Getting Started Getting Students Modeling: Designing and Facilitating Open-ended Math Modeling Experiences

Ben Galluzzo, Katie Kavanagh
Clarkson University

bgalluzz@clarkson.edu, kkavanag@clarkson.edu

Abstract
“Modeling” is a term that has several meanings in general, but particularly in mathematics. Here math modeling refers to the process of creating a mathematical representation of a real-world scenario to make a prediction or provide insight. There is a distinction between using a formula that arises from an application (for example, distance equals rate times time) and the actual creation of a mathematical relationship itself that can be useful in an applied setting. In this two part workshop, we demonstrate how to develop authentic math modeling challenge problems that are accessible and relevant to students. In the second part of the workshop we talk about how to facilitate math modeling so that students have an opportunity to be creative and innovative in their modeling process while having ownership over their solution.

Introduction
Real-world, messy problems can be approached with mathematics, resulting in a range of possible solutions to help guide decision making. Something as (seemingly) simple as the question, “When should a person replace their car?” could be answered with a wide range of mathematical justifications, each with different strengths and weaknesses. Both students and teachers are often uncomfortable with the notion of math modeling because it so open-ended. Text book “word problems” give you all the information you need to derive a specific solution. Real-world problems are open-ended with lots of unknown information that seems prohibitive. But it is this open-ended nature of real-world problems that leads to building and applying problem solving skills, creativity, innovation, and mathematics.
Mathematical modeling can be thought of as an iterative process made up of the following components. (Note that the word ``steps" is intentionally avoided to highlight the lack of a prescribed ordering of these components, as some may occur simultaneously and some may be repeated.)

- Identify the Problem Because modeling problems are open-ended, the modeler must be specific in defining what it is they would like to find out.
- Make Assumptions and Identify Variables Since it is impossible to account for all the important factors in a given situation, the modeler must make choices about what to incorporate in their representation of the real-world. Making assumptions helps reveal the variables that will be considered and also reduce the number of them by deciding not to include everything. Within this process, relationships between variables will emerge based on observations, physical laws, or simplifications.
- Do the Math Eventually, a relationship between input and output will allow for a solution to be found.
- Analyze and Assess the Solution When considering the results and insights gained from the model, one asks if the answer makes sense.
- Iterate Usually, the model can be refined and the process can be repeated to improve the model's performance.
- Implement the Model and Report Results A clear report on the model and its implementation makes the model understandable to others.

In this two-part workshop, we give a brief introduction to math modeling and focus on two main objectives; (1) developing authentic math modeling experiences for students and (2) facilitating math modeling so that students gain confidence and expertise.

Part I: Designing Math Modeling Challenges

What is a Math Modeling Problem? To get a better understanding of the difference between math modeling and typical math problems, consider the question "When should a person replace their car?" compared to the following; "I want to replace my car when it reaches 87,000 miles. The car currently has 45,000 miles on it. I drive 65 miles/day. How long will it be until I replace my car?"

There are obvious differences between these two questions. The first one seems to give no information (it doesn't even seem like a math problem) while the second one has actual numbers (with units!) in them and tells you the I would replace my car based on reaching a certain mileage. This example illustrates the open-endedness of modeling problems. To get started on the first question, a student would need to do some brainstorming and make some assumptions. They would need to think about important variables and what contributes to making the decision to replace a car. An entire classroom of students could come up with very different solutions based on costs, mileage, lifestyles, etc. For the second problem, every student would (hopefully get) 646 days (or possibly 647 depending on how they rounded the answer).

We will discuss how to develop math modeling questions so that students can engage in a problem-solving experience much different from the "plug and chug" approach to solving word problems. The key is to pose an exciting, open-ended question that students can explore and tackle using mathematics that they know and perhaps even need to discover on their own. We consider the following approaches;

- Investigate the progression of a problem from a "classical" mechanical calculation to an open-ended, curricularly relevant modeling problem,
- Consider typical math problems from the text; do they exhibit characteristics that allow students to explore? Are they "open"? And, if not, can they be altered to become more authentic?
- Build known real world situations "up" and "down" to fit the needs and abilities of your students,
- Share "starter" resources that may serve as inspiration for developing your own math modeling problems.

We encourage all participants to create a modeling challenge appropriate for their classroom audience.

Part II: Facilitating Math Modeling Experiences

When are students modeling? And, are your students doing it? During the second half of this workshop we will explore techniques instructors can use to help students successfully navigate the modeling process. For example, consider two student reactions to the problem statement: "How much does it cost to own a car for one year?"

Student A: "I'm not sure I have enough information to get started."
Student B: "$9,122. I Googled it."

While seemingly disparate, both responses provide a good entry point for developing a modeling solution to the problem. The facilitator is now able to guide both students, and the class, by asking questions that promote continued inquiry and highlight the need for a process when model building. In response to Student A, it would be appropriate to ask "what do you think is important to know in order to get started? In response to Student B, one can ask "Is this the only result of your Google search? How do you think "they" arrived at this answer?" In both cases, (and perhaps with a little more questioning from the instructor) students will understand the need to further define the question and, as

a result, will begin to “brainstorm” and start to make decisions about variables and parameters that they consider important to know in order to find solutions.

It is challenging for a teacher to provide authentic modeling experiences, but worth the effort. You may need to take a risk and try something, see how the students respond and what they come up with, and then try something different the next time or build on what you did even more. Students often appreciate that their instructor is also learning something new and going through the problem solving experience with them! As you begin to create modeling experiences for your students and engage them in the process, the following questions may help guide you in your own assessment of the problem and process that you are designing;

- Did they start with a big, open-ended, real-world question that is relevant to the students?
- Did they ask questions and make assumptions to get a more concise problem statement?
- Are they using mathematical tools to develop relationships between variables and to solve the problem?
- Did they test their model and ask if their answer makes sense?
- Did they identify strengths and weaknesses to possibly improve their model?
- Are they communicating their approach and solution to someone else?

Summary and Workshop Outcomes

A collection of useful questions to ask students modelers will be shared and discussed with participants. Specifically we will introduce modeling situations and explore student solutions from the perspective of the classroom facilitator. We will also highlight how different aspects of the modeling process naturally overlap and intersect; providing instructors with multiple occasions to develop entry and exit points for classroom problem solving. Participants at the workshop will gain experience developing and infusing math modeling into the classroom.

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