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Preventive Maintenance Subject to Equipment Unavailability

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***Abstract*—Preventive maintenance has received considerable attention in industries and the literature. Conventional preventive maintenance models often assume that equipment is always available for maintenance activities. However, in many mission-critical industries, equipment may not be available for scheduled maintenance due to busy operational schedules. Forced shutdown of the equipment may incur extra costs that cannot be offset by the benefits from preventively maintaining the equipment. In this paper, we propose innovative preventive maintenance policies to address the challenges caused by equipment unavailability. Maintenance models with possible rescheduling are developed for both time-based and condition-based maintenance policies, and the objective is to minimize the long-run cost rate of all maintenance activities. The proposed policies, with consideration of equipment unavailability for prescheduled PM, are compared with the policies that ignore this unavailability. Numerical examples are provided to illustrate the proposed policies.**

***Index Terms*—Age-based preventive maintenance (PM), condition-based maintenance (CBM), equipment unavailability, maintenance rescheduling.**

I. Introduction

PREVENTIVE maintenance (PM) has been extensively studied in the literature. PM is often implemented to prevent or delay equipment failure. However, the majority of the existing PM models assume that systems can be shut down for maintenance whenever maintenance is needed. While this assumption is appropriate in some industries, it is not valid in many mission-critical industries. For example, it might not be economical to shut down machines for maintenance activities during production runs in many chemical plants, since such an interruption will waste all raw materials being processed. Additionally, some equipment in busy shipping ports/terminals may not be readily available for PM due to the heavy traffic. Thus, the assumption that equipment is always available for prescheduled maintenance needs to be re-examined. In this paper, we develop PM policies with rescheduling due to system unavailability, and investigate the tradeoffs between failure costs, PM rescheduling, and potential losses due to forced shutdown for PM activities.

The literature on the use of mathematical modeling for analyzing and optimizing scheduled maintenance plans is abundant. In order to keep the system operating at a desirable condition, PM is often used to delay or prevent system failure. PM activities can be generally classified into two categories: time-based maintenance (TBM) and condition-based maintenance (CBM). TBM schedules are typically determined based on a probabilistic model of system failure. Reviews on TBM can be found in [1]–[7]. CBM is a maintenance approach that relies on advances in sensor technology to create data-driven reliability models to develop strategies for condition monitoring and maintenance. Reviews of CBM can be found in [8]–[10]. Most of the existing PM research overlooks the interrelationship between maintenance planning and equipment work schedules. As a result, maintenance planning is often done without considering the interactions between these two activities.

The interdependence between PM planning and production scheduling has received some attention in manufacturing industries. If PM planning does not take the production schedule into consideration, equipment unavailability during production may result in unsatisfied demands. On the other hand, unnecessary failure costs may be incurred if production service does not consider the time needed for maintenance activities and PM is not performed in a timely fashion. Cassady *et al.* [11], [12] propose integrated models that simultaneously determine production scheduling and PM planning decisions. These models investigate how maintenance activities affect available production time and how elapsed production time affects the probability of machine failure. Fitouhi *et al.* [13] and Nourelfath *et al.* [14] develop joint PM and production planning models with the objective of minimizing maintenance- and production-related costs while satisfying all demands. Other literature that addresses the interrelationship between PM planning and production scheduling can be found in [15]–[17]. Similarly, the interdependence of production and maintenance has been recognized in process and railway industries. Ashayeri *et al.* [18] show that the nature of a production process such as routing, flexibility, change-over time, and production equipment makes the maintenance activities associated with these processes more complicated. And therefore, highly automated production

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environments combined with complicated maintenance functions necessitate the need to simultaneously plan production and maintenance activities to improve productivity and efficiency. According to Gorman *et al.* [19], the scheduling of railway track maintenance projects is extremely complex, with numerous job scheduling constraints, and this scheduling requires consideration of efficient maintenance production routing around the rail network while paying close attention to any train delay such maintenance projects might cause. Several models have been proposed to resolve the conflicts between train operations and the scheduling of maintenance activities [19]–[21].

However, the majority of integrated production and maintenance models either coordinate the production with maintenance decisions, or plan maintenance activities with fixed production/work schedules. The uncertainty in work schedules is largely ignored. For example, production schedules are difficult to predict when big variabilities in demands exist, causing equipment proudction schedule to be less predictable. Consequently, equipment may have to be in operation when PM actions need to be performed according to predetermined schedules. When equipment is unavailable for scheduled PM activities, it may not be feasible to simply wait until the equipment is available, since many maintenance activities are outsourced and require maintenance service providers to send a team to the site, and it takes time to mobilize the team and finish all necessary preparatory work (e.g., set up tools) before performing maintenance actions. All these activities require advanced scheduling. The uncertainty of equipment availability for PM poses great challenges on maintenance planning and requires that maintenance planning cope with work schedules to ensure the continuous operations demanded. There are few studies that combine maintenance planning with uncertainty in work schedules. As more maintenance activities are being outsourced, especially for complex equipment, models addressing PM with consideration of equipment unavailability become more relevant.

To meet this need, we propose innovative PM policies that allow rescheduling to deal with uncertainty in equipment availability. We consider the case in which equipment may be busy and not available for scheduled maintenance activities and it is not economical to immediately shut down equipment for maintenance. We propose two types of rescheduling policies which are summarized as follows.

1) *Type 1 Rescheduling:* Reschedule PM action until the equipment is available or the equipment fails, whichever occurs first. A rescheduling cost is incurred every time PM is delayed.
2) *Type 2 Rescheduling:* Reschedule PM actions to a later time if the equipment is not available for maintenance. However, unlike Type 1 rescheduling policy that allows infinite number of PM attempts, this policy has an upper limit on equipment operation time or on the degradation level. Once the time limit or degradation level is reached, the equipment is forced to shut down for PM. An additional cost is incurred if the equipment is mandatorily stopped for PM.

The two types of PM rescheduling are developed for both age-based PM and CBM, separately. The main contribution of this paper lies in the new, novel PM policies that incorporate rescheduling PM actions for equipment that face uncertain work schedules. To the best of our knowledge, this is among the first efforts that analyze the impacts of equipment unavailability on maintenance strategies and coordinate maintenance activities with uncertain work schedules.

The rest of this paper is organized as follows. Section II provides a detailed system description. Section III develops age-based PM policies for both Type 1 and Type 2 rescheduling with illustrative examples. Section IV develops CBM-based policies for both Type 1 and Type 2 rescheduling. Section V concludes this paper.

II. System Description

Notation

c_{pm}	Cost of PM.
c_{cm}	Cost of CM.
c_r	Cost of PM rescheduling.
c_i	Cost of inspection.
c_m	Cost of mandatory shutdown.
p	Probability that equipment is available for PM at any point in time.
$h(p)$	PDF of p.
γ	Cost rate.
η	Cycle length.
C	Total cost in a renewal cycle.
s	Failure threshold.
τ_1	Time when PM is first attempted under age-based PM.
τ_2	Time when equipment is mandatorily shut down for PM under age-based PM with Type 2 rescheduling.
ω	PM reschedule time interval.
δ	Inspection interval under CBM.
ξ	PM threshold under CBM.
ζ	Forced shutdown threshold.
$g(t)$	Probability density function (PDF) of time to failure.
$X(t)$	Random wear accumulated in time interval $[0,t]$.
$F(x; x_0, t)$	Cumulative distribution function (CDF) of $X(t)$, $\Pr\{X(t) \leq x, X(0) = x_0\}$.
$f(x; x_0, t)$	Probability density function (PDF) of $X(t)$.
$T(x)$	Random time for the wear to reach level x.
$R(t)$	Reliability Function.
p_n	Probability that PM is successfully performed upon the nth PM attempts under age-based PM.
q_n	Probability that failure is detected upon the nth PM attempts under age-based PM.
$\kappa_{\lambda,n}$	Probability that PM is first attempted upon the λth inspection and the PM is performed upon the nth PM attempt under CBM with Type 1 rescheduling.
$\kappa'_{\lambda,n}$	Probability that PM is first attempted upon the λth inspection and the PM is performed upon the nth PM attempt under CBM with Type 2 rescheduling.
$\theta_{\lambda,n}$	Probability that PM is first attempted upon the λth inspection and failure is detected upon the nth PM attempt under CBM with Type 1 rescheduling.

$\theta'_{\lambda,n}$ Probability that PM is first attempted upon the λth inspection and failure is detected at the *n*th PM attempt under CBM with Type 2 rescheduling.

$q'_{\lambda,n}$ Probability that PM is first attempted upon the λth inspection and forced to shut down at the *n*th PM attempt under CBM with Type 2 rescheduling.

In this paper, we consider equipment with uncertain work schedules. Specifically, there is a probability *p* that the equipment is available for PM at any point in time. This probability can be a constant or modeled by some probability distribution. In practice, the availability probability *p* is often associated with some uncertainties, and therefore we assume that *p* is a random variable with a PDF of *h*(*p*). For example, if *p* is normally distributed, then *h*(*p*) is the PDF of a truncated normal distribution over (0,1)

$$h(p) = \frac{\varphi(p;\mu,\sigma^2)}{\psi(1;\mu,\sigma^2) - \psi(0;\mu,\sigma^2)}, \quad p \in [0,1]$$

where $\varphi(x;\mu,\sigma^2)$ and $\psi(x;\mu,\sigma^2)$ are normal distribution PDF and CDF with mean μ and variance σ^2.

Two types of PM rescheduling policies are proposed to resolve issues caused by uncertainties in work schedules. Let ω denote the interval between two PM attempts. Under Type 1 rescheduling, if equipment is not available for a scheduled PM, a PM action will be reattempted after a prespecified interval (ω), and a rescheduling cost of c_r is incurred every time the PM is rescheduled. The PM action will be reattempted every ω time units until either PM is successfully performed or the equipment fails, whichever occurs first. Under Type 2 rescheduling, a similar delay scheme is developed for PM actions. However, under Type I policy, after the number of PM attempts reaches either a maximum time or degradation threshold, the equipment is forced to shut down for PM. If a forced shutdown occurs, an additional shutdown cost (c_m) is incurred. We integrate these two types of rescheduling policies with TBM and CBM, respectively. For TBM, a widely used age-based PM policy is chosen. For CBM, a periodic inspection/PM policy is considered.

III. Age-Based PM With Rescheduling

Under a traditional age-based PM policy, equipment is preventively maintained if it functions without failure for τ time units. If a failure occurs before the scheduled PM, then CM is performed immediately. Failures are assumed to be self-announcing, and both PM and CM restore the equipment to an as-good-as-new state. We first extend this traditional age-based PM policy by incorporating Type 1 rescheduling. Under Type 1 rescheduling policy, the first PM attempt occurs when the equipment has functioned without failure for τ_1 time units. If the equipment is not available for PM at time τ_1, the PM action is delayed by ω time units, and a rescheduling cost of c_r is incurred. The delay interval ω can be a random variable. However, for mathematical simplicity, we assume the delay interval ω is fixed.

Fig. 1 illustrates some sample scenarios of the PM process under Type 1 rescheduling. Fig. 1(a) shows that the equipment

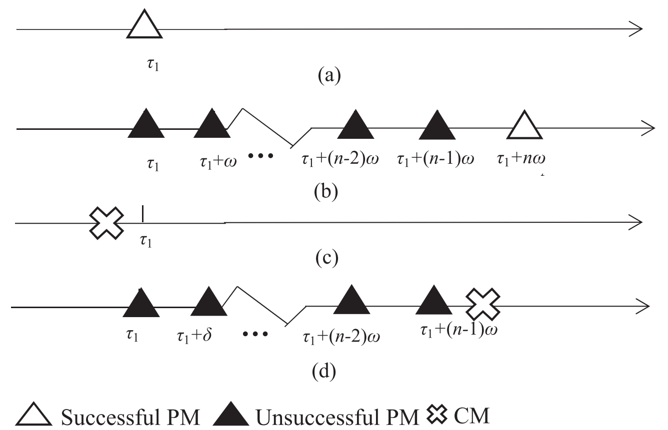

Fig. 1. Age-based PM with Type 1 Rescheduling. (a) PM successfully performed at the first PM attempt. (b) PM successfully performed at the $(n+1)$th attempt. (c) Failure before the first PM attempt. (d) Failure between the $(n-1)$th and *n*th PM attempts.

is available upon the first PM attempt. Fig. 1(b) illustrates that the equipment is not available until the *n*th PM attempt, and PM is successfully performed upon the *n*th attempt. Fig. 1(c) demonstrates a sample scenario where the equipment fails before the first PM attempt; and Fig. 1(d) shows that a failure occurs after several PM attempts.

If a PM is successfully performed upon the *n*th attempt, the renewal cycle time is $\tau_1 + (n-1)\omega$. If a failure occurs between the $(n-1)$th and *n*th PM attempts, the cycle time is $\tau_1 + (n-1)\omega + t$, where *t* is the uptime of the equipment during the failure interval. Let *C* represent the total cost in a renewal cycle that includes PM, CM, and rescheduling costs, and let η represent the cycle length which is the time interval between two successive maintenance actions, either PM or CM. Our objective is to minimize the expected cost rate by optimizing the first PM attempt interval τ_1 and the PM rescheduling/delay interval ω.

Model P1

$$\tau_1^*, \omega^* = \arg\min \left\{ \gamma = \frac{E(C)}{E(\eta)} \right\},$$
$$\tau_1 > 0, \omega > 0$$

where $E(C) = \int_0^1 E(C|p)h(p)dp$ and $E(\eta) = \int_0^1 E(\eta|p) h(p)dp$. $E(C|p)$ and $E(\eta|p)$ are the expected cost and expected renewal cycle length for a given *p*, respectively.

Given the probability of equipment availability *p*, the expected maintenance-related cost is as follows:

$$E(C|p) = \sum_{n=1}^{\infty} ((n-1)\ c_r + c_{\text{pm}})\, p_n + \sum_{n=1}^{\infty} ((n-1)\ c_r + c_{\text{cm}})\, q_n$$

where the probability of PM successfully performed upon the *n*th attempt is

$$p_n = p(1-p)^{n-1} R((n-1)\,\omega + \tau_1)$$

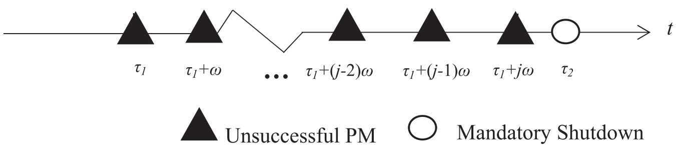

Fig. 2. Age-based PM with a mandatory shutdown.

and the probability of a failure between the $(n-1)$th and the nth attempts is

$$q_n = \begin{cases} 1 - R(\tau_1), & n = 1 \\ (1-p)^{n-1}\{R((n-2)\omega + \tau_1) \\ \quad -R((n-1)\omega + \tau_1)\}, & n > 1 \end{cases}.$$

Next, we derive the expected cycle length for a given p. The cycle length has four scenarios.

1) If the equipment is available for PM upon the first attempt, the cycle time is τ_1.
2) If a failure occurs before τ_1, the cycle time is the actual operational time t.
3) The cycle time is $\tau_1 + (n-1)\omega$ if the equipment is available for PM upon the nth attempt.
4) The cycle time is $(\tau_1 + (n-2)\omega + t)$ if the equipment fails between the $(n-1)$th and nth PM attempts for any $n > 1$.

The expected cycle length given p is computed as follows:

$$E(\eta|p) = \sum_{n=1}^{+\infty}(\tau_1 + (n-1)\omega)p_n + \int_0^{\tau_1} tg(t)dt + \sum_{n=2}^{\infty}(1-p)^{n-1}\int_0^{\omega}(\tau_1 + (n-2)\omega + t) \times g(\tau_1 + (n-2)\omega + t)dt.$$

We next develop an age-based PM policy with Type 2 rescheduling. Under this policy, the PM will first be attempted at time τ_1 provided the equipment has not previously failed. Similarly as in Type 1 rescheduling, if the equipment is unavailable for PM at time τ_1, it will be rescheduled every ω time units. However, with Type 2 rescheduling, there is a maximum operation time τ_2 $(\tau_2 > \tau_1)$. That is, if the equipment functions without failure and is not available for PM prior to reaching the operational threshold, the equipment will be forced to shut down for PM at time τ_2. Fig. 2 provides an illustrative example of Type 2 rescheduling.

The relationship between τ_1 and τ_2 can be expressed in the following equation:

$$\tau_2 = \tau_1 + j\omega + t \tag{1}$$

where j denotes the number of PM attempts between τ_1 and τ_2, and t is the equipment operational time between the $(j+1)$th PM attempt and time τ_2. With (1), the maximum operation time allowed is converted to the maximum number of PM attempts.

The problem above is formulated as an optimization model using renewal theory. The decision variables are τ_1, ω, τ_2 for age-based PM with Type 2 rescheduling. Therefore, we have

Model P2

$$\tau_1^*, \tau_2^*, \omega^* = \arg\min\left\{\gamma = \frac{E(C)}{E(\eta)}\right\},$$
$$\tau_1 > 0, \tau_2 > 0, \omega > 0$$

where $E(C) = \int_0^1 E(C|p)h(p)dp$ and $E(\eta) = \int_0^1 E(\eta|p)h(p)dp$. $E(C|p)$ and $E(\eta|p)$ are the expected cost and expected renewal cycle length for a given p, respectively. We first derive the expected cost given p. In addition to the rescheduling cost (c_r), a mandatory shutdown cost (c_m) is incurred if the equipment is forced to shut down for PM. The total maintenance costs for the cycle ending with a forced shutdown is $(j+1)c_r + c_{\rm pm} + c_m$. The expected cost given p in a renewal cycle is

$$E(C|p) = \sum_{n=1}^{j+1}((n-1)\ c_r + c_{\rm pm}) \times p_n + \sum_{n=1}^{j+1}((n-1)\ c_r + c_{\rm cm}) \times q_n + ((j+1)c_r + c_{\rm pm} + c_m) \times p_M + ((j+1)c_r + c_{\rm cm}) \times q_M$$

where the probability of having a forced shutdown is

$$p_M = (1-p)^{j+1}R(\tau_2)$$

and the probability that the system fails after the $(j+1)$th PM attempt and before the forced shutdown

$$q_M = (1-p)^{j+1}(R(\tau_1 + j\omega) - R(\tau_2)).$$

Similarly, we derive the expected cycle length. If PM is successfully performed on the nth attempt, the cycle time is $\tau_1 + (n-1)\omega$, $n \le (j+1)$. If the equipment operates without failure until time τ_2, the cycle time is τ_2. If a failure occurs between the $(n-1)$th and nth PM attempts, the cycle time is $\tau_1 + (n-2)\omega + t$, where t is the uptime of the equipment during the failure interval. The expected cycle length given p is given as follows:

$$E(\eta|p) = \sum_{n=1}^{j+1}(\tau_1 + (n-1)\omega)p_n + \tau_2 p_M + \int_0^{\tau_1} tg(t)dt + \sum_{n=2}^{j+1}(1-p)^{n-1}\int_0^{\omega}(\tau_1 + (n-2)\omega + t)g(t)dt + (1-p)^{j+1}\int_0^{\tau_2 - j\omega - \tau_1}(\tau_1 + j\omega + t)\, g(\tau_1 + j\omega + t)dt.$$

To assess the effect of equipment unavailability on PM policies, the policies with rescheduling are compared with the policies that ignore equipment unavailability. The age-based PM model without considering equipment unavailability is given by

Model P3

$$\tau_1^* = \arg\min\left\{\gamma = \frac{E(C)}{E(\eta)}\right\}, \quad \tau_1 > 0$$

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where

$$E(C) = c_{\rm cm}(1 - R(\tau_1)) + c_{\rm pm}R(\tau_1)$$

and

$$E(\eta) = \int_0^{\tau_1} tf(t)\,dt + \tau_1 R(\tau_1).$$

The optimization problems (**P1** and **P2**) are mathematically complex, and derivatives of the objective functions are difficult to obtain. Therefore, numerical search without utilizing derivatives is used to find optimal solutions. Rosenbrock's algorithm is used to search for optimal burn-in and maintenance policies. The method of Rosenbrock does not employ line searches but rather takes discrete steps along the search directions. At each iteration, the procedure searches iteratively along *n* linearly independent and orthogonal directions (*n* is the number of decision variables). When a new point is reached at the end of an iteration, a new set of orthogonal vectors is constructed. An acceleration feature is also incorporated by suitably increasing or decreasing the step lengths as the method proceeds [22]. The method of Rosenbrock using line searches converges to a stationary point if the following assumptions are true: 1) The minimum of *f* along any line in R_n is unique; and 2) the sequence of points generated by the algorithm is contained in a compact subset of R_n. Since it is mathematically difficult to obtain the first and second partial derivatives of the objective function, we do not analytically demonstrate whether the two assumptions are satisfied in this study. Therefore, Rosenbrock's algorithm does not guarantee the global optimality in this paper. For detailed steps of the method, we refer the readers to [22] and [23].

A. Illustrative Example

In this section, we provide two numerical examples to illustrate the two proposed rescheduling policies. Based on its wide application in practice, we assume that the equipment's time to failure follows a Weibull distribution. Suppose the shape and scale parameters of the Weibull distribution are 2 and 50, respectively. The $c_{\rm cm}$, $c_{\rm pm}$, c_r, and c_m are 1000, 200, 40, and 400, respectively.

Example 1: Assume that the distribution of *p* is a truncated normal distribution with μ and σ^2 equal to 0.5 and 0.5. The optimal results are

1) *P1:* $\tau_1 = 30.24$, $\omega = 2.65$, and corresponding $\gamma = 18.98$ for age-based PM with Type 1 rescheduling.
2) *P2:* $\tau_1 = 29.94$, $\omega = 0.05$, $\tau_2 = 50.94$, and $\gamma = 18.38$ for age-based PM with Type 2 rescheduling.
3) *P3:* $\tau_1 = 25.53$.

We can see that the first PM attempt time (τ_1) is different for the policies that consider equipment unavailability. This indicates that equipment unavailability for PM cannot be ignored and PM policies should be developed with the consideration of uncertainty of equipment availability for PM activities.

Example 2: Assume that the distribution of *p* is a truncated normal distribution with μ and σ^2 equal to 0.2 and 0.5.

1) *P1:* $\tau_1 = 32.14$, $\omega = 3.16$, and corresponding $\gamma = 19.75$ for age-based PM with Type 1 rescheduling.
2) *P2:* $\tau_1 = 29.87$, $\omega = 0.07$, $\tau_2 = 69.93$, and $\gamma = 19.08$ for age-based PM with Type 2 rescheduling.
3) *P3:* $\tau_1 = 25.53$.

We make similar observations in Example 2. The first PM attempt time (τ_1) is different from the policies that consider equipment unavailability. This further confirms the necessity of developing appropriate rescheduling policies when equipment may be unavailable for scheduled maintenance activities. From Example 2, we can also see that a lower equipment availability for PM leads to larger rescheduling intervals. This is because the incentive for rescheduling is less when the equipment is more likely to be unavailable for scheduled maintenance.

B. Sensitivity Analysis

In this section, we conduct a sensitivity analysis to examine the impacts of important model inputs. The parameters of interests are rescheduling cost (c_r), corrective maintenance cost ($c_{\rm cm}$), and forced shutdown cost (c_m). Numerical Example 1 in Section III-A will serve as a baseline model to conduct the sensitivity analysis. Policies 1, 2, and 3 in the sensitivity analysis are associated with models **P1**, **P2**, and **P3**, respectively.

1) Effects of Rescheduling Cost: Three different levels of rescheduling costs are examined $c_r = 4, 20$ and 100. Table I summarizes the optimal results from the two types of policies under different rescheduling costs. From Table I, we can see that the rescheduling interval ω increase as c_r increases. This is straightforward, since the increase in c_r indicates rescheduling is more expensive. The age-based PM interval (τ_1) increases as the rescheduling cost increases, in order to reduce the number of rescheduling attempts. This confirms the necessity of accounting for the possibility that equipment may be unavailable for prescheduled PM activities.

2) Effects of CM Cost: We now examine the impacts of CM cost. Three levels of CM costs are considered: low ($c_{\rm cm} = 400$), medium ($c_{\rm cm} = 1000$), and high ($c_{\rm cm} = 2000$). Results of optimal policies under different CM costs are summarized in Table II. From Table II, we can see that as $c_{\rm cm}$ increases, both τ_1 and ω decrease rapidly. The PM interval (τ_1) is reduced, and PM is rescheduled more frequently, in order to avoid high failure costs. For policy 2, the forced shutdown interval (τ_2) also decreases to prevent failures from occurring. It is also observed that Policy 3 is significantly different from Policy 1 and Policy 2, implying that it is imperative to consider equipment unavailability when scheduling PM.

3) Effects of Forced Shutdown Cost: Similarly, we investigate optimal policies under three different forced shutdown costs. Table III presents analysis for different forced shutdown costs. The effects of c_m are mainly on the intervals of forced shutdown and rescheduling. Naturally, the more expensive a forced shutdown is, the more likely we want to avoid such actions. As c_m increases, rescheduling is attempted more frequently, with the intention of performing PM action before reaching the forced shutdown limit. And τ_2 also increases to delay the costly forced shutdown.

Note that optimal cost rates from the two types of rescheduling policies are also summarized and compared for all cases

TABLE I
ANALYSIS OF c_r UNDER AGE-BASED PM

c_r	Policy 1			Policy 2				Policy 3	
	τ_1	ω	γ	τ_1	τ_2	ω	γ	τ_1	γ
4	26.66	0.05	16.78	25.81	53.32	0.03	17.03	25.53	16.34
20	27.25	0.11	17.87	27.19	51.39	0.04	17.66	25.53	16.34
100	35.82	8.04	20.53	36.15	50.73	5.26	20.28	25.53	16.34

TABLE II
ANALYSIS OF c_{cm} UNDER AGE-BASED PM

c_{cm}	Policy 1			Policy 2				Policy 3	
	τ_1	ω	γ	τ_1	τ_2	ω	γ	τ_1	γ
400	70.25	20.23	8.98	70.11	383.84	20.21	8.98	54.54	8.73
1000	30.24	2.65	18.98	29.94	50.94	0.05	18.38	25.53	16.34
2000	17.47	0.04	29.16	16.88	31.17	0.01	27.85	16.82	24.22

TABLE III
ANALYSIS OF c_{m} UNDER AGE-BASED PM

c_{m}	Policy 2				Policy 3	
	τ_1	τ_2	ω	γ	τ_1	γ
100	29.90	32.48	1.44	18.20	25.53	16.34
400	29.94	50.94	0.05	18.38	25.53	16.34
700	31.14	87.57	0.04	18.41	25.53	16.34

considered in the sensitivity analysis. The optimal cost rate is dependent on maintenance-related cost parameters, such as PM/CM costs, rescheduling cost, and forced shutdown cost. When both types of rescheduling are allowed, the one with a lower cost rate will be more cost beneficial.

IV. CONDITION-BASED MAINTENANCE WITH RESCHEDULING

Advances in censor technology and data analytics have resulted in an increase in the implementation of CBM which aims to reduce maintenance wastes. It can be more beneficial to incorporate rescheduling into CBM for equipment that cannot strictly follow predetermined PM schedules. We consider a periodic inspection policy with possible rescheduling. We assume that equipment failure can only be detected by inspection or during PM rescheduling. The equipment is periodically inspected with a fixed interval δ. Upon each inspection, if the equipment deterioration is above the maintenance threshold (ξ) but below the failure threshold (s), PM is performed if the equipment is available. If the equipment deterioration is below the maintenance threshold, we do nothing. To address the issue of equipment unavailability for PM, two policies that are similar to those developed for the age-based PM are proposed. The first type of rescheduling policy attempts the PM every ω time units until the equipment is available—if no failure occurs before a successful PM. For the first policy, there is no limit on the number of PM attempts. The second type of rescheduling policy is similar to the first type of policy except that a forced shutdown threshold is allowed. The equipment is mandatorily shut down for PM if the equipment degradation level is between the mandatory shutdown threshold (ζ) and failure threshold (s). In addition to the rescheduling cost, a forced shutdown cost is incurred. Fig. 3 illustrates some scenarios for CBM with rescheduling. Fig. 3(a) shows that the equipment failure is detected upon the λth inspection and no PM has been yet attempted. Fig. 3(b) shows that PM is first attempted and performed upon the λth inspection. No rescheduling is needed in the scenario illustrated in Fig. 3(b). Fig. 3(c) shows that failure occurs during the rescheduling process, Fig. 3(d) demonstrates that PM is performed after

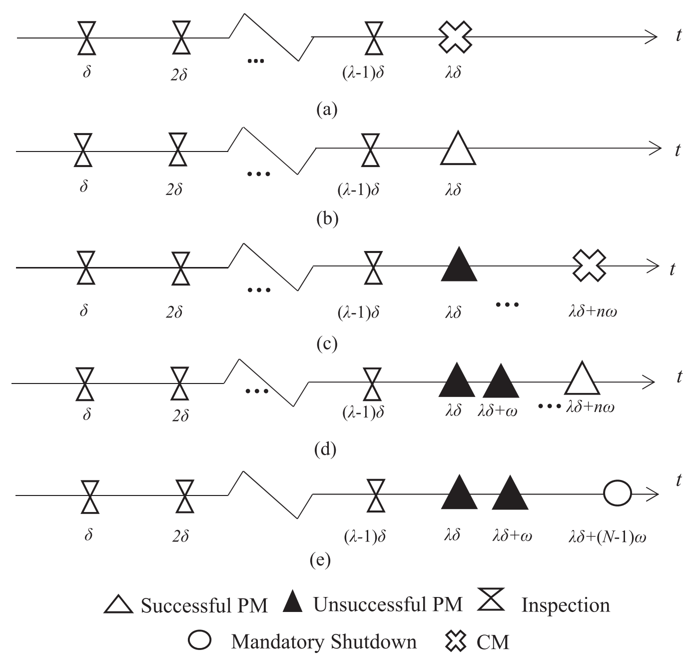

Fig. 3. CBM with rescheduling. (a) Failure detected at the λth inspections. (b) PM succeeds at the λth inspection. (c) Failure detected at the $(n+1)$th PM attempt. (d) PM succeeds at the $(n+1)$th attempts. (e) PM forcibly performed at the Nth attempt.

several unsuccessful attempts. Fig. 3(e) illustrates the forced shutdown.

We now develop a mathematical model for CBM with Type 1 rescheduling policy. Our objective is to minimize the expected total cost rate by optimizing the inspection interval δ, maintenance threshold ξ, and the PM rescheduling/delay interval ω.

Model P4:

$$\delta^*, \xi^*, \omega^* = \arg\min \left\{ \gamma = \frac{E(C)}{E(\eta)} \right\},$$
$$\delta > 0, \xi > 0, \omega > 0$$

where $E(C) = \int_0^1 E(C|p)h(p)dp$ and $E(\eta) = \int_0^1 E(\eta|p)h(p)dp$. $E(C|p)$ and $E(\eta|p)$ are the expected cost and expected renewal cycle length for a given p, respectively. We first derive the expected cost in a renewal cycle given the equipment availability p. Let $\kappa_{\lambda,n}$ denote the probability that the first PM is attempted upon the λth inspection and PM is successfully performed at the nth attempt, and let $\theta_{\lambda,n}$ denote the probability that the first PM is attempted upon the λth inspection and CM is performed at the nth inspection. For a given λ, there are two scenarios for maintenance costs. If the renewal cycle ends with a successful PM, maintenance-related costs include inspection costs, PM cost, and any rescheduling costs. If the renewal cycle ends with a failure, maintenance costs include inspection costs, CM cost, and any rescheduling costs. The maintenance-related costs given p is as follows:

$$E(C|p) = \sum_{\lambda=1}^{+\infty}\sum_{n=1}^{+\infty}((\lambda c_i + (n-1)\,c_r + c_{\text{pm}}) \times \kappa_{\lambda,n} + (\lambda c_i + (n-1)\,c_r + c_{\text{cm}}) \times \theta_{\lambda,n})$$

where the probability that PM is successfully performed at the nth attempt is

$$\kappa_{\lambda,n} = \begin{cases} p\int_0^{\xi} f\left(u; 0, (\lambda-1)\,\delta\right) \left(F\left(s; u, \delta\right) - F\left(\xi; u, \delta\right)\right) du, & n = 1 \\ (1-p)^{n-1} p \int_0^{\xi} f\left(u; 0, (\lambda-1)\,\delta\right) \int_{\xi}^{s} f\left(v; u, \delta\right) F\left(s; v, (n-1)\,\omega\right) dv du, & n > 1 \end{cases}$$

and the probability that CM is performed between the nth and $(n+1)$th attempts is

$$\theta_{\lambda,n} = \begin{cases} \int_0^{\xi} f\left(u; 0, (\lambda-1)\,\delta\right)\left(1 - F\left(s; u, \delta\right)\right) du, & n = 1 \\ (1-p)^{n-1} \int_0^{\xi} f\left(u; 0, (\lambda-1)\,\delta\right) \int_{\xi}^{s} f\left(v; u, \delta\right) \int_{v}^{s} f\left(x; v, (n-2)\,\omega\right)\left(1 - F\left(s; x, \omega\right)\right) dx dv du, & n > 1 \end{cases}.$$

The derivations for $\kappa_{\lambda,n}$ and $\theta_{\lambda,n}$ under Type 1 rescheduling policy are provided in Appendix I. Next, we derive the expected renewal cycle length given the available probability. Given that the first PM is attempted at the λth inspection. If the renewal cycle ends with a successful PM or CM upon the nth attempt, the cycle length is $(\lambda\delta + (n-1)\omega)$. The expected cycle length given p is as follows:

$$E(\eta|p) = \sum_{\lambda=1}^{+\infty}\sum_{n=1}^{+\infty}((\lambda\delta + (n-1)\,\omega) \times \kappa_{\lambda,n} + (\lambda\delta + (n-1)\,\omega) \times \theta_{\lambda,n}).$$

Next, we develop the mathematical model for CBM with Type 2 rescheduling. We have the same objective of minimizing the expected cost rate, and the decision variables are inspection interval (δ), maintenance threshold (ξ), rescheduling interval (ω), and forced shutdown threshold (ζ).

Model P5:

$$\delta^*, \xi^*, \omega^*, \zeta^* = \arg\min \left\{ \gamma = \frac{E(C)}{E(\eta)} \right\},$$
$$\delta > 0, \omega > 0, \zeta >, \xi > 0$$

where $E(C) = \int_0^1 E(C|p)h(p)dp$ and $E(\eta) = \int_0^1 E(\eta|p)h(p)dp$. $E(C|p)$ and $E(\eta|p)$ are the expected cost and expected renewal cycle length for a given p, respectively. Similarly, we first derive the maintenance cost given the probability of equipment unavailability p. The maintenance cost in a renewal cycle where at least one PM is attempted under Type 2 rescheduling is derived as follows. Given that the first PM is attempted upon the λth inspection, there are three scenarios for maintenance-related costs. If the renewal cycle ends with a successful PM, which means the degradation level is between ξ and ζ, the cost is $\lambda c_i + (n-1)c_r + c_{\text{pm}}$. If the degradation level is found between ζ and s, a mandatory PM is performed along with the cost $\lambda c_i + (n-1)c_r + c_{\text{pm}} + c_m$. If equipment failed upon inspection, then CM is performed and the cost is $\lambda c_i + (n-1)c_r + c_{\text{cm}}$

$$E(C|p) = \sum_{\lambda=1}^{+\infty}\sum_{n=1}^{+\infty} \begin{pmatrix} (\lambda c_i + (n-1)\,c_r + c_{\text{pm}}) \times {\kappa'}_{\lambda,n} \\ + (\lambda c_i + (n-1)\,c_r + c_{\text{pm}} + c_m) \times {q'}_{\lambda,n} \\ + (\lambda c_i + (n-1)\,c_r + c_{\text{cm}}) \times {\theta'}_{\lambda,n} \end{pmatrix}$$

where the probability that PM is successfully performed at nth attempt is

$$\kappa'_{\lambda,n} = \begin{cases} p\int_0^{\xi} f\left(u; 0, (\lambda-1)\,\delta\right) \left(F\left(\zeta; u, \delta\right) - F\left(\xi; u, \delta\right)\right) du, & n = 1 \\ (1-p)^{n-1} p \int_0^{\xi} f\left(u; 0, (\lambda-1)\,\delta\right) \int_{\xi}^{\zeta} f\left(\nu; u, \delta\right) F\left(\zeta; \nu, (n-1)\,\omega\right) d\nu du, & n > 1 \end{cases}$$

the probability that PM is successfully and forcibly performed at the nth attempt is

$$q'_{\lambda,n} = \begin{cases} \int_0^{\xi} f\left(u; 0, (\lambda-1)\,\delta\right)\left(F\left(s; u, \delta\right) - F\left(\zeta; u, \delta\right)\right) du, & n = 1 \\ (1-p)^{n-1} \times \int_0^{\xi} f\left(u; 0, (\lambda-1)\,\delta\right) \int_{\xi}^{\zeta} f\left(\nu; u, \delta\right) \int_{\nu}^{\zeta} f\left(x; \nu, (n-2)\,\omega\right)(F\left(s; x, \omega\right) - F\left(\zeta; x, \omega\right)) dx d\nu du, & n > 1 \end{cases}$$

and the probability that CM is performed at the nth attempt is

$$\theta'_{\lambda,n} = \begin{cases} \int_0^{\xi} f(u;0,(\lambda-1)\delta)(1-F(s;u,\delta))\,du, & n=1 \\ (1-p)^{n-1}\int_0^{\xi} f(u;0,(\lambda-1)\delta)\int_{\xi}^{\zeta} f(\nu;u,\delta) & \\ \quad \int_{\nu}^{\zeta} f(x;\nu,(n-2)\omega)(1-F(s;x,\omega))\,dx\,d\nu\,du, & n>1 \end{cases}.$$

Next, we derive the expected renewal cycle length give the available probability. The maintenance cost in a renewal cycle where at least one PM is attempted under Type 2 rescheduling is derived as follows. Given that the first PM is attempted upon the λth inspection, there are three scenarios for cycle length. Regardless whether the renewal cycle ends with a successful PM, or ends with mandatory shutdown, or ends with failure, the cycle lengths all equal to $\lambda\delta + (n-1)\omega$. The expected cycle length for a given p is as follows:

$$E(\eta|p) = \sum_{\lambda=1}^{+\infty}\sum_{n=1}^{+\infty}(\lambda\delta+(n-1)\omega)\times(\kappa'_{\lambda,n}+\theta'_{\lambda,n}+q'_{\lambda,n}).$$

To investigate the effects of equipment unavailability on CBM policies, the policies with rescheduling allowed are also compared with the policies developed without considering equipment unavailability. The benchmark model considers a conventional periodic inspection/PM policy with the same objective of minimizing the long-run cost rate.

Model P6:

$$\delta^*,\xi^* = \arg\min\left\{\gamma = \frac{E(C)}{E(\eta)}\right\},$$
$$\delta>0,\xi>0.$$

The expected cost and cycle length are given by

$$E(C) = \sum_{\lambda=1}^{\infty}(c_{\mathrm{pm}}+\lambda c_i)\times\int_0^{\xi} f(u;0,(\lambda-1)\delta)\,(F(s;u,\delta) - F(\xi;u,\delta))du + \sum_{\lambda=1}^{\infty}(c_{\mathrm{cm}}+\lambda c_i) \times\int_0^{\xi} f(u;0,(\lambda-1)\delta)(1-F(\xi;u,\delta))du$$

and

$$E(\eta) = \sum_{\lambda=1}^{\infty}\lambda\delta\times\int_0^{\xi} f(u;0,(\lambda-1)\delta)(1-F(\xi;u,\delta))du.$$

A. Illustrative Examples

Stochastic processes that are commonly used to describe deterioration processes include Brownian motion, geometric Brownian motion, and gamma processes. For illustrative purposes, we consider a Gamma-process-based degradation model. We assume that equipment deterioration can be described by a Gamma process with shape parameter α and scale parameter β. Let $X(t)$ represent the degradation increment over time interval $[0,t]$. The CDF of $X(t)$ is given by

$$F(s;x_0,t) = \int_0^{s-x_0}\frac{1}{\Gamma(\alpha t)\beta^{\alpha t}}x^{\alpha t-1}\exp\left(-\frac{x}{\beta}\right)dx,$$

and the PDF of $X(t)$ is

$$f(s;x_0,t) = \frac{1}{\Gamma(\alpha t)\beta^{\alpha t}}x^{\alpha t-1}\exp\left(-\frac{x}{\beta}\right).$$

The mean and variance of $X(t)$ can be expressed as $\alpha\beta$ and $\alpha\beta^2$. The CDF of the associated failure times is given by

$$G(t;s,x_0) = \frac{\Gamma(\alpha t,\beta(s-x_0))}{\Gamma(\alpha t)}$$

where $\Gamma(\alpha t,\beta(s-x_0)) = \int_{\beta(s-x_0)}^{\infty} y^{\alpha t-1}e^{-y}dy$ is an incomplete gamma function.

The PDF of failure times can be expressed as [24]

$$g(t;x_0,s) = \alpha\left\{\rho(\alpha t)-\log\left(\frac{s-x_0}{\beta}\right)\right\} \left\{1-\frac{\Gamma\left(\alpha t,\frac{s-x_0}{\beta}\right)}{\Gamma(\alpha t)}\right\} + \frac{\alpha\left(\frac{s-x_0}{\beta}\right)^{\alpha_i t}}{\Gamma(\alpha t)(\alpha t)^2}\cdot {}_2F_2$$

where $\rho(z) = d/dz\log\Gamma(z)$ is the digamma function and ${}_2F_2$ represents the generalized hypergeometric function or Barnes extended hypergeometric function ${}_2F_2 = 1+\sum_{k=1}^{\infty}\left(\frac{\alpha_i t}{\alpha_i t+k}\right)^2\frac{((s-x_0)/\beta_i)^k}{k!}$.

Two numerical examples are provided to illustrate the proposed CBM with rescheduling. Suppose $\alpha=1$ and $\beta=3$. The c_{cm}, c_{pm}, c_r, c_i, and c_m are 1000, 200, 40, 20, and 400, respectively. The failure threshold s is 200. Rosenbrock's method is applied to search for the optimal solution.

Example 3: Assume that p follows a truncated normal distribution with μ and σ^2 equal to 0.5 and 0.5. The optimal results are as follows.

1) *P4:* $\xi=87.03$, $\delta=26.32$, $\omega=1.07$, and corresponding $\gamma=6.53$ for CBM with Type 1 rescheduling.
2) *P5:* $\xi=75.36$, $\delta=26.65$, $\omega=1.02$, $\zeta=195.14$, and $\gamma=7.11$for CBM with Type 2 rescheduling.
3) *P6:* $\xi=104.41$, $\delta=26.20$, and $\gamma=4.93$.

Example 4: Assume that p follows a truncated normal distribution with μ and σ^2 equal to 0.5 and 0.5. The optimal results are as follows.

1) *P4:* $\xi=85.47$, $\delta=52.80$, $\omega=1.5$, and corresponding $\gamma=6.74$ for CBM with Type 1 rescheduling.
2) *P5:* $\xi=88.09$, $\delta=26.57$, $\omega=2.15$, $\zeta=197.4$, and $\gamma=7.18$ for CBM with Type 2 rescheduling.
3) *P6:* $\xi=104.41$, $\delta=26.20$, and $\gamma=4.93$.

Results from these two numerical examples show that the inspection intervals (δ) and maintenance thresholds (ξ) with consideration of equipment uncertainty are different from the policies that ignore equipment unavailability (P6).

B. Sensitivity Analysis

In this section, we conduct a sensitivity analysis to examine the impacts of important model inputs on CBM. The parameters of interests are c_r, c_{cm}, and c_m. The numerical example in Section IV-A will serve as a baseline model to conduct sensitivity analysis. Policies 4, 5, and 6 are corresponding to models **P4**, **P5**, and **P6**, respectively.

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

TABLE IV
ANALYSIS OF RESCHEDULING COST UNDER CBM

c_{r}	Policy 4				Policy 5					Policy 6		
	ξ	δ	ω	γ	ξ	δ	ω	ζ	γ	ξ	δ	γ
4	99.98	23.63	1.08	4.96	82.93	23.06	1.08	187.54	5.37	104.40	26.2	4.93
20	85.62	24.38	1.03	5.80	78.93	24.48	1.01	193.75	6.15	104.4	26.2	4.93
100	62.22	54.29	1.08	7.29	59.47	48.81	1.02	198.48	6.84	104.4	26.2	4.93

TABLE V
ANALYSIS OF CM COST UNDER CBM

c_{cm}	Policy 4				Policy 5					Policy 6		
	ξ	δ	ω	γ	ξ	δ	ω	ζ	γ	ξ	δ	γ
400	99.92	55.53	69.98	4.06	86.19	27.70	69.98	172.49	5.01	117.13	28.64	4.55
1000	87.03	26.32	1.07	6.53	75.36	26.65	1.02	195.14	7.11	104.42	26.20	4.93
2000	84.94	24.78	1.03	6.75	71.98	23.24	1.02	198.86	7.24	99.89	25.15	5.12

TABLE VI
ANALYSIS OF FORCED SHUTDOWN COST UNDER CBM

c_{m}	Policy 5					Policy 6		
	ξ	δ	ω	ζ	γ	ξ	δ	γ
100	103.65	27.01	5.77	171.42	5.74	104.42	26.20	4.93
400	75.36	26.65	1.02	195.14	7.11	104.42	26.20	4.93
700	72.56	20.95	1.24	199.61	7.23	104.42	26.20	4.93

1) Effects of Rescheduling Cost: Table IV summarizes the optimal results of the various policies under different values of c_r. From Table IV, we can see that the rescheduling cost has a large impact on inspection interval (δ) and PM threshold (ξ). As c_r increases, the inspection interval (δ) increases and the PM threshold (ξ) decreases. While this appears counter-intuitive at first, our conjecture is that a higher rescheduling cost makes PM more expensive and thereby provides less incentive to frequently inspect the system. The rescheduling cost does not have the same impact on the rescheduling interval length under CBM as it does under age-based PM.

2) Effects of CM Cost: We now examine the impacts of CM cost. Results under different levels of CM cost for CBM are summarized in Table V. From Table V, we can see that the rescheduling interval (ω) decreases quickly as the CM cost increases. More frequent PM rescheduling attempts are needed to avoid failures and reduce high CM costs. The inspection interval (δ) and PM threshold (ξ) also decrease as a result of the increase in CM cost.

3) Effects of Forced Shutdown Cost: Similar analysis is performed on the impacts of cost of forced shutdown c_m on optimal policies of CBM. As c_m increases, the inspection interval (δ) and PM threshold (ξ) decrease to initiate the PM attempts early, so forced shutdown can be potentially avoided. The rescheduling interval also decreases consequently to increase the chance of performing PM rather than forced shutdown. The forced shutdown threshold (ζ) also increases for the same purpose.

A common observation from Tables IV–VI is that optimal results under Policy 4 and Policy 5 are different from those under Policy 6. This shows that optimal CBM policies also need be reoptimized when equipment may be unavailable for scheduled PM activities.

V. CONCLUSION

In this paper, we proposed—a set of innovative PM models that consider uncertainty in equipment availability when attempting to perform scheduled PM. Two types of rescheduling policies are developed. A Type 1 rescheduling policy delays a PM action repeatedly if the equipment is not available for PM. Under this policy, an infinite number of delays are allowed. Type 2 rescheduling policy was similar to Type 1 policy in terms of delaying the PM action when the equipment is not available for PM. However, under Type 2 policy, the equipment will be forced to shut down for PM if a threshold on the time or deterioration level is reached. The two types of rescheduling policies are integrated with both age-based PM and CBM. Numerical examples and sensitivity analysis are provided to illustrate the proposed PM strategies. In order to examine the potential benefits of the proposed models, the optimal policies with the consideration of equipment unavailability are compared with the ones that ignore such availability. Our results show that the policies that consider equipment unavailability are significantly different from those that do not. Thus, PM strategies should consider equipment availability.

The present model assumes that the probability model for equipment unavailability remains the same over time. One extension is to consider time-dependent equipment unavailability. For example, equipment may experience different work schedules at different times, and consequently equipment unavailability is different over time. Another useful extension is to dynamically reschedule PM actions rather than utilizing a fixed delay interval. Future work will examine a wider range of model parameters to better assess the impact of model parameters on PM rescheduling due to unavailable equipment. A full experimental analysis would be needed in such a case.

APPENDIX I

1) Derivation of the probability that PM is successfully performed at the nth attempt ($\kappa_{\lambda,n}$).

Suppose that the first PM action is attempted upon the λth inspection. The probability that PM is successfully performed upon the first attempt is straightforward, and is given by

$$\begin{aligned}\kappa_{\lambda,1} &= \Pr\{\text{PM successfully performed at the first attempt}\}\\ &= p\Pr\{X((\lambda-1)\delta)\le\xi\cap\xi<X(\lambda\delta)\le s\}\\ &= p\int_0^{\xi} f(u;0,(\lambda-1)\delta)\int_{\xi}^{s} f(x;u,\delta)dxdu\\ &= p\int_0^{\xi} f(u;0,(\lambda-1)\delta)\left(F(s;u,\delta)-F(\xi;u,\delta)\right)du.\end{aligned}$$

If equipment's first PM action is attempted upon the λth inspection, but not successful due to unavailable equipment, and the equipment operates without failure until the second attempt and becomes available for PM, then PM is successfully performed at the second attempt. The probability that PM is performed at the second attempt is given by

$$\begin{aligned}\kappa_{\lambda,2} &= \Pr\{\text{PM successfully performed at the second attempt}\}\\ &= (1-p)\,p\Pr\{X((\lambda-1)\delta)\le\xi\cap\xi<X(\lambda\delta)\\ &\qquad\le s\cap\xi<X(\lambda\delta+\omega)\le s\}\\ &= (1-p)\,p\int_0^{\xi} f(u;0,(\lambda-1)\delta)\int_{\xi}^{s} f(v;u,\delta)\\ &\qquad\int_{v}^{s} f(x;v,\omega)dxdvdu\\ &= (1-p)\,p\int_0^{\xi} f(u;0,(\lambda-1)\delta)\int_{\xi}^{s}\\ &\qquad f(v;u,\delta)\,F(s;v,\omega)\,dvdu.\end{aligned}$$

Next, we generalize it to the probability that PM is successfully performed at the nth attempt

$$\kappa_{\lambda,n} = \Pr\{\text{PM successfully performed at the } n\text{th attempt}\} = \begin{cases} p\int_0^{\xi} f(u;0,(\lambda-1)\delta)\\ \quad(F(s;u,\delta)-F(\xi;u,\delta))\,du, & n=1\\ (1-p)^{n-1}p\int_0^{\xi} f(u;0,(\lambda-1)\delta)\int_{\xi}^{s}\\ \quad f(v;u,\delta)\,F(s;v,(n-1)\omega)\,dvdu, & n>1\end{cases}.$$

2) Derivation of the probability that CM is successfully performed at the nth attempt ($\theta_{\lambda,n}$).

Suppose that the first maintenance action is attempted upon the λth inspection. The probability that CM is successfully performed upon the first inspection is straightforward, and is given by

$$\begin{aligned}\theta_{\lambda,1} &= \Pr\{\text{CM performed at the first inspection}\}\\ &= \Pr\{X((\lambda-1)\delta)\le\xi\cap X(\lambda\delta)>s\}\\ &= \int_0^{\xi} f(u;0,(\lambda-1)\delta)\int_{s}^{\infty} f(x;u,\delta)dxdu\\ &= \int_0^{\xi} f(u;0,(\lambda-1)\delta)\left(1-F(s;u,\delta)\right)du.\end{aligned}$$

Similarly, the probability that CM is performed at the second inspection is given by

$$\begin{aligned}\theta_{\lambda,2} &= \Pr\{\text{CM performed at the second inspection}\}\\ &= (1-p)\Pr\{X((\lambda-1)\delta)\le\xi\cap\xi<X(\lambda\delta)\\ &\qquad\le s\cap X(\lambda\delta+\omega)>s\}\\ &= (1-p)\int_0^{\xi} f(u;0,(\lambda-1)\delta)\int_{\xi}^{s} f(v;u,\delta)\\ &\qquad\int_{s}^{\infty} f(x;u,\omega)dxdvdu\\ &= (1-p)\int_0^{\xi} f(u;0,(\lambda-1)\delta)\int_{\xi}^{s}\\ &\qquad f(v;u,\delta)\left(1-F(s;v,\omega)\right)dvdu.\end{aligned}$$

Similarly, the probability that CM is performed at the third inspection is given by

$$\begin{aligned}\theta_{\lambda,3} &= \Pr\{\text{CM performed at the third inspection}\}\\ &= (1-p)^2\Pr\{X((\lambda-1)\delta)\le\xi\cap\xi<X(\lambda\delta)\\ &\qquad\le s\cap\xi<X(\lambda\delta+\omega)\le s\cap X(\lambda\delta+2\omega)>s\}\\ &= (1-p)^2\int_0^{\xi} f(u;0,(\lambda-1)\delta)\int_{\xi}^{s}\\ &\qquad f(v;u,\delta)\int_{v}^{s} f(x;v,\omega)\int_{s}^{\infty} f(y;u,\omega)dydxdvdu\\ &= (1-p)^2\int_0^{\xi} f(u;0,(\lambda-1)\delta)\int_{\xi}^{s}\\ &\qquad f(v;u,\delta)\int_{v}^{s} f(x;v,\omega)\left(1-F(s;v,\omega)\right)dxdvdu.\end{aligned}$$

Next, we generalize it to the probability that CM is successfully performed at the nth inspection

$$\theta_{\lambda,n} = \Pr\{\text{CM successfully performed at the } n\text{th inspection}\} = \begin{cases}\int_0^{\xi} f(u;0,(\lambda-1)\delta)(1-F(s;u,\delta))\,du, & n=1\\ (1-p)^{n-1}\int_0^{\xi} f(u;0,(\lambda-1)\delta)\int_{\xi}^{s}\\ \quad f(v;u,\delta)\int_{v}^{s} f(x;v,(n-2)\omega)\\ \quad(1-F(s;v,\omega))\,dxdvdu, & n>1\end{cases}.$$

3) Derivation of the probability that PM is successfully performed at the nth attempt when forced shutdown is allowed (${\kappa'}_{\lambda,n}$).

Suppose that the first PM action is attempted upon the λth inspection. Considering the forced shutdown threshold ζ, the probability that PM is successfully performed upon the first attempt is straightforward, and is given by

$${\kappa'}_{\lambda,1} = \Pr\{\text{PM successfully performed at the first attempt}\}$$

$$= p \Pr\{X((\lambda-1)\delta) \le \xi \cap \xi < X(\lambda\delta) \le \zeta\}$$
$$= p \int_0^{\xi} f(u;0,(\lambda-1)\delta) \int_{\xi}^{\zeta} f(x;u,\delta) dx du$$
$$= p \int_0^{\xi} f(u;0,(\lambda-1)\delta)(F(\zeta;u,\delta) - F(\xi;u,\delta)) du.$$

If equipment's first PM action is attempted upon the λth inspection, but not successful due to equipment unavailability, and the equipment operates without failure until the second attempt and becomes available for PM, then PM is successfully performed at the second attempt. The probability that PM is performed at the second attempt is given by

$$\begin{aligned}
&{\kappa'}_{\lambda,2} \\
&= \Pr\{\text{PM successfully performed at the second attempt}\} \\
&= (1-p) p \Pr\{X((\lambda-1)\delta) \le \xi \cap \xi \le X(\lambda\delta) \le \zeta \cap \xi \\
&\quad \le X(\lambda\delta+\omega) \le \zeta\} \\
&= (1-p) p \int_0^{\xi} f(u;0,(\lambda-1)\delta) \int_{\xi}^{\zeta} f(\nu;u,\delta) \\
&\quad \int_{\nu}^{\zeta} f(x;\nu,\omega) dx d\nu du \\
&= (1-p) p \int_0^{\xi} f(u;0,(\lambda-1)\delta) \int_{\xi}^{\zeta} \\
&\quad f(\nu;u,\delta) F(\zeta;\nu,\omega) d\nu du.
\end{aligned}$$

Next, we generalize it to the probability that PM is successfully performed at the *n*th attempt

$$\begin{aligned}
&{\kappa'}_{\lambda,n} \\
&= \Pr\{\text{PM successfully performed at the } n\text{th attempt}\} \\
&= \begin{cases} p \int_0^{\xi} f(u;0,(\lambda-1)\delta) \\ \quad (F(\zeta-u;0,\delta) - F(\xi-u;0,\delta)) du, & n=1 \\ (1-p)^{n-1} p \int_0^{\xi} f(u;0,(\lambda-1)\delta) \int_{\xi}^{\zeta} f(\nu;u,\delta) \\ \quad F(\zeta;\nu,(n-1)\omega) d\nu du, & n>1 \end{cases}.
\end{aligned}$$

4) Derivation of the probability that CM is successfully performed at the *n*th attempt considering mandatory shutdown scenario (${\theta'}_{\lambda,n}$).

Suppose that the first maintenance action is attempted upon the λth inspection. The probability that CM is successfully performed upon the first inspection is straightforward, and is given by

$$\begin{aligned}
{\theta'}_{\lambda,1} &= \Pr\{\text{CM performed at the first inspection}\} \\
&= p \Pr\{X((\lambda-1)\delta) \le \xi \cap X(\lambda\delta) > s\} \\
&= p \int_0^{\xi} f(u;0,(\lambda-1)\delta) \int_s^{\infty} f(x;u,\delta) dx du \\
&= p \int_0^{\xi} f(u;0,(\lambda-1)\delta)(1 - F(s;u,\delta)) du.
\end{aligned}$$

Similarly, the probability that CM is performed at the second inspection is given by

$$\begin{aligned}
{\theta'}_{\lambda,2} &= \Pr\{\text{CM performed at the second inspection}\} \\
&= (1-p) \Pr\{X((\lambda-1)\delta) \le \xi \cap \xi \le X(\lambda\delta) \\
&\quad \le \zeta \cap X(\lambda\delta+\omega) > s\} \\
&= (1-p) p \int_0^{\xi} f(u;0,(\lambda-1)\delta) \int_{\xi}^{\zeta} \\
&\quad f(\nu;u,\delta) \int_s^{\infty} f(x;\nu,\omega) dx d\nu du \\
&= (1-p) p \int_0^{\xi} f(u;0,(\lambda-1)\delta) \int_{\xi}^{\zeta} f(\nu;u,\delta) \\
&\quad (1 - F(s;\zeta,\omega)) d\nu du.
\end{aligned}$$

Similarly, the probability that CM is performed at the third inspection is given by

$$\begin{aligned}
{\theta'}_{\lambda,3} &= \Pr\{\text{CM performed at the third inspection}\} \\
&= (1-p)^2 \Pr\{X((\lambda-1)\delta) \le \xi \cap \xi \le X(\lambda\delta) \le \zeta \cap \xi \\
&\quad \le X(\lambda\delta+\omega) \le \zeta \cap X(\lambda\delta+2\omega) > s\} \\
&= (1-p)^2 \int_0^{\xi} f(u;0,(\lambda-1)\delta) \int_{\xi}^{\zeta} f(\nu;u,\delta) \\
&\quad \int_{\nu}^{\zeta} f(x;\nu,\delta) \int_s^{+\infty} f(y;x,\omega) dy dx d\nu du \\
&= (1-p)^2 \int_0^{\xi} f(u;0,(\lambda-1)\delta) \int_{\xi}^{\zeta} f(\nu;u,\delta) \\
&\quad \int_{\nu}^{\zeta} f(x;\nu,\omega)(1 - F(s;x,\omega)) dx d\nu du.
\end{aligned}$$

Next, we generalize it to the probability that CM is successfully performed at the *n*th inspection

$$\begin{aligned}
&{\theta'}_{\lambda,n} \\
&= \Pr\{\text{CM successfully performed at the } n\text{th inspection}\} \\
&= \begin{cases} \int_0^{\xi} f(u;0,(\lambda-1)\delta) \\ \quad (1 - F(s-u;0,\delta)) du, & n=1 \\ (1-p)^{n-1} \int_0^{\xi} f(u;0,(\lambda-1)\delta) \\ \quad \int_{\xi}^{\zeta} f(\nu;u,\delta) \int_{\nu}^{\zeta} f(x;\nu,(n-2)\omega) \\ \quad (1 - F(s;x,\omega)) dx d\nu du, & n>1 \end{cases}.
\end{aligned}$$

5) Derivation of the probability that forced shutdown is performed at the *n*th attempt (${q'}_{\lambda,n}$).

Suppose that the first maintenance action is attempted upon the λth inspection. The probability that forced shutdown is performed upon the first PM attempt is given by

$$\begin{aligned}
&{q'}_{\lambda,1} \\
&= \Pr\{\text{forced shutdown performed at the first attempt}\} \\
&= \Pr\{X((\lambda-1)\delta) \le \xi \cap \zeta < X((\lambda-1)\delta) \le s\} \\
&= \int_0^{\xi} f(u;0,(\lambda-1)\delta) \int_{\zeta}^{s} f(\nu;u,\delta) d\nu du \\
&= \int_0^{\xi} f(u;0,(\lambda-1)\delta)(F(s;u,\delta) - F(\zeta;u,\delta)) du.
\end{aligned}$$

Similarly, the probability that forced shutdown is performed at the second inspection is given by

$${q'}_{\lambda,2}$$

$$
\begin{aligned}
&= \Pr\{\text{forced shutdown performed at the second attempt}\}\\
&= (1-p)\Pr\{X((\lambda-1)\delta) \le \xi \cap \xi < X(\lambda\delta) \le \zeta \cap \zeta\\
&\qquad < X(\lambda\delta+\omega) \le s\}\\
&= (1-p)\int_0^{\xi} f(u;0,(\lambda-1)\delta)\int_{\xi}^{\zeta} f(\nu;u,\delta)\\
&\qquad \int_{\zeta}^{s} f(x;u,\omega)dxdodu\\
&= (1-p)\int_0^{\xi} f(u;0,(\lambda-1)\delta)\int_{\xi}^{\zeta} f(\nu;u,\delta)\\
&\qquad (F(s;\nu,\omega)\,F(\zeta;\nu,\omega))\,d\nu du.
\end{aligned}
$$

Similarly, the probability that CM is performed at the third PM attempt is given by

$$
\begin{aligned}
&q'_{\lambda,3}\\
&= \Pr\{\text{forced shutdown performed at the third attempt}\}\\
&= (1-p)^2\Pr\{X((\lambda-1)\delta) \le \xi \cap \xi < X(\lambda\delta) \le \zeta \cap \xi\\
&\qquad < X(\lambda\delta+\omega) \le \zeta \cap \zeta < X(\lambda\delta+\omega) \le s\}\\
&= (1-p)^2\int_0^{\xi} f(u;0,(\lambda-1)\delta)\int_{\xi}^{\zeta} f(\nu;u,\delta)\int_{z}^{\zeta} f(x;\nu,\delta)\\
&\qquad \int_{\zeta}^{s} f(y;x,\omega)dydxd\nu du\\
&= (1-p)^2\int_0^{\xi} f(u;0,(\lambda-1)\delta)\int_{\xi}^{\zeta} f(\nu;u,\delta)\int_{\nu}^{\zeta} f(x;\nu,\omega)\\
&\qquad (F(s;x,\omega)-F(\zeta;x,\omega))\,dxd\nu du.
\end{aligned}
$$

Next, we generalize it to the probability that forced shutdown is performed at the *n*th PM attempt

$$
\begin{aligned}
&q'_{\lambda,n}\\
&= \Pr\{\text{forced shutdown performed at the } n\text{th attempt}\}\\
&= \begin{cases}
\int_0^{\xi} f(u;0,(\lambda-1)\delta) & \\
\quad (F(s;u,\delta)-F(\zeta;u,\delta))\,du, & n=1\\
(1-p)^{n-1}\int_0^{\xi} f(u;0,(\lambda-1)\delta)\int_{\xi}^{\zeta} f(\nu;u,\delta) & \\
\int_{z}^{\zeta} f(x;\nu,(n-2)\omega)\,(F(s;u,\omega) & \\
\quad -F(\zeta;u,\omega))dxd\nu du, & n>1
\end{cases}.
\end{aligned}
$$

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