the intuition, in the high RES cost regime, i.e., when the unit cost of renewable energy is larger than the unit price of energy in spot market, no investment in RES should be made. This observation should be true even if the assumptions of uniformly distributed renewable generation and exponential demand response functions made in Section II do not hold. This is because the operator always prefers deterministic energy procurement (spot market) to random energy generation (RES), as long as the price of the former is lower than that of the latter. This holds true

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Algorithm 1: Computing the optimal decisions.

Inputs:
  The unit (infrastructure) cost of RES: α
  The unit energy price of the spot market: β
  The price-demand parameters of users: (ξ_i, ϕ_i), $i = 1, ..., n$
  The realization factor (revealed after r is determined): θ
Outputs:
  The optimal RES penetration: r^*
  The optimal energy purchase from the spot market: s^*
  The optimal pricing vector: $\boldsymbol{p}^*$

1: Compute the constants $\mu_1, \mu_2, \pi_1, \pi_2, \epsilon$ according to (26), (27) and (30).
  ///////////////////////////Computation of r^*///////////////////////////
2: **if** $\alpha \le \frac{\beta\mu_1^2+2\epsilon}{\mu_2^2}$ **then**
3:   Compute r^* as in (34).
4: **else if** $\frac{\beta\mu_1^2+2\epsilon}{\mu_2^2} < \alpha \le \beta$ **then**
5:   Solve (36) by using bisection method to get x^*. Then compute r^* according to (35).
6: **else**
7:   Set $r^* = 0$.
8: **end if**
  ///////////////////////////Computation of s^*///////////////////////////
9: The realization factor θ is realized.
10: Compute $s^* = \left[\sum_{i=1}^n \xi_i e^{-\phi_i\beta-1} - \theta r^*\right]^+$.
  ///////////////////////////Computation of $\boldsymbol{p}^*$///////////////////////////
11: **if** $\theta r^* + s^* \le e^{-1}\sum_{i=1}^n \xi_i$ **then**
12:   Solve $\sum_{i=1}^n \xi_i e^{-\phi_i\lambda^*-1} = \theta r^* + s^*$ using bisection method to obtain $\lambda^* \ge 0$. Set $p_i^* = \lambda^* + \frac{1}{\phi_i}$, $i = 1, ..., n$.
13: **else**
14:   Set $p_i^* = \frac{1}{\phi_i}$, $i = 1, ..., n$.
15: **end if**

---

regardless of the specific distribution of renewable generation and the functional forms of demand response characteristics.

D. Summary of the Algorithm

Based on Propositions 1, 2 and 3, we summarize the computation procedure of the optimal decisions r^*, s^* and $\boldsymbol{p}^*$ in Algorithm 1. We note that, in practice, r^*, s^*, $\boldsymbol{p}^*$ are computed at different timescales. s^* and $\boldsymbol{p}^*$ are computed once in each time frame, e.g., each hour or each day, while r^* is computed in the planning phase and remains the same for multiple time frames, e.g., one year. For instance, in the planning phase, the private party (an investor or planner) uses Algorithm 1 to compute the optimal r^* and constructs the corresponding amount of RES infrastructures. Then, this r^* remains the same for the following whole year. In each day of the following year, after the realization of θ in this day is revealed, the operator uses Algorithm 1 to compute the optimal s^* and $\boldsymbol{p}^*$, which are used in this day only. In the next day, the realization of θ changes and the operator uses Algorithm 1 again to compute the new optimal s^* and $\boldsymbol{p}^*$.

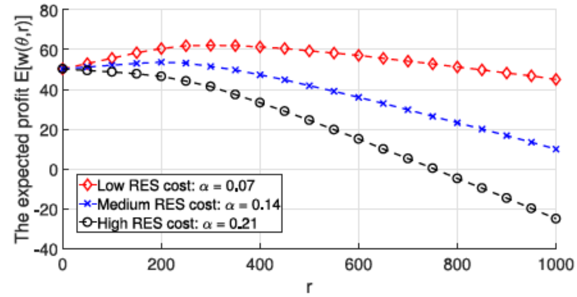
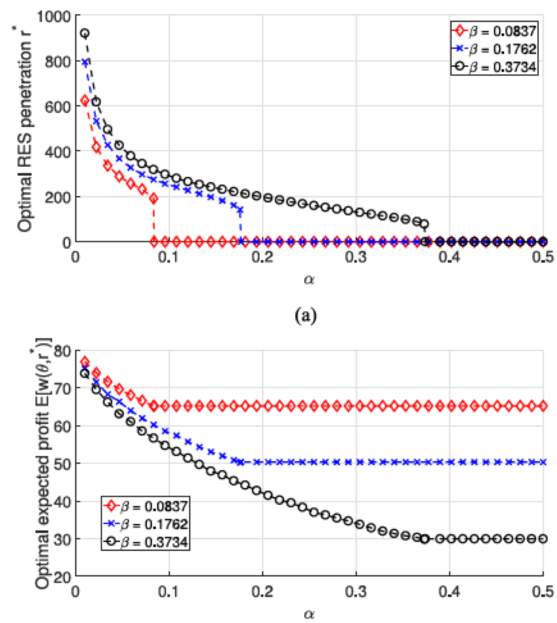


Fig. 3. The impact of the RES penetration r on the expected profit in low, medium and high RES cost regimes.



(a)

(b)

Fig. 4. The impact of the unit cost α of renewable energy and the unit price β of the energy from the spot market. (a) The impact of α, β on the optimal RES penetration r^*. (b) The impact of α, β on the optimal expected profit $\mathbb{E}(w(\theta, r^*))$

Further, as stated in Section II, the model parameters ξ_i, ϕ_i and β can be time-varying in practice. In such a case, some minor modifications of Algorithm 1 are needed. Specifically, denote the values of these parameters at time frame t as $\xi_{i,t}$, $\phi_{i,t}$ and β_t. Denote their time-averages as $\bar{\xi}_i$, $\bar{\phi}_i$ and $\bar{\beta}$, respectively (defined in Section II). In the part *Computation of* r^* of Algorithm 1 (planning phase), we replace ξ_i, ϕ_i and β with their time-average versions $\bar{\xi}_i$, $\bar{\phi}_i$ and $\bar{\beta}$, respectively. Additionally, in each time frame t, the parts *Computation of* s^* and *Computation of* $\boldsymbol{p}^*$ of Algorithm 1 are executed by replacing ξ_i, ϕ_i and β with their current values at time frame t, i.e., $\xi_{i,t}$, $\phi_{i,t}$ and β_t.

IV. Numerical Experiments

In this section, simulations are implemented to verify the optimality of Algorithm 1 and the solution structures of problems (1) and (2). In all experiments, we consider a power system depicted in Fig. 1 with $n = 100$ end users and set the price-demand parameters as $\xi_i = \sqrt{i}$, $\phi_i = 1 + \frac{4(i-1)}{99}$, $i = 1, ..., 100$. These user parameters are chosen for demonstration purpose only and our observations in this section hold with general parameters.

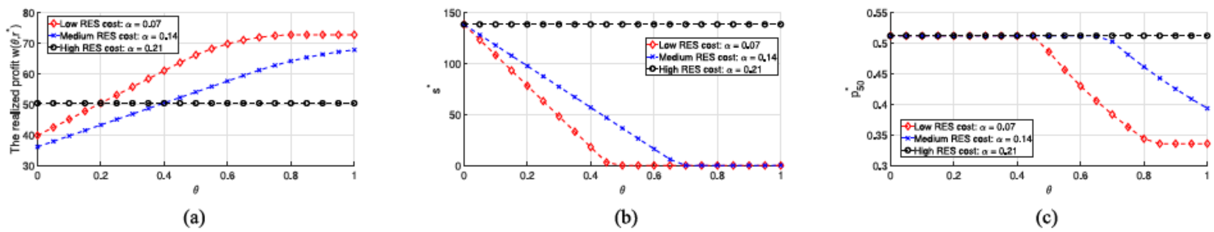


Fig. 5. Impact of the realization factor θ. (a) The impact of θ on the realized profit $w(\theta, r^*)$. (b) The impact of θ on the optimal spot market purchase s^*. (c) The impact of θ on the optimal price p^*_{50} for user 50.

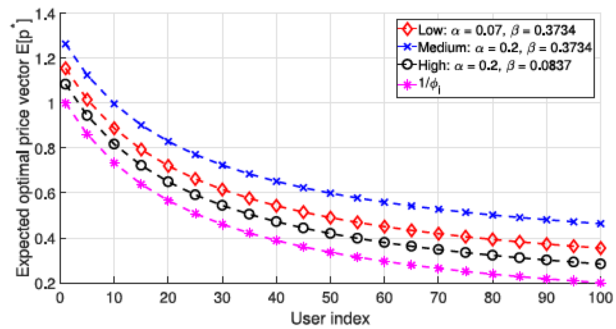


Fig. 6. The impact of α and β on the expected optimal pricing vector $\mathbb{E}[\boldsymbol{p}^*]$.

First, we demonstrate the importance of the *optimal* design of RES penetration. To this end, we vary the RES penetration r and study its impact on the expected profit $\mathbb{E}[w(\theta, r)]$. From [17], we know that the electricity rate of New York residents is \$0.1762 per kW·h. Thus, we choose the unit price of energy from the spot market to be $\beta = 0.1762$. We consider three possible values of the unit cost of renewable energy: $\alpha = 0.07$, $\alpha = 0.14$ and $\alpha = 0.21$, which correspond to the low, medium, and high RES cost regimes, respectively, according to Proposition 3. As explained in Section II, we note that the typical cost of renewable energy from solar panels is around \$0.1218 per kW·h, which is at the same scale of the chosen values for α. For these three values of α, the plots of the expected profit versus the values of r are given in Fig. 3, in which the expectation is computed as the average of 10000 Monte Carlo trials. According to Proposition 3, for $\alpha = 0.07$, 0.14, 0.21, the values of the optimal r^* are 300.5321, 203.445 and 0, respectively, which are in accordance with the maximal points of the three curves in Fig. 3. This confirms the optimality of Algorithm 1 in choosing r. Moreover, we observe the importance of optimal design of RES penetration from Fig. 3. Specifically, in the low and medium RES cost regimes, some renewables should be incorporated to improve the profit, e.g., when $\alpha = 0.07$, appropriate RES penetration can improve the expected profit from $\mathbb{E}[w(\theta, 0)] = 50.31$ to $\mathbb{E}[w(\theta, r^*)] = 62.13$. The reason is that, in low and medium RES cost regimes, the unit cost of RES is lower than the unit price in the spot market. However, renewable energy should not be incorporated too much since its realization is uncertain and the extra supply, if realized, can only be sold at low prices or even wasted (c.f. Proposition 1). In the following experiments, the renewable deployment r is chosen to be the optimal r^* by Algorithm 1 unless otherwise noted.

Next, we investigate the impact of the unit cost α of renewable energy and the unit price β of the energy in the spot market. From [17], we know that, in the US, Hawaii residents have the highest electricity rate (\$0.3734 per kW·h) while Louisiana residents have the lowest (\$0.0837 per kW·h). Togerther with New York residents' rate of (\$0.1762 per kW·h, we consider three possible values of the unit price of energy from the spot market: $\beta = 0.0837$, $\beta = 0.1762$ and $\beta = 0.3734$. For each of these three values of β, we plot the optimal RES penetration r^* versus the unit cost α of renewables, as illustrated in Fig. 4(a). For fixed β, we observe a threshold structure of r^* when α varies. When α increases, r^* first decreases smoothly (since the renewable energy becomes more and more expensive) and then drops to zero suddenly. From Fig. 4(a), we see that the threshold of α for r^* to drop to zero is the corresponding β of the curve. The reason is that, when α is larger than β, the system enters the high RES cost regime and r^* becomes zero (c.f. Proposition 3). In addition, for fixed α, by comparing the three curves in Fig. 4, we observe that r^* increases with β. The reason is that, as the energy from the spot market becomes more expensive, more investment should be made in RES. Furthermore, we examine the impact of α, β on the optimal expected profit $\mathbb{E}[w(\theta, r^*)]$ in Fig. 4(b). For fixed β, by increasing α, we see an analogous threshold structure of the profit, which first decreases and then remains constant. The reason is that, as long as α enters into the high RES cost regime, no investment in RES should be made, i.e., $r^* = 0$, and further increase in α will not affect the profit. Additionally, we note that r^* is zero in the high RES cost regime regardless of the specific distribution of the renewable generation and the functional forms of the demand response characteristics, i.e., robustness to the model assumptions in Section II. Thus, even if these assumptions do not hold, the aforementioned threshold structures of the optimal solution are still true.

We further study the influence of the realization factor θ. Recall that a new value of θ is realized in each time frame, e.g., each day or each hour, while the RES penetration r^* remains the same for multiple time frames, e.g., one year. So, the impact of θ reflects how the power system varies daily or hourly for a given RES penetration level r^*. Specifically, we set $\beta = 0.1762$ and consider three values of α: 0.07, 0.14 and 0.21, which are in the low, medium and high RES cost regimes, respectively. The impact of θ on the realized profit $w(\theta, r^*)$ is plotted in Fig. 5(a). We observe that, in the high RES cost regime, $w(\theta, r^*)$ is a constant independent of θ. The reason is that, in high RES cost regime, the optimal r^* is zero and the realization factor θ affects nothing. We also observe that, in the low RES cost regime, $w(\theta, r^*)$ first increases with θ and then saturates, i.e., remains constant, for large enough θ. The reason can be explained as

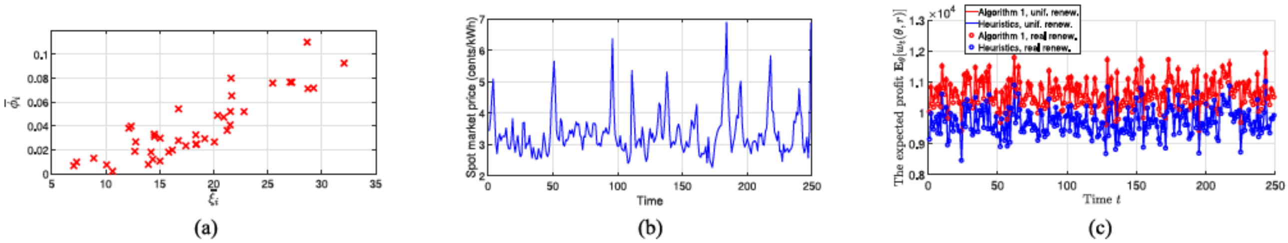


Fig. 7. Real data based experiment and comparison with heuristic method. (a) Distribution of price-demand parameters of users estimated from real data. (b) Daily spot prices of wholesale electricity of PJM West (an electricity hub) in 2017. (c) Comparison between Algorithm 1 and the heuristic method.

follows. In the low RES cost regime, the optimal r^* is no less than μ_2 (c.f. Proposition 3). Thus, according to (29), when $\theta > \frac{\mu_2}{r^*}$, $w(\theta, r^*) = \pi_2 - \frac{\alpha r^*}{2}$, which does not depend on θ (note that r^* is determined before θ is realized, i.e., r^* does not depend on the realization of θ). Actually, in such a case, the total supply t falls into case 2 of Proposition 1 so that the supply is larger than demand and any extra supply will be wasted. Thus, further increase in θ will not enhance the realized profit any more. Furthermore, we plot the impact of θ on the optimal purchase s^* from the spot market in Fig. 5(b). We observe a similar threshold structure: for large enough θ, s^* is zero, i.e., the operator does not purchase any energy from the spot market. This verifies the threshold solution structure of s^* in Proposition 2. Additionally, in Fig. 5(c), we examine the impact of θ on the optimal price p^*_{50} for user 50. We remark that, in all RES cost regimes, p^*_{50} remains constant for small θ. The reason is that, for small θ, from Proposition 2, we know that $s^* = \sum_{i=1}^n \xi_i e^{-\phi_i \beta - 1} - \theta r^*$ so that the total supply $t = s^* + \theta r^*$ is a constant $\sum_{i=1}^n \xi_i e^{-\phi_i \beta - 1}$. Hence, the optimal pricing vector p^* also remains constant. In low and medium RES cost regimes, when θ is larger than a certain threshold, p^* decreases with θ, i.e., more realized renewable energy helps reduce energy prices at the user side. We further observe that, as θ keeps increasing, in the low RES cost regime, p^*_{50} finally saturates at a constant. This corresponds to case 2 of Proposition 1 and can be explained as follows. In the low RES cost regime, high penetration of renewables is favored, i.e., r^* is large. Thus, when θ is large, the realized renewable energy θr^* is large so that the total supply satisfies case 2 of Proposition 1. In such a case, the optimal price of each user i is a constant $p^*_i = \frac{1}{\phi_i}$ ($p^*_{50} = 1/\phi_{50} = 0.3356$ in our case).

Next, we investigate the impact of α and β on the expected optimal prices of users. We consider two values of β: 0.3734 (Hawaii's rate) and 0.0837 (Louisiana's rate). We also consider two values of α: 0.07 and 0.2, which are of the same scale as the typical unit cost of solar panels (0.1218). Then, we consider three values of (α, β): (0.07, 0.3734), (0.2, 0.3734), (0.2, 0.0837), which are in the low, medium and high RES cost regimes, respectively. The corresponding expected optimal pricing vectors are shown in Fig. 6. Comparison between the three pricing vectors suggest that the prices at the user side increase with both α and β, i.e., the higher the energy cost/price is, the higher the prices at the user side are. We further plot the vector $[1/\phi_i]_{i=1,\ldots,100}$ in Fig. 6 and observe that all pricing vector are its constant shifts (all curves are parallel). The reason is that the difference between the optimal prices of two arbitrary users i, j is always $p^*_i - p^*_j = \frac{1}{\phi_i} - \frac{1}{\phi_j}$ (c.f. Proposition 1), which is a constant related to ϕ only. Thus, $\forall i, j$: $p^*_i - \frac{1}{\phi_i} = p^*_j - \frac{1}{\phi_j}$. So, $p^*_i - \frac{1}{\phi_i}$ does not depend on i.

Finally, we apply the proposed Algorithm 1 to real data and compare it with a heuristic method, which determines renewable deployments based on heuristics. To this end, we make use of the annual retail sales data of electricity in the U.S. (with state labels) in 2016 [41] (also shown in Fig. 2(b)) and fit an exponential form demand response function $d_i(p_i) = \bar{\xi}_i e^{-\bar{\phi}_i p_i}$ for each state i, $i = 1, \ldots, 42$ (only $n = 42$ states have enough data to fit relatively accurately). Here, each state is regarded as a user with a specific demand response function. The distribution of the price-demand parameters $(\bar{\xi}_i, \bar{\phi}_i)$ of the 42 states is shown in Fig. 7(a). In addition, the daily spot prices of wholesale electricity of PJM West (an electricity hub) in 2017 are shown in Fig. 7(b) [53] and are regarded as the time-varying prices $\{\beta_t\}_{t=1}^T$ of the spot market, where $T = 249$ (the prices are only available on weekdays). The time-average spot market price is $\bar{\beta} = \frac{1}{T}\sum_{t=1}^T \beta_t = 3.3745$ and the cost of renewable energy is set to be $\alpha = 2.5$. In practice, the price-demand parameters of users can also be time-varying across days. Thus, for $t = 1, \ldots, T$, we set the time-varying price-demand parameters as $\xi_{i,t} = \bar{\xi}_i + u_{i,t}$ and $\phi_{i,t} = \bar{\phi}_i + v_{i,t}$, where $u_{i,t}$ and $v_{i,t}$ are uniformly distributed on $\left[-\frac{1}{5}\bar{\xi}_i, \frac{1}{5}\bar{\xi}_i\right]$ and $\left[-\frac{1}{5}\bar{\phi}_i, \frac{1}{5}\bar{\phi}_i\right]$, respectively. The tuple $(\alpha, \{\beta_t\}, \{\xi_{i,t}\}, \{\phi_{i,t}\})$ based on real data defines the model setup. According to the discussion in the end of Section III, time-varying model parameters can be handled by minor modification of Algorithm 1 as follows. In the planning phase, the optimal renewable deployment r^* is computed by Algorithm 1 based on the time-average parameters $(\alpha, \bar{\beta}, \{\bar{\xi}_i\}, \{\bar{\phi}_i\})$. In this case, $r^* = 319$. On the other hand, without the optimal design of renewable deployment, one may use a simple heuristic method to determine r_{heu} as follows. If the price to users is set to be $p = 1.5\alpha$ (this price is used only to get a rough estimate of typical user demands and the real prices of the heuristic method will be set optimally by using Algorithm 1), then the average total demand from users is $\sum_{i=1}^n \bar{\xi}_i e^{-\bar{\phi}_i p} = 669$. If we match the expected renewable generation $\frac{r}{2}$ with this total demand, the heuristic renewable deployment is $r_{\text{heu}} = 1338$. Then, in each time t, for both choices of r, i.e., r^* of Algorithm 1 and r_{heu} of the heuristic method, we use Algorithm 1 to compute the optimal prices p^*_t and the optimal procurement from spot market s^*_t based on the current model parameters at time t. For both Algorithm 1 and the heuristic method, the time-varying expected profit $\mathbb{E}_\theta[w_t(\theta, r)]$ is plotted in Fig. 7(c), where w_t is a time-varying version of the function w defined in (2) by replacing β, ξ_i, ϕ_i with $\beta_t, \xi_{i,t}, \phi_{i,t}$, respectively. The expected profits are computed with respect to two distributions of the renewable realization factor θ, namely the uniform distribution on $[0, 1]$ and the non-uniform distribution of real renewable generation in Australia (c.f. Fig. 2(a)) [40]. We observe that, for both distributions of θ, the expected profit of Algorithm 1 is higher than that of the heuristic method, highlighting the importance of optimal design of renewable deployment advocated in this paper. Moreover, the expected profits computed from these two distributions of θ are very close. This further confirms that the uniform distribution can approximate the distribution of real-world renewable generation well, as has been justified in Fig. 2(a).

V. Conclusion

In this paper, we have studied the joint energy procurement and demand response in a power system. The operator procures energy from RESs and the spot market and then sells it to end users through differentiated pricing. Unlike most existing works on RES, we take into account the (infrastructure) cost of renewables and design optimal renewable deployment by controlling the construction of RES facilities in the planning phase. Formulating the energy procurement and pricing procedure as a two-stage optimization problem, we have derived optimal decisions on RES penetration, purchase from the spot market and user pricing. A computationally efficient algorithm involving only closed form computation and simple bisection search has been presented to compute the optimal decisions. Numerical experiments have been carried out to corroborate the optimality of the proposed algorithm and the solution structure of the optimal decisions. This work sheds some light on the optimal deployment of renewables in the context of energy procurement and demand response in power grids. One prospective future direction is to take into account the temporal variations of user demands and spot market prices more systematically by resorting to practical stochastic models such as Markov processes. Another promising future direction is to examine the effects of power network constraints on the optimal design of renewable deployment.

Appendix

A. Proof of Proposition 1

The (partial) Lagrangian of problem (14) can be written as:

$$\mathfrak{L}(p,\lambda) = -\sum_{i=1}^{n} p_i \xi_i e^{-\phi_i p_i} + \lambda \left(\sum_{i=1}^{n} \xi_i e^{-\phi_i p_i} - t \right), \quad (37)$$

where λ is the associated multiplier. For $\lambda \geq 0$, the dual function is given by:

$$g(\lambda) = \inf_{p \succeq 0} \mathfrak{L}(p,\lambda) \quad (38)$$

$$= \sum_{i=1}^{n} \inf_{p_i \geq 0} \left\{ (-p_i + \lambda) \xi_i e^{-\phi_i p_i} \right\} - \lambda t. \quad (39)$$

We note:

$$\frac{d}{dp_i} \left[(-p_i + \lambda) e^{-\phi_i p_i} \right] = \phi_i e^{-\phi_i p_i} \left(p_i - \lambda - \frac{1}{\phi_i} \right). \quad (40)$$

Thus, the p_i that achieves the infimum in (39) is given by:

$$p_i = \lambda + \frac{1}{\phi_i},\ i = 1, \ldots, n, \quad (41)$$

and for $\lambda \geq 0$, the dual function is given by:

$$g(\lambda) = -\sum_{i=1}^{n} \frac{\xi_i}{\phi_i} e^{-\phi_i \lambda - 1} - \lambda t. \quad (42)$$

The dual problem of problem (14) is:

$$\begin{aligned} &\text{Maximize } g(\lambda) \\ &\text{subject to } \lambda \geq 0. \end{aligned} \quad (43)$$

Denote the dual optimal point and the dual optimal value as λ^* and $d^* = g(\lambda^*)$, respectively. For $\lambda \geq 0$, we compute the derivative of $g(\lambda)$ as:

$$g'(\lambda) = \sum_{i=1}^{n} \xi_i e^{-\phi_i \lambda - 1} - t. \quad (44)$$

We observe that $g'(\lambda)$ is a strict increasing function for $\lambda \geq 0$ with $g'(0) = e^{-1} \sum_{i=1}^{n} \xi_i - t$ and $\lim_{\lambda \to +\infty} g'(\lambda) = -t < 0$. Therefore, we distinguish two situations. (i) If $g'(0) \geq 0$, i.e., $t \leq e^{-1} \sum_{i=1}^{n} \xi_i$, then the dual optimal point $\lambda^* \geq 0$ is the unique solution of $g'(\lambda^*) = 0$, i.e.,

$$\sum_{i=1}^{n} \xi_i e^{-\phi_i \lambda^* - 1} = t. \quad (45)$$

(ii) If $g'(0) < 0$, i.e., $t > e^{-1} \sum_{i=1}^{n} \xi_i$, then $g'(\lambda) < 0, \forall \lambda \geq 0$. So the dual optimal point is $\lambda^* = 0$. Since the primal objective function $f(p)$ is non-convex, the primal problem (14) is a non-convex problem, for which strong duality does not hold in general [51]. Therefore, it is not rigorous to directly use (41) to obtain the primal optimal point from the dual optimal point λ^*. In the following, we verify that strong duality indeed holds for problem (14) and we can indeed compute the primal optimal point from λ^* by using (41). Again, we distinguish two cases.

Case (i): $t \leq e^{-1} \sum_{i=1}^{n} \xi_i$. In this case, λ^* is determined by (45) and the dual optimal value is $d^* = -\sum_{i=1}^{n} \frac{\xi_i}{\phi_i} e^{-\phi_i \lambda^* - 1} - \lambda^* t$. Define a price vector $\widetilde{p}$ as $\widetilde{p}_i = \lambda^* + \frac{1}{\phi_i}, i = 1, \ldots, n$. Thus,

$$\sum_{i=1}^{n} \xi_i e^{-\phi_i \widetilde{p}_i} = \sum_{i=1}^{n} \xi_i e^{-\phi_i \lambda^* - 1} = t, \quad (46)$$

where the last step results from (45). Hence, $\widetilde{p}$ is primal feasible. Furthermore, we have:

$$f(\widetilde{p}) = -\sum_{i=1}^{n} \left(\lambda^* + \frac{1}{\phi_i} \right) \xi_i e^{-\phi_i \left(\lambda^* + \frac{1}{\phi_i} \right)} \quad (47)$$

$$= -\lambda^* \sum_{i=1}^{n} \xi_i e^{-\phi_i \lambda^* - 1} - \sum_{i=1}^{n} \frac{\xi_i}{\phi_i} e^{-\phi_i \lambda^* - 1} \quad (48)$$

$$= d^*, \quad (49)$$

where in the last equality we make use of (45). Therefore,

$$d^* = f(\widetilde{p}) \overset{(a)}{\geq} f^* \overset{(b)}{\geq} d^*, \quad (50)$$

in which (a) is due to the primal feasibility of $\widetilde{p}$ and (b) results from the weak duality of any optimization problems (not necessarily convex) [51]. So, $f(\widetilde{p}) = f^* = d^*$. In other words, strong duality holds and $\widetilde{p}$ is primal optimal.

Case (ii): $t > e^{-1}\sum_{i=1}^n \xi_i$. In such a case $\lambda^* = 0$ and $d^* = -e^{-1}\sum_{i=1}^n \frac{\xi_i}{\phi_i}$. Consider the price vector $\widetilde{p}$ defined as $\widetilde{p}_i = \frac{1}{\phi_i}$, $i = 1, \ldots, n$. Thus,

$$\sum_{i=1}^n \xi_i e^{-\phi_i \widetilde{p}_i} = \sum_{i=1}^n \xi_i e^{-1} < t, \tag{51}$$

which implies the primal feasibility of $\widetilde{p}$. Moreover, $f(\widetilde{p}) = -\sum_{i=1}^n \frac{\xi_i}{\phi_i} e^{-1} = d^*$. Thus, we have $d^* = f(\widetilde{p}) \geq f^* \geq d^*$, in which the first inequality is due to the primal feasibility of $\widetilde{p}$ and the second inequality means weak duality. Therefore, $f(\widetilde{p}) = f^* = d^*$, i.e., strong duality holds and $\widetilde{p}$ is primal optimal.

Summarizing cases (i) and (ii) concludes Proposition 1.

B. Proof of Proposition 2

We first distinguish two cases.

Case (1): $\theta r > e^{-1}\sum_{i=1}^n \xi_i$. In this case, for any $s \geq 0$, we have $h(\theta r + s) = e^{-1}\sum_{i=1}^n \frac{\xi_i}{\phi_i}$ and $q(s) = e^{-1}\sum_{i=1}^n \frac{\xi_i}{\phi_i} - \beta s$, which decreases with s. Thus, $s^* = 0$ and $q^* = e^{-1}\sum_{i=1}^n \frac{\xi_i}{\phi_i}$.

Case (2): $\theta r \leq e^{-1}\sum_{i=1}^n \xi_i$. In this case, for $s > e^{-1}\sum_{i=1}^n \xi_i - \theta r$, we have $q(s) = e^{-1}\sum_{i=1}^n \frac{\xi_i}{\phi_i} - \beta s$. Thus, when s is already larger than $e^{-1}\sum_{i=1}^n \xi_i - \theta r$, further increasing s will make $q(s)$ to decrease. Therefore, to find the maximal point, we only need to focus on the interval $0 \leq s \leq e^{-1}\sum_{i=1}^n \xi_i - \theta r$. When s is within this interval, we have:

$$q(s) = \sum_{i=1}^n \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(\theta r + s) - 1} + (\theta r + s)\delta(\theta r + s) - \beta s. \tag{52}$$

Taking derivative, we obtain:

$$\begin{aligned} q'(s) = &-\sum_{i=1}^n \xi_i e^{-\phi_i \delta(\theta r + s) - 1} \delta'(\theta r + s) + \delta(\theta r + s) \\ &+ (\theta r + s)\delta'(\theta r + s) - \beta \qquad (53) \\ &\overset{(a)}{=} \delta(\theta r + s) - \beta, \qquad (54) \end{aligned}$$

where in (a) we make use of the fact $\sum_{i=1}^n \xi_i e^{-\phi_i \delta(\theta r + s) - 1} = \theta r + s$ from Definition 1. Thus, $q'(s)$ is a strictly decreasing function over the interval $s \in [0, e^{-1}\sum_{i=1}^n \xi_i - \theta r]$. In addition, we know that $q'(0) = \delta(\theta r) - \beta$ and $q'(e^{-1}\sum_{i=1}^n \xi_i - \theta r) = -\beta < 0$. As such, we further distinguish case (2) into two cases as follows.

- *Case (2a): $\sum_{i=1}^n \xi_i e^{-\phi_i \beta - 1} \leq \theta r$.* In such a case, from the monotonicity of δ, we have $\delta(\theta r) \leq \beta$, i.e., $q'(0) \leq 0$. Thus, $q'(s) \leq 0$ for any $s \in [0, e^{-1}\sum_{i=1}^n \xi_i - \theta r]$. So, $s^* = 0$ and the corresponding optimal value is:

$$q^* = \sum_{i=1}^n \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(\theta r) - 1} + \theta r \delta(\theta r). \tag{55}$$

- *Case (2b): $\sum_{i=1}^n \xi_i e^{-\phi_i \beta - 1} > \theta r$.* In such a case, we have $\delta(\theta r) > \beta$, i.e., $q'(0) > 0$. So the optimal point s^* is the unique solution of $q'(s^*) = 0$ over the interval $[0, e^{-1}\sum_{i=1}^n \xi_i - \theta r]$. Thus, solving for s^* gives:

$$s^* = \sum_{i=1}^n \xi_i e^{-\phi_i \beta - 1} - \theta r. \tag{56}$$

The corresponding optimal value $q^* = q(s^*)$ is computed as:

$$q^* = \sum_{i=1}^n \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(\theta r + s^*) - 1} + (\theta r + s^*)\delta(\theta r + s^*) - \beta s^* \tag{57}$$

$$\begin{aligned} &\overset{(a)}{=} \sum_{i=1}^n \frac{\xi_i}{\phi_i} e^{-\phi_i \beta - 1} + \beta \sum_{i=1}^n \xi_i e^{-\phi_i \beta - 1} \\ &\quad - \beta\left(\sum_{i=1}^n \xi_i e^{-\phi_i \beta - 1} - \theta r\right) \qquad (58) \\ &= \sum_{i=1}^n \frac{\xi_i}{\phi_i} e^{-\phi_i \beta - 1} + \beta \theta r, \qquad (59) \end{aligned}$$

where in (a) we make use of the fact $\delta(\theta r + s^*) = \beta$ and (56).

Summarizing cases (1), (2a) and (2b) gives the result in Proposition 2.

C. Proof of Lemma 1

We distinguish three cases.

Case (i): $r > \mu_2$. In such a case, according to (29), we compute:

$$\begin{aligned} &\mathbb{E}[w(\theta, r)] \qquad (60) \\ &= \int_0^1 w(\theta, r) d\theta \qquad (61) \\ &= \int_0^{\frac{\mu_1}{r}} \left(\pi_1 + \beta \theta r - \frac{\alpha r}{2}\right) d\theta \\ &\quad + \int_{\frac{\mu_1}{r}}^{\frac{\mu_2}{r}} \left[\sum_{i=1}^n \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(\theta r) - 1} + \theta r \delta(\theta r) - \frac{\alpha r}{2}\right] d\theta \\ &\quad + \int_{\frac{\mu_2}{r}}^1 \left(\pi_2 - \frac{\alpha r}{2}\right) d\theta \qquad (62) \\ &\overset{(a)}{=} \pi_2 - \frac{\pi_2 \mu_2}{r} - \frac{\alpha r}{2} + \frac{\pi_1 \mu_1}{r} + \frac{\beta \mu_1^2}{2r} \\ &\quad + \frac{1}{r}\int_{\mu_1}^{\mu_2} \sum_{i=1}^n \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(\mu) - 1} d\mu + \frac{1}{r}\int_{\mu_1}^{\mu_2} \mu \delta(\mu) d\mu, \qquad (63) \end{aligned}$$

where in (a) we change the integral variable from θ to $\mu = \theta r$. With integration by parts, we obtain:

$$\begin{aligned} &\int_{\mu_1}^{\mu_2} \sum_{i=1}^n \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(\mu) - 1} d\mu \\ &= \mu \sum_{i=1}^n \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(\mu) - 1}\Big|_{\mu_1}^{\mu_2} - \int_{\mu_1}^{\mu_2} \mu d\left(\sum_{i=1}^n \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(\mu) - 1}\right). \qquad (64) \end{aligned}$$

Making use of (28) and the definitions in (26), (27), we can compute:

$$\mu \sum_{i=1}^{n} \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(\mu)-1} \Bigg|_{\mu_1}^{\mu_2} = \mu_2 \pi_2 - \mu_1 \pi_1. \tag{65}$$

Taking derivatives of $\sum_{i=1}^{n} \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(\mu)-1}$, we get:

$$\frac{d}{d\mu}\left(\sum_{i=1}^{n} \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(\mu)-1}\right)$$

$$= -\left(\sum_{i=1}^{n} \xi_i e^{-\phi_i \delta(\mu)-1}\right)\delta'(\mu) \tag{66}$$

$$= -\mu\delta'(\mu). \tag{67}$$

Substituting (65) and (67) into (64) yields:

$$\int_{\mu_1}^{\mu_2} \sum_{i=1}^{n} \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(\mu)-1} d\mu \tag{68}$$

$$= \mu_2\pi_2 - \mu_1\pi_1 + \int_{\mu_1}^{\mu_2} \mu^2 \delta'(\mu) d\mu \tag{69}$$

$$\overset{\text{(a)}}{=} \mu_2\pi_2 - \mu_1\pi_1 - \mu_1^2\beta - 2\int_{\mu_1}^{\mu_2} \mu\delta(\mu)d\mu, \tag{70}$$

where in (a) we use integration by parts. Furthermore,

$$\int_{\mu_1}^{\mu_2} \mu\delta(\mu)d\mu \tag{71}$$

$$\overset{\text{(a)}}{=} \int_{\beta}^{0} \left(\sum_{i=1}^{n} \xi_i e^{-\phi_i x-1}\right) x d\left(\sum_{i=1}^{n} \xi_i e^{-\phi_i x-1}\right) \tag{72}$$

$$= e^{-2} \sum_{i,j=1}^{n} \xi_i \xi_j \phi_j \int_0^{\beta} x e^{-(\phi_i+\phi_j)x} dx \tag{73}$$

$$\overset{\text{(b)}}{=} \epsilon, \tag{74}$$

where in (a) we change the integral variable from μ to $x = \delta(\mu)$, i.e., $\mu = \sum_{i=1}^{n} \xi_i e^{-\phi_i x-1}$; in (b) we make use of the definition of ϵ in (30). Substituting (70) and (74) into (63), we obtain:

$$\mathbb{E}[w(\theta, r)] = \pi_2 - \frac{\alpha r}{2} - \frac{\beta\mu_1^2}{2r} - \frac{\epsilon}{r}, \tag{75}$$

which is clearly concave over the interval $r \in [\mu_2, +\infty)$.

Case (ii): $\mu_1 \le r \le \mu_2$. In such a case, according to (29), we compute:

$$\mathbb{E}[w(\theta, r)] \tag{76}$$

$$= \int_0^{\frac{\mu_1}{r}} \left(\pi_1 + \beta\theta r - \frac{\alpha r}{2}\right) d\theta$$

$$+ \int_{\frac{\mu_1}{r}}^{1} \left[\sum_{i=1}^{n} \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(\theta r)-1} + \theta r \delta(\theta r) - \frac{\alpha r}{2}\right] d\theta \tag{77}$$

$$= -\frac{\alpha r}{2} + \frac{\pi_1\mu_1}{r} + \frac{\beta\mu_1^2}{2r}$$

$$+ \frac{1}{r}\int_{\mu_1}^{r} \left[\sum_{i=1}^{n} \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(\mu)-1} + \mu\delta(\mu)\right] d\mu. \tag{78}$$

Moreover,

$$\int_{\mu_1}^{r} \left(\sum_{i=1}^{n} \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(\mu)-1}\right) d\mu \tag{79}$$

$$\overset{\text{(a)}}{=} \mu \sum_{i=1}^{n} \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(\mu)-1} \Bigg|_{\mu_1}^{r} - \int_{\mu_1}^{r} \mu d\left(\sum_{i=1}^{n} \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(\mu)-1}\right) \tag{80}$$

$$\overset{\text{(b)}}{=} r \sum_{i=1}^{n} \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(r)-1} - \mu_1\pi_1 + \int_{\mu_1}^{r} \mu^2 \delta'(\mu) d\mu \tag{81}$$

$$\overset{\text{(c)}}{=} r \sum_{i=1}^{n} \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(r)-1} - \mu_1\pi_1 + r^2\delta(r) - \mu_1^2\beta$$

$$- 2\int_{\mu_1}^{r} \delta(\mu)\mu d\mu, \tag{82}$$

where in (a) and (c) we use integration by parts; in (b) we make use of the facts $\frac{d}{d\mu}(\sum_{i=1}^{n} \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(\mu)-1}) = -\mu\delta'(\mu)$ and $\delta(\mu_1) = \beta$. We want to transform the first summation in (82) into a form more amenable to differentiation. To this end, we derive:

$$\int_{\mu_1}^{r} \delta(\mu)d\mu \tag{83}$$

$$= \int_{\beta}^{\delta(r)} x d\left(\sum_{i=1}^{n} \xi_i e^{-\phi_i x-1}\right) \tag{84}$$

$$= \sum_{i=1}^{n} \xi_i\phi_i \int_{\delta(r)}^{\beta} x e^{-\phi_i x-1} dx] \tag{85}$$

$$= -\sum_{i=1}^{n} \xi_i \Bigg[\beta e^{-\phi_i\beta-1} - \delta(r) e^{-\phi_i \delta(r)-1} + \frac{1}{\phi_i} e^{-\phi_i\beta-1}$$

$$- \frac{1}{\phi_i} e^{-\phi_i \delta(r)-1}\Bigg] \tag{86}$$

$$= -\beta\mu_1 + r\delta(r) - \pi_1 + \sum_{i=1}^{n} \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(r)-1}. \tag{87}$$

Substituting (87) into (82) gives:

$$\int_{\mu_1}^{r} \sum_{i=1}^{n} \frac{\xi_i}{\phi_i} e^{-\phi_i \delta(\mu)-1} d\mu$$

$$= \beta\mu_1 r + \pi_1 r - \mu_1\pi_1 - \mu_1^2\beta + r\int_{\mu_1}^{r} \delta(\mu)d\mu - 2\int_{\mu_1}^{r} \delta(\mu)\mu d\mu. \tag{88}$$

Substitiuting (88) into (78), we obtain:

$$\mathbb{E}[w(\theta, r)]$$

$$= -\frac{\alpha r}{2} - \frac{\beta\mu_1^2}{2r} + \beta\mu_1 + \pi_1 + \int_{\mu_1}^{r} \delta(\mu)d\mu - \frac{1}{r}\int_{\mu_1}^{r} \delta(\mu)\mu d\mu. \tag{89}$$

In addition, taking twice derivative, we get:

$$\frac{d^2}{dr^2}\mathbb{E}[w(\theta, r)] = -\frac{v(r)}{r^3}, \tag{90}$$

where the function $v(r)$ is defined as:

$$v(r) = \beta\mu_1^2 + 2\int_{\mu_1}^{r} \delta(\mu)\mu d\mu - \delta(r)r^2. \tag{91}$$

The derivative of $v(r)$ is:

$$v'(r) = -\delta'(r)r^2 \geq 0, \tag{92}$$

where we make use of the fact that δ is a monotonically decreasing function (and thus $\delta'(r) \leq 0$). So, $v(r)$ is a monotonically increasing function over the interval $[\mu_1, \mu_2]$. Note that $v(\mu_1) = 0$. Therefore, $v(r) \geq 0$ for any $v \in [\mu_1, \mu_2]$. So, $\frac{d^2}{dr^2}\mathbb{E}[w(\theta, r)] \leq 0$ for any $v \in [\mu_1, \mu_2]$, which implies that $\mathbb{E}[w(\theta, r)]$ is concave over the interval $[\mu_1, \mu_2]$.

Case (iii): $0 \leq r < \mu_1$. In such a case, according to (29), we have:

$$\mathbb{E}[w(\theta, r)] \tag{93}$$

$$= \int_0^1 \left(\pi_1 + \beta\theta r - \frac{\alpha r}{2}\right) d\theta \tag{94}$$

$$= \pi_1 + \frac{(\beta - \alpha)r}{2}. \tag{95}$$

D. Proof of Proposition 3

We first study the derivative of $\mathbb{E}[w(\theta, r)]$ in the three cases identified in Lemma 1.

(i) When $r > \mu_2$, from (31), we have:

$$\frac{d}{dr}\mathbb{E}[w(\theta, r)] = -\frac{\alpha}{2} + \frac{\beta\mu_1^2}{2r^2} + \frac{\epsilon}{r^2}, \tag{96}$$

which is monotonically decreasing over the interval $(\mu_2, +\infty)$. In addition, we know $\frac{d}{dr}\mathbb{E}[w(\theta, r)]\big|_{\mu_2^+} = -\frac{\alpha}{2} + \frac{\beta\mu_1^2}{2\mu_2^2} + \frac{\epsilon}{\mu_2^2}$ and $\lim_{r\to+\infty} \frac{d}{dr}\mathbb{E}[w(\theta, r)] = -\frac{\alpha}{2} < 0$.

(ii) When $\mu_1 \leq r \leq \mu_2$, we have:

$$\frac{d}{dr}\mathbb{E}[w(\theta, r)] = -\frac{\alpha}{2} + \frac{\beta\mu_1^2}{2r^2} + \frac{1}{r^2}\int_{\mu_1}^{r} \delta(\mu)\mu d\mu. \tag{97}$$

$\frac{d}{dr}\mathbb{E}[w(\theta, r)]$ is monotonically decreasing over the interval $[\mu_1, \mu_2]$ as $\mathbb{E}[w(\theta, r)]$ is concave over this interval (Lemma 1). In addition, we have:

$$\frac{d}{dr}\mathbb{E}[w(\theta, r)]\bigg|_{\mu_2^-} = -\frac{\alpha}{2} + \frac{\beta\mu_1^2}{2\mu_2^2} + \frac{1}{\mu_2^2}\int_{\mu_1}^{\mu_2} \delta(\mu)\mu d\mu$$

$$\overset{(a)}{=} \frac{d}{dr}\mathbb{E}[w(\theta, r)]\bigg|_{\mu_2^+}, \tag{98}$$

where in (a) we make use of (74). So, $\frac{d}{dr}\mathbb{E}[w(\theta, r)]$ is continuous at $r = \mu_2$. Furthermore, we know $\frac{d}{dr}\mathbb{E}[w(\theta, r)]\big|_{\mu_1^+} = \frac{\beta - \alpha}{2}$.

(iii) When $0 \leq r < \mu_1$, we have $\frac{d}{dr}\mathbb{E}[w(\theta, r)] = \frac{\beta - \alpha}{2}$, which is a constant. So, $\frac{d}{dr}\mathbb{E}[w(\theta, r)]$ is continuous at $r = \mu_1$ as well.

Combining the aforementioned three cases, we see that $\frac{d}{dr}\mathbb{E}[w(\theta, r)]$ is continuous and monotonically decreasing over the entire interval $[0, +\infty)$. Hence, $\mathbb{E}[w(\theta, r)]$ is concave over the entire interval $[0, +\infty)$. Thus, in what follows, we find the minimal point of $\mathbb{E}[w(\theta, r)]$ over $[0, +\infty)$ by distinguishing three regimes based on the signs of $\frac{d}{dr}\mathbb{E}[w(\theta, r)]$ at the boundary points μ_1 and μ_2.

1) In the *Low RES Cost Regime* of Proposition 3, we have $\frac{d}{dr}\mathbb{E}[w(\theta, r)]\big|_{\mu_2} \geq 0$. In such a case, the optimal r^* is in the interval $[\mu_2, +\infty)$ and is determined by (according to (96)):

$$-\frac{\alpha}{2} + \frac{\beta\mu_1^2}{2(r^*)^2} + \frac{\epsilon}{(r^*)^2} = 0, \tag{99}$$

which leads to $r^* = \sqrt{\frac{\beta\mu_1^2 + 2\epsilon}{\alpha}}$.

2) In the *Medium RES Cost Regime* of Proposition 3, we have $\frac{d}{dr}\mathbb{E}[w(\theta, r)]\big|_{\mu_2} < 0$ and $\frac{d}{dr}\mathbb{E}[w(\theta, r)]\big|_{\mu_1} \geq 0$. In such a case, the optimal point r^* is in the interval $[\mu_1, \mu_2]$ and is determined as the unique solution of the following equation over $[\mu_1, \mu_2]$ (according to (97)):

$$-\frac{\alpha}{2} + \frac{\beta\mu_1^2}{2(r^*)^2} + \frac{1}{(r^*)^2}\int_{\mu_1}^{r^*} \delta(\mu)\mu d\mu = 0. \tag{100}$$

With similar calculations as in achieving (74), we can demonstrate that, for any $r \in [\mu_1, \mu_2]$:

$$\int_{\mu_1}^{r} \delta(\mu)\mu d\mu$$

$$= e^{-2}\sum_{i,j=1}^{n} \frac{\xi_i\xi_j\phi_j}{\phi_i + \phi_j}\bigg[\delta(r)e^{-(\phi_i+\phi_j)\delta(r)} - \beta e^{-(\phi_i+\phi_j)\beta}$$

$$+ \frac{1}{\phi_i + \phi_j}\left(e^{-(\phi_i+\phi_j)\delta(r)} - e^{-(\phi_i+\phi_j)\beta}\right)\bigg]. \tag{101}$$

Substituting (101) into (100) and defining $x^* = \delta(r^*)$, we know that $r^* = \sum_{i=1}^{n} \xi_i e^{-\phi_i x^* - 1}$ and x^* is determined as the unique solution of the following equation over the interval $x \in [0, \beta]$:

$$e^{-2}\sum_{i,j=1}^{n} \frac{\xi_i\xi_j\phi_j}{\phi_i + \phi_j}\left[\left(x + \frac{1}{\phi_i + \phi_j}\right)e^{-(\phi_i+\phi_j)x}\right.$$

$$\left. - \left(\beta + \frac{1}{\phi_i + \phi_j}\right)e^{-(\phi_i+\phi_j)\beta}\right]$$

$$-\frac{\alpha}{2}\left(\sum_{i=1}^{n} \xi_i e^{-\phi_i x - 1}\right)^2 + \frac{\beta\mu_1^2}{2} = 0. \tag{102}$$

3) In the *High RES Cost Regime* of Proposition 3, $\frac{d}{dr}\mathbb{E}[w(\theta, r)] < 0$ for any $r \geq 0$. Thus, the optimal point is $r^* = 0$.

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