

Optimal Path Planning for Unmanned Aerial Systems to Cover Multiple Regions

Junfei Xie^{*}, Jin Lei[†], Luis Rodolfo Garcia Carrillo[‡]
Texas A&M University-Corpus Christi, Corpus Christi, TX, 78412, USA

In many unmanned aerial systems (UAS) applications such as land assessment, search and rescue, UASs are often required to survey or scan multiple spatially distributed regions. To realize these applications, one of the most critical challenges to address is how to find the optimal paths for the UASs to cover multiple regions. This problem can be considered as a variant of the traveling salesmen problem (TSP) integrated with the coverage path planning (CPP) problem, which has been rarely studied in the literature. In this paper, we conduct a systematic investigation on this problem. The mathematical problem formulation is first provided, followed by two intra-regional path planning algorithms of different capabilities. An advanced dynamic programming (DP) based approach is then introduced that finds (sub-) optimal solutions for the integrated TSP-CPP problem. Numerical analysis and comparative simulation studies demonstrate the optimality of the proposed approaches.

I. Introduction

In recent years, unmanned aerial systems (UASs) have gained great popularity in both military and commercial applications. The unique features of UASs including low cost, high maneuverability, and fast speed, have made them promising solutions to perform missions in hazardous environments such as post-disaster surveillance, searching for victims in a battle field, and exploration of Mars.¹ In these and many other UAS applications, one of the most critical challenges to address is the path planning for the UASs, i.e., finding an optimal path from the UAS's present location to a target.

Path planning for UASs has been actively studied in the literature. Many works are aimed at finding the shortest path to a target while avoiding static or dynamic obstacles.²⁻⁸ For instance, the search for the optimal UAS flight path that minimizes the total travel distance and maximizes the distance to radar sites over a hostile territory is studied in reference.² This problem is solved through modifying a sub-optimal path to derive the optimal solution efficiently. Additional methods to generate collision-free flight paths in presence of static obstacles include the reinforcement learning based approach,³ the Tangent-plus-Lyapunov Vector Field Guidance-based approach,⁵ and the path planning based on attractive and repulsive potential fields.⁸ To avoid dynamic obstacles, a Closed-Loop Rapidly exploring Random Tree (RRT) based approach is developed in reference,⁴ which has been tested in actual flights. Other path planning approaches with dynamic obstacle avoidance capability include the chance-constrained RRT-based approach⁷ and the A* based approach.⁶

In cases when the UAS is required to visit multiple targets, the path planning problem falls into the category of the well known traveling salesman problem (TSP) or more generally the vehicle routing problem (VRP).^{9,10} In the TSP, a vehicle aims to visit a set of locations at the minimal cost. The basic VRP generalizes the TSP and further requires the vehicle to fulfill the demand of each target. It is also frequently referred to as the vehicle scheduling, truck dispatching or delivery problem. Extended from these baseline problems, many variants of TSP and VRP have been proposed and researched. For instance, a survey of MTSP that generalizes TSP for multiple vehicles can be found in reference.¹¹ VRP with time windows are investigated in references,¹²⁻¹⁴ which restricts the time periods when deliveries can be made to the targets. VRP for heterogeneous vehicles with different characteristics such as size, capacities, and costs, has been

^{*} Assistant Professor, Department of Computing Sciences, and AIAA Member. Email: Junfei.Xie@tamucc.edu

[†] Associate Professor, Department of Mathematics & Statistics. Email: lei.jin@tamucc.edu

[‡] Assistant Professor, Department of Electrical Engineering. Email: luis.garcia@tamucc.edu

studied in references.^{15–17} Reference¹⁸ studies the drone delivery problem and takes the unique features of drones including limited flight range and carrying capacity into the consideration.

Path planning for the UAS to cover a single region, which is known as the coverage path planning (CPP) problem, has also been studied extensively. The main objective of the CPP problem¹⁹ is to find a collision-free path so that the vehicle will cover the whole region after it completes following this path. CPP methods can be classified into two categories based on whether full coverage can be provably achieved: (i) the heuristic or randomized approaches, and (ii) the complete approaches. The heuristic or randomized approaches are simple to implement, but may require large amount of time to achieve full coverage. The complete approaches achieve full coverage by first decomposing the region into sub-regions (or called cells) and then performing coverage. Commonly used cellular decomposition methods include the Trapezoidal decomposition,²⁰ Boustrophedon decomposition²¹ and Morse decomposition.²² Typical coverage algorithms include the grid-based wavefront algorithm,²³ the Spiral-Spanning Tree Coverage algorithm,²⁴ and the Backtracking Spiral Algorithm.²⁵

Despite the extensive works on path planning, most existing studies focus on finding the shortest path to a target, the optimal path to visit a set of targets, or the path to completely cover a single region. To the best of our knowledge, the coverage of multiple spatially separated regions has been rarely investigated, which is however commonly encountered in many UAS applications. For instance, in post-disaster management, UASs may need to collect disaster information from multiple affected areas for damage assessment, search and rescue missions, targets may be distributed over multiple isolated regions. The scenarios when UASs need to cover multiple regions are also common in agricultural applications such as crop scouting, precision agriculture, and livestock management. To address this problem, simply assigning one UAS for each region or randomly selecting the region to be covered next may generate unaffordable cost, or may result in long mission completion time. Innovative approaches that generate optimal paths to cover multiple regions are needed to reduce the mission cost and improve the efficiency of the system.

The path planning problem for a single UAS to fully cover multiple spatially distributed regions can be considered as a combination of the TSP and CPP problems, which aims to optimize both inter- and intra-regional paths and is given the name TSP-CPP here. To the best of our knowledge, this integrated TSP-CPP problem is new to the literature. It is more challenging than the TSP or CPP problem as the optimizations of the inter- and intra-regional paths are nested and thus cannot be considered separately. Furthermore, the locations to enter and exit a region impact both inter- and intra-regional paths, which are not considered in the CPP problem and add significant complexities to the problem. In our initial investigation,²⁶ we developed two approaches to this TSP-CPP problem for rectangular regions. One is the grid-based approach that converts the TSP-CPP problem into a TSP problem by decomposing regions into a set of uniform grids that are smaller than or equal to the sensor range of the UAS. Another one is the dynamic programming (DP) based approach that explores the joint optimization of both inter- and intra-regional paths.

In this paper, we conduct a comprehensive investigation on the TSP-CPP problem and extend the DP-based approach for irregularly shaped convex polygonal regions. In particular, a more complete mathematical formulation of the TSP-CPP problem that includes the optimization of intra-regional paths, selection of entrance and exit locations in each region and optimization of the visiting order for regions is first provided. Two intra-regional path planning algorithms of different capabilities are then developed. We then introduce an advanced DP-based approach that finds the (sub-) optimal path to cover multiple convex polygonal regions. Systematic numerical and simulation-based studies are conducted to prove and illustrate the optimality of the proposed approaches. Of note, while we were developing this paper based on our earlier results, we noticed a recent development that solves the TSP-CPP problem heuristically.²⁷ This approach, called Two Steps Path Planning (TSPP), first determines the visiting order for the regions using their centroids, and then plans the coverage path for each region. Although simple, this heuristic approach does not guarantee optimal paths by ignoring the correlations between inter- and intra-regional path planning. In this paper, we investigate the TSP-CPP problem in a systematic way, provide mathematical formulations, and develop approaches that are proved to be optimal under mild assumptions.

In the rest of the paper, we first formulate the integrated TSP-CPP problem mathematically in Section II. Two intra-regional path planning algorithms and the advanced DP-based approach are then introduced in Section III. Simulation results are provided in Section IV to demonstrate the performance of the proposed approaches. Section V includes a brief conclusion and discussions on future works.

II. Problem Formulation

Consider the TSP-CPP problem for a single multirotor UAS that can turn with an arbitrary radius of curvature. Suppose a UAS, which carries a sensor with a coverage range of $a \times a$, $a > 0$, is initially located at a depot and then dispatched to survey N non-overlapping spatially distributed convex polygonal regions. The regions are labeled with numbers $1, \dots, N$, and the depot with number 0. Suppose the location of the depot, denoted as v_0 , and the locations of each region's vertices, denoted as $v_{i1}, v_{i2}, \dots, v_{ir_i}$, are known, where r_i is the total number of vertices of region i , $i \in N = \{1, 2, \dots, N\}$. The UAS is assumed to fly at a constant altitude with a constant speed, and has sufficient power to complete the mission.

The objective of the TSP-CPP problem considered here is to find the optimal path that starts and ends at the depot, such that each region is completely covered by the UAS and the total travel cost is minimized. To describe an intra-regional path that leads to full coverage of a region, we let $S = \{s_{im}\}$, $m = 1, 2, \dots, n$ be the set of locations to visit in region $i \in N$, and let y_{pq}^i , $p, q \in \{1, 2, \dots, n\}$, be a decision variable to capture the visiting order of locations in S_i , such that $y_{pq}^i = 1$ if the UAS moves from location s_{ip} to location s_{iq} and $y_{pq}^i = 0$ otherwise, where $n_i = |S_i|$ is the cardinality of set S_i . The UAS covers region i completely if it moves through all locations in S_i according to the visiting order specified by set $\{y_{pq}^i\}$. To capture the entrance and exit locations in region i , we introduce decision variables e_p^i and t_p^i , $p \in \{1, 2, \dots, n\}$, where $e_p^i = 1$ if the UAS enters region i from location s_{ip} , and $e_p^i = 0$ otherwise. Similarly, $t_p^i = 1$ if the UAS exits region i from location s_{ip} , and $t_p^i = 0$ otherwise. In the special case when region i can be fully covered by the UAS' sensor range, $n = 1$, $y_{11}^i = 1$, $e_1^i = 1$, $t_1^i = 1$, and s_{i1} is the centroid of this region. To capture the order of the regions to visit, we introduce a decision variable x_{ij} , $i, j \in N_0 = N \cup \{0\}$, where $x_{ij} = 1$ if the UAS moves from region (or depot) i to region (or depot) j , and $x_{ij} = 0$ otherwise. A complete path, often referred to as a *tour*, to cover all regions can then be described by S , y_{pq}^i , e_p^i , t_p^i , $\forall i \in N$, $p, q \in S_i$, and x_{ij} , $\forall i, j \in N_0$. The total travel cost J can be calculated by the following equation

$$J = \sum_{i \in N} \sum_{j \in N, j \neq i} \sum_{p=1}^{n_i} \sum_{q=1}^{n_j} x_{ij} t_p^i e_q^j d(s_{ip}, s_{jq}) + \sum_{i \in N} \sum_{p=1}^{n_i} \sum_{q=1}^{n_i} y_{pq}^i d(s_{ip}, s_{iq}) + \sum_{i \in N} \sum_{p=1}^{n_i} x_{0i} e_p^i d(v_0, s_{ip}) \quad (1)$$

$$+ \sum_{i \in N} \sum_{p=1}^{n_i} x_{i0} t_p^i d(s_{ip}, v_0)$$

where $d(s_p, s_q)$ represents the cost to travel from location s_p to location s_q . Here we adopt the Euclidean distance to measure the travel cost.

In equation (1), the first two terms evaluate the total travel cost for inter- and intra-regional movements. The last two terms calculate the travel cost for moving from the depot to the first region in the tour and from the last region back to the depot respectively. The validity of the tour can be ensured by introducing the following constraints

$$\sum_{j \in N, j \neq i} x_{ij} = 1, \quad \forall i \in N_0 \quad (2)$$

$$\sum_{i \in N_0, i \neq j} x_{ij} = 1, \quad \forall j \in N_0 \quad (3)$$

$$\sum_{q=1, q \neq p}^{n_i} y_{pq}^i = 1, \quad \sum_{p=1, p \neq q}^{n_i} y_{pq}^i = 1, \quad \forall i \in N, n_i > 1 \quad (4)$$

$$\sum_{p=1}^{n_i} e_p^i = 1, \quad \sum_{p=1}^{n_i} t_p^i = 1, \quad \forall i \in N \quad (5)$$

$$\sum_{i,j \in M_1} x_{ij} \leq |M_1| - 1, \quad \forall M_1 \subset N_0, |M_1| \geq 2 \quad (6)$$

$$\sum_{p,q \in M_2} y_{pq}^i \leq |M_2| - 1, \quad \forall i \in N, M_2 \subset \{1, 2, \dots, n_i\}, |M_2| \geq 2 \quad (7)$$

where $|M|$ above is the cardinality of a set M . Equations (2)-(3) ensure that each region is only visited once. Equation (4) ensures that each location $s_p \in S_i$ in region i is only visited once. Equation (5) restricts

the UAS to enter or exit each region only once. Equations (6)-(7) eliminate sub-tours and maintain the continuity of the tour. Therefore, the TSP-CPP problem can be expressed as

$$\min_{S_i, y_{pq}^i, e_p^i, t_p^i, \forall i \in N, p, q \in S_i, x_{ij}, \forall i, j \in N} J \quad (8)$$

under the constraints of equations (2)-(7). Notice that the TSP-CPP problem is reduced to a TSP when all regions are small enough and each can be covered by the UAS' sensor range. In addition, when there is a single region to visit and this region is larger than the UAS' sensor range, the TSP-CPP problem is reduced to a CPP problem.

III. Advanced Dynamic Programming based Approach to the TSP-CPP Problem

To address the TSP-CPP problem in equation (8), it is essential to find the optimal intra-regional paths (captured by S_i and $\{y_{pq}^i\}$), the optimal entrance and exit locations (captured by $\{e_p^i\}$ and $\{t_p^i\}$), and the optimal visiting order for the regions (captured by $\{x_{ij}\}$). Notice that the optimizations of the intra-regional paths (a CPP problem) and the visiting order for regions (a TSP problem) are both impacted by the entrance and exit locations in each region. Entrance and exit locations that minimize the distance between two regions may not lead to the shortest intra-regional paths in these regions, and vice versa. Therefore, the optimizations of the intra-regional paths, visiting order for regions, and the entrance and exit locations should be considered simultaneously.

In this section, we introduce an advanced dynamical programming (DP) based approach, extended from our earlier development²⁶ to solve the TSP-CPP problem for irregularly shaped convex polygonal regions. This approach jointly optimizes the intra-regional paths, visiting order for regions and the entrance and exit locations. Before we describe the detailed procedures of the DP-based approach, let us first introduce two approaches of different capabilities for intra-regional path planning.

A. Intra-Regional Path Planning

There are two main approaches to find the path that covers a region completely. One is to decompose the region into cells (or sub-regions) and then find the path to cover each cell in back-and-forth patterns. Another one is to discretize the region into a collection of grids, so that each grid is smaller than or equal to the UAS' sensor range. The region can then be completely covered by traversing through all the grids. In this section, we investigate both approaches and explore a modified cellular decomposition method to generate (sub-) optimal intra-regional paths. Heuristic or randomized CPP approaches are not considered here, as they do not guarantee complete area coverage.

1. Back-and-Forth Based Coverage (BFC)

Consider a convex polygonal region without exclusion zones that are forbidden from visiting, the UAS flying in a back-and-forth motion has parallel flight lines. These flight lines can be found by cellular decomposition using a set of parallel *lines of support*, which intersect the region. The distance between the lines of support that intersect the region at a single vertex or edge is called the *span* of the region. Figure 1 shows two lines of support, l_1 and l_2 , that intersect a pentagon at its vertices, and the associated span W .

Let us next describe the approach to find the back-and-forth based coverage path for a convex polygonal region using a modified cellular decomposition method, given the lines of support, l_1 and l_2 , and the corresponding span W . In the simplest case when $d \frac{W}{a} e = 1$ (i.e., $W \leq a$), no decomposition is required and the UAS can fully cover the region without making any turns. Figure 2(a) shows the smallest rectangular cell to fully cover a region with $d \frac{W}{a} e = 1$. The optimal coverage flight line^a is highlighted in red, which is $\frac{W}{2}$ away from the two lines of support and has the end-points $\frac{a}{2}$ away from the cell's boundaries along the direction of the span W . In cases when $d \frac{W}{a} e > 1$, the region is decomposed into $d \frac{W}{a} e$ cells as illustrated in Figures 2(b)-2(c). To minimize the travel distance, we let the width of the two outermost cells that are bounded by l_1 or l_2 to be $\frac{a+b}{2}$, and their coverage flight lines to be $\frac{a}{2}$ away from l_1 or l_2 , where $b = \frac{W-a}{dW/ae-1}$.

^aIn this paper, the *flight line* refers to the path segment along the direction of the lines of support in a cell.

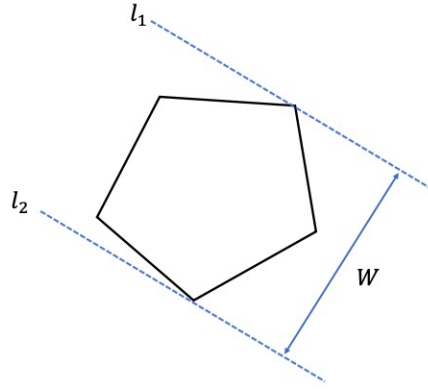


Figure 1. Span of a convex polygonal region.

If $d \frac{W}{a} e > 2$, we let the width of the intermediate cells to be b and their coverage flight lines to be $\frac{b}{2}$ away from the boundaries of the cells. Therefore, the distance between adjacent parallel flight lines is the same and equal to b . The end-points of the flight line in each cell are $\frac{a}{2}$ away from the cell's boundaries along the direction of the span W . The closest end-points in adjacent cells are connected as illustrated in Figure 2(b)-2(c). The following two lemmas illustrate the performance of the BFC.

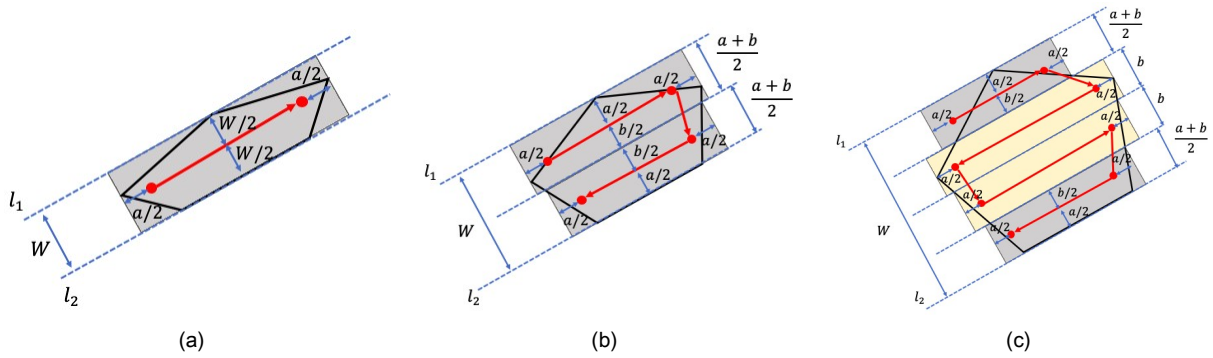


Figure 2. Illustration of the BFC that adopts a modified cellular decomposition method to find the path that covers a region completely in cases when a) $d \frac{W}{a} e = 1$, b) $d \frac{W}{a} e = 2$, and c) $d \frac{W}{a} e > 2$. The rectangular cells of different sizes are highlighted in grey or light yellow. The intra-regional paths are highlighted in red.

Lemma 1. Given the vertices of a convex polygonal region and the UAS' sensor range $a \times a$, the path generated by the BFC leads to full coverage of the region.

Proof. For a particular span W , as $b = \frac{W-a}{dW/ae-1} = a \frac{W/a-1}{dW/ae-1} \leq a$, the width of each cell ($\frac{a+b}{2}$ or b) is smaller than or equal to the sensor range a . Furthermore, as the end-points of the flight line in each cell are $\frac{a}{2}$ away from the boundaries along the direction of the span W , the UAS following this flight line will fully cover the cell. Hence, by following the flight lines in all cells, the UAS will cover the whole region completely. $\square$

Lemma 2. Given the UAS' sensor range $a \times a$, the BFC generates the shortest path to cover a region, if the region is rectangular and the lines of support are parallel to the region's edges of length L_1 , where $d \frac{L_1}{a} e(L_2 - a) + L_1 \leq d \frac{L_2}{a} e(L_1 - a) + L_2$ and L_2 is the length of the other two edges of the rectangular region.

The proof of Lemma 2 can be easily obtained according to Lemma 1 in reference,²⁶ which provides the shortest coverage distance for a rectangular region, and is thus omitted here. Note that the value of the span, W , and the length of the coverage path generated by the BFC varies as the direction of the lines of support changes. The minimum span, W_{min} , which is the width of the convex polygonal region, is achieved when the lines of support are parallel to one of the region's edges. It is proved in reference²⁸ that the coverage

path with the least number of turns is achieved when the UAS flies in the direction perpendicular to the width W_{min} . Reference²⁸ also provides an efficient algorithm of time complexity $O(n)$ to find the span and width of a convex polygon with n vertices, which is adopted in this paper to find the span W for the given lines of support.

As the tour in our TSP-CPP problem is not only impacted by intra-regional paths, but also the entrance and exit locations and the visiting order for regions, we allow the directions of the lines of support, denoted as α , to vary while searching for the optimal tour. Given the direction α of the lines of support, the candidate pairs of entrance and exit locations, denoted as set $G(\alpha) = \{(s_{ie}, s_{it})\}$, include all possible combinations of the end-points of the BFC paths, where $s_{ie}, s_{it} \in S_i(\alpha)$ and $e \neq t$ if $|S_i(\alpha)| \geq 2$. Note that $|C_i(\alpha)| = 4$ if $|S_i(\alpha)| \geq 2$.

2. Grid-Based Coverage (GC)

The BFC can find the (sub-) optimal intra-regional paths efficiently, but it places constraints on the entrance and exit locations. The grid-based coverage (GC) approach relaxes these constraints. In particular, it decomposes a region into a collection of grids and then finds the intra-regional path by traversing through all the grids. The UAS can enter or exit the region through any of the grids.

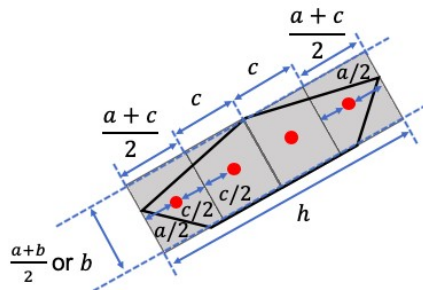


Figure 3. Decomposition of a cell into grids.

To decompose a convex polygonal region into grids, we first apply the modified cellular decomposition method described in the previous section to decompose the region into $\frac{W}{a}$ cells, given the lines of support, l_1 and l_2 . Each cell is then further decomposed into $\frac{h}{a}$ grids, where h is the height of the cell (see Figure 3 for an illustration). If $d_{\frac{h}{a}}^h > 1$, the height of the two outermost grids is $\frac{a+c}{2}$, and the height of the other grids is c , where $c = \frac{h-a}{dh/ae-1}$. As $c = \frac{h-a}{dh/ae-1} = a \frac{h/a-1}{dh/ae-1} \leq a$, the size of each grid is smaller than or equal to the UAS' sensor range $a \times a$. The UAS can thus fully cover the region by traversing through all the grids. To reduce the travel cost, we select the location to visit in each grid (i.e., s_{ip}) by applying the BFC to generate flight lines in each cell and then uniformly discretizing the flight lines with an interval of c , as illustrated in Figure 3. As the UAS will always pass through the boundaries of a region when entering or exiting from this region, we let the candidate pairs of entrance and exit locations in each region, $C_i(\alpha) = \{(s_{ie}, s_{it})\}$, include all possible combinations of the locations at the boundaries, where $s_{ie}, s_{it} \in S_i(\alpha)$ and $e \neq t$ if $|S_i(\alpha)| \geq 2$.

With the set of locations $S_i(\alpha)$ to visit in a region determined, the GC then searches for the optimal path to traverse through all locations in $S_i(\alpha)$, given the start (entrance) location $s_{ie} \in S_i(\alpha)$ and the end (exit) location $s_{it} \in S_i(\alpha)$. This problem can be formulated as a minimum Hamiltonian path problem with specified start and end points.^{29,30} The problem can be addressed by converting it into a TSP problem through introducing a dummy point with zero distance to both the start and end points, and then applying DP³¹ to find the optimal solution.

Lemma 3. Given the UAS' sensor range $a \times a$, and the direction of the lines of support, α , the path generated by the GC is shorter than or equal to the one generated by the BFC if the same pair of entrance and exit locations is selected.

Proof. Note that the GC finds the set of locations $S_i(\alpha)$ to visit in a region by discretizing the path found by the BFC. As the GC searches for the shortest path to visit all locations in $S_i(\alpha)$, the generated path must be shorter than or equal to the one generated by the BFC. $\square$

Although the GC may generate shorter tours compared with the BFC, due to more flexibility on entrance and exit locations, it is not scalable to the number of grids and thus the size of the regions.

B. Algorithm Description

Define $f_1(\alpha) = \{S_i(\alpha), C_i(\alpha)\}$ as the function that takes the direction of the lines of support, α , as the input and produces the set of locations to visit, $S_i(\alpha)$, and the candidate entrance and exit locations, $C_i(\alpha)$, in region i as the outputs. Define $f_2(S_{ie}, S_{it}) = \{y_{pq}^i\}$ as the function that takes a candidate pair of entrance and exit locations, $(S_{ie}, S_{it}) \in C_i(\alpha)$, as the input and produces the optimal visiting order $\{y_{pq}^i\}$ of locations in set $S_i(\alpha)$. The corresponding travel cost is denoted as $g(S_{ie}, S_{it}) = \sum_{p=1}^{n_i} \sum_{q=1}^{n_i} y_{pq}^i d(S_{ip}, S_{iq})$. Both functions, f_1 and f_2 , can be achieved using the BFC or GC. By running f_1 and f_2 at all possible values of α , we can find the complete set of candidate pairs of entrance and exit locations, denoted as $C_i = \{C_i(\alpha)\}$. Let us next describe the use of DP to solve the TSP-CPP problem.

Let $T \subseteq N$ be a subset of target regions, and define $D(T, i, S_{ie}, S_{it})$, $i \in T$, $(S_{ie}, S_{it}) \in C_i$, as the travel cost of the (sub-) optimal path that fully covers each region in T , starts at the depot, enters the last region i from S_{ie} and ends at S_{it} . To calculate the value of $D(T, i, S_{ie}, S_{it})$, we consider the path that ends at the region visited prior to region i . In particular, suppose the UAS visits region $k \in T$ prior to region i and $(S_{ke}, S_{kt}) \in C_k$. Therefore, $D(T, i, S_{ie}, S_{it})$ can be computed by adding $D(T - \{i\}, k, S_{ke}, S_{kt})$, the cost of traveling from S_{kt} to S_{ie} , and the cost of the optimal intra-regional path from S_{ie} to S_{it} . As k could be any region in $T - \{i\}$ and (S_{ke}, S_{kt}) could be any pair in C_k , $D(T, i, S_{ie}, S_{it})$ is solved by considering all $k \in T - \{i\}$

Algorithm 1: DP-based Approach to the TSP-CPP Problem

Input: Depot v_0 , vertices of all regions in N , and sensor range $r \times r$ of the UAS.

Output: A (sub-) optimal path that covers all regions in N starting and ending at depot v_0 with the minimum cost, and the minimum cost.

```

1 foreach  $i \in N$  do
2   foreach possible  $\alpha \in [0, \pi)$  do
3      $\{S_i(\alpha), C_i(\alpha)\} \leftarrow f_1(\alpha)$ ;
4     foreach  $(S_{ie}, S_{it}) \in C_i(\alpha)$  do
5        $\{y_{pq}^i\} \leftarrow f_2(S_{ie}, S_{it})$ ;
6        $g(S_{ie}, S_{it}) \leftarrow \sum_{p=1}^{n_i} \sum_{q=1}^{n_i} y_{pq}^i d(S_{ip}, S_{iq})$ ;
7        $D(\{i\}, i, S_{ie}, S_{it}) \leftarrow d(v_0, S_{ie}) + g(S_{ie}, S_{it})$ ;
8        $P(\{i\}, i, S_{ie}, S_{it}) \leftarrow (v_0, S_i(\alpha), \{y_{pq}^i\})$ ;
9      $C_i \leftarrow \{C_i, C_i(\alpha)\}$ ;
10 foreach  $t = 2, \dots, N$  do
11   foreach  $T \subseteq N$  where  $|T| = t$  do
12     foreach  $i \in T$  do
13       foreach  $(S_{ie}, S_{it}) \in C_i$  do
14         foreach  $k \in T - \{i\}$  do
15           foreach  $(S_{ke}, S_{kt}) \in C_k$  do
16              $dist \leftarrow D(T - \{i\}, k, S_{ke}, S_{kt}) + d(S_{kt}, S_{ie}) + g(S_{ie}, S_{it})$ ;
17             if  $dist < D(T, i, S_{ie}, S_{it})$  then
18                $D(T, i, S_{ie}, S_{it}) \leftarrow dist$ ;
19                $\{S_i, \{y_{pq}^i\}\} \leftarrow P(\{i\}, i, S_{ie}, S_{it})$ ;
20                $P(T, i, S_{ie}, S_{it}) \leftarrow (S_{ke}, S_{kt}, S_i, \{y_{pq}^i\})$ ;
21 return  $\min_{i \in N, (S_{ie}, S_{it}) \in C_i} D(N, i, S_{ie}, S_{it}) + d(v_0, S_{ie})$  and the optimal path by backtracking over arcs in table  $P$ .
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and all $(s_{ke}, s_{kt}) \in C_k$ for each k . The mathematical formulation of $D(T, i, s_{ie}, s_{it})$ is given by

$$D(T, i, s_{ie}, s_{it}) = \begin{cases} d(v_0, s_{ie}) + g(s_{ie}, s_{it}) & \text{if } T = \{i\} \\ \infty & \text{if } i \notin T \\ \min_{\substack{k \in T - \{i\} \\ (s_{ke}, s_{kt}) \in C_k}} D(T - \{i\}, k, s_{ke}, s_{kt}) + d(s_{kt}, s_{ie}) + g(s_{ie}, s_{it}) & \text{otherwise} \end{cases} \quad (9)$$

The pseudo-code in Algorithm 1 illustrates the proposed DP-based approach, which finds the candidate pairs of entrance and exit locations and the (sub-) optimal intra-regional paths for each region first (Lines 1-9), and then performs the DP to search for the (sub-) optimal tour (Lines 10-21). In Line 2 of Algorithm 1, $\alpha \in [0, \pi)$ has infinite number of possible values and thus its range needs to be discretized. Here, we consider the directions along each region's edges, as these directions typically lead to intra-regional paths of less turns. Table P in Lines 8 and 21 is used to track the locations to visit along the path.

Theorem 4. Consider the TSP-CPP problem formulated in equation (8). With intra-regional paths determined by BFC, the DP-based approach shown in Algorithm 1 finds the shortest tour, if all regions are rectangular and the UAS covers each region by moving along the directions of the edges in back-and-forth motions.

Proof. According to Lemma 2, the BFC finds the shortest path to fully cover a rectangular region when moving along the region's edges. Based on this, we can then prove the optimality of the DP-based approach by induction.³¹ In particular, as $D(T, i, s_{ie}, s_{it})$ calculated in each recursion step is the length of the shortest path that fully covers each region in T , and ends at region i with s_{ie} and s_{it} as the entrance and exit locations respectively, Line 21 in Algorithm 1 finds the shortest path to cover all regions. $\square$

Theorem 5. Consider the TSP-CPP problem formulated in equation (8). Under the same settings, the DP-based approach of GC option always generates tours of shorter or same length compared with the DP-based approach of BFC option.

The proof of Theorem 5 can be directly derived from Lemma 3 and thus is omitted here.

IV. Simulation Studies

In this section, we conduct a series of simulation studies to illustrate the performance of the proposed approaches.

A. Performance Evaluation of the BFC

In the first study, we show that the BFC described in Section 1 generates the shortest intra-regional path when the region is rectangular. For comparison, we also implement two alternative approaches that adopt traditional cellular decomposition methods. The first alternative approach decomposes a region into cells of the same width equal to $w = \frac{W}{\lceil W/a \rceil}$. The second alternative approach decomposes a region into cells of width $w = a$. For both methods, flight lines with equal distance to the boundaries of the cells are found to cover the region. Figure 4 shows the intra-regional paths with back-and-forth patterns found by three approaches to cover a rectangular region of size 15×10 . With the UAS' sensor range set to $a = 2.4$, the area covered by the UAS is highlighted in light yellow. As shown in the figure, all three paths lead to complete coverage of the region. The path found by BFC has the shortest length of 70.6. The paths found by the first and second alternative approaches have a length of 71 and 72.6, respectively.

In cases when the region is not a rectangle, the BFC cannot guarantee the shortest coverage paths, as illustrated in Figure 5. In this example scenario, a triangular region is considered and the sensor range is set to $a = 2.4$. The coverage path found by the second alternative approach in Figure 5(c) has the shortest length of 22.6802. The BFC in Figure 5(a) and the first alternative approach in Figure 5(b) find relatively longer paths of 23.3703 and 23.6854 respectively. Comparing Figure 5(c) with Figure 5(a), we can observe that, as the cells have a larger width, the flight line generated by the second alternative approach in the smaller cell is much shorter than the one generated by the BFC, leading to a shorter coverage path. However, a larger cell width does not impact the length of the flight lines when the region is a rectangle. On the contrary, a larger cell width will lead to a larger distance between adjacent flight lines and thus longer coverage paths for rectangular regions. This explains the optimality of the BFC for rectangular regions.

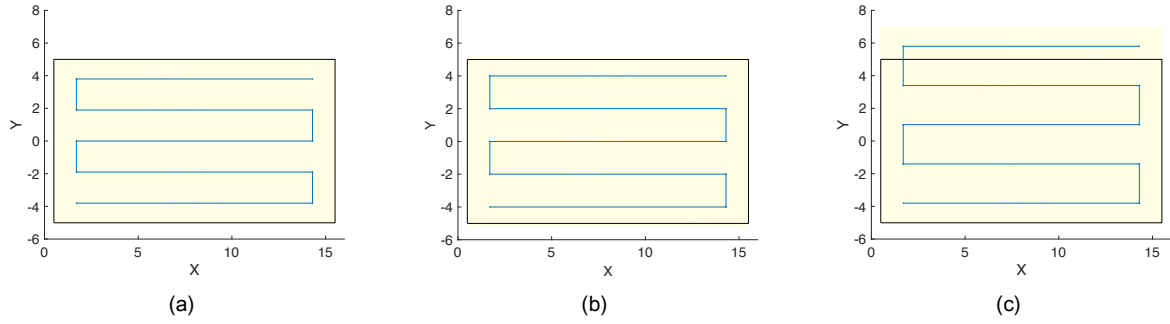


Figure 4. The intra-regional paths (blue lines) of back-and-forth pattern to cover a rectangular region found by a) the BFC; b) the first alternative approach that decomposes a region into cells of width $w = \frac{W}{dW/ae}$; and c) the second alternative approach that decomposes a region into cells of width $w = a$. The area covered by the UAS is highlighted in light yellow.

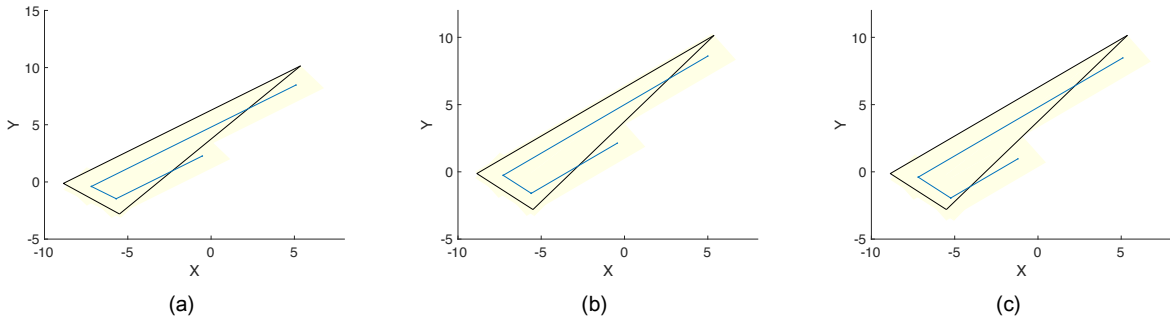


Figure 5. The intra-regional paths of back-and-forth pattern to cover a triangular region found by a) the BFC; b) the first alternative approach that decomposes a region into cells of width $w = \frac{W}{dW/ae}$; and c) the second alternative approach that decomposes a region into cells of width $w = a$.

B. Performance Evaluation of the GC

In this study, we evaluate the performance of the GC, by comparing it with the BFC. As the GC provides more choices for the entrance and exit locations, it typically generates shorter tours. On the other hand, more choices also mean more computational time required to find the best choice. Figures 6(a)-6(b) demonstrate

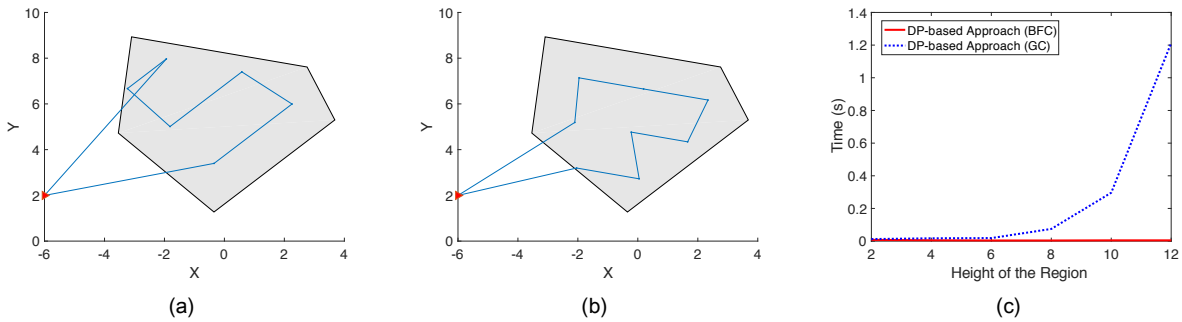


Figure 6. The tours generated by the DP-based approach with a) BFC option and b) GC option. c) Comparison of the execution time required by the two approaches with the increase of the height of the region. Shaded grey areas and the red triangle represent regions and the depot respectively.

the performance of the two intra-regional path planning algorithms when a single pentagonal region is considered with the sensor range set to $a = 3$. As expected, the tour found by the GC, which has a length of 23.6047, is shorter than the one found by the BFC, which has a length of 26.3118. Of note, the BFC generates a shorter intra-regional path of length 13.2677, compared with the intra-regional path of length 14.4361 generated by the GC. This demonstrates the significant role of entrance and exit locations in determining the optimal tour.

To show the demerit of the GC in efficiency, we vary the region size, so that a larger region leads to more pairs of entrance and exit locations to evaluate. In particular, we consider a single region of size 2×2 and then increase the height of the region. The sensor range is set to $a = 2$. To reduce uncertainty, we repeat each experiment for 10 times and take the average. As shown in Figure 6(c), the average execution time required by the GC grows exponentially with the increase of the region size. On the contrary, the BFC shows good scalability to the region size.

C. Performance Evaluation of the DP-based Approach

In the last study, we demonstrate the performance of the proposed DP-based approach to the TSP-CPP problem, by comparing it with the two steps path planning (TSPP) algorithm²⁷ that was developed very recently. For fair comparison, we replace the intra-regional path planning approach in the original TSPP algorithm with the proposed BFC. We then generate five convex polygonal regions of random size, which are distributed randomly over the space. The sensor range of the UAS is set to $a = 3$. Figures 7(a)-7(c) visualize the tours generated by the DP-based approaches and the TSPP. Among these three tours, the one generated by the DP-based approach of GC option has the shortest length of 102.7972. The TSPP generates the longest tour of length 106.0364.

Now let's compare the tour generated by the DP-based approach of BFC option in Figure 7(a), which has a length of 103.7193, with the one generated by the TSPP in Figure 7(c). Although both approaches adopt the same intra-regional path planning algorithm, i.e., BFC, the TSPP considers the optimization

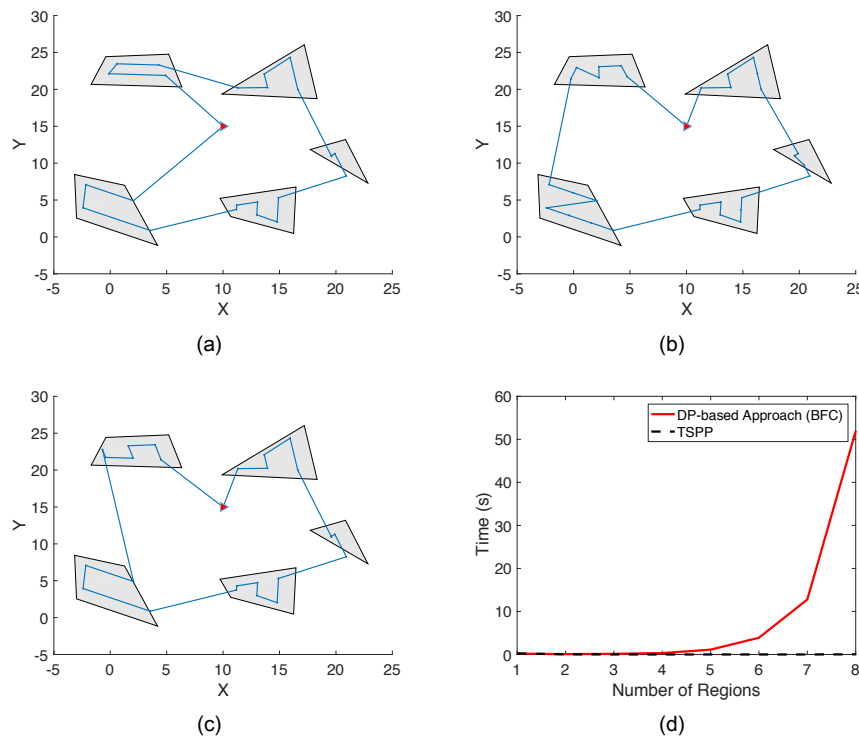


Figure 7. The tours generated by the a) proposed DP-based approach of BFC option; b) proposed DP-based approach of GC option; and c) TSPP. Comparison of the execution time required by the DP-based approach of BFC option and the TSPP with the increase of the number of regions.

of the visiting order for regions and the optimization of intra-regional paths separately, which leads to a longer tour. On the other hand, ignoring the inter-dependencies between the inter- and intra-regional path plans makes the TSP more efficient than the DP-based approach, as shown in Figure 7(d), which plots the execution time of the two approaches with the increase of the number of regions. The size of each region is 4×6 and the sensor range is $a = 2$. We will leave the enhancement of the DP-based approach in efficiency to the future work.

V. Conclusion

This paper explores a new UAS path planning problem, named TSP-CPP, which seeks the optimal tour for UAS to cover multiple spatially distributed regions. To solve this problem, a novel mathematical formulation was provided first. Two intra-regional path planning algorithms of different capabilities were then introduced, which generate (sub-) optimal paths through a modified cellular decomposition method. Based on the two intra-regional path planning algorithms, an advanced DP-based approach was then developed, which exploits the joint optimization of the intra-regional paths, visiting order for the regions, and the entrance and exit locations in each region. We proved that the proposed DP-based approach finds the optimal path under certain assumptions. Numerical analysis and comparative simulation studies demonstrate the good performance of the proposed approaches in finding (sub-) optimal paths. In the future, we will explore intelligent optimization algorithms such as genetic algorithm³⁰ or neuromorphic learning³² to improve the efficiency of the DP-based approach, which is not scalable to the number of regions in its present form.

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