Effective Heuristics for Multi-Robot Path Planning in Warehouse Environments

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***Abstract*— In this preliminary study, we propose a new centralized decoupled algorithm for solving one-shot and dynamic optimal multi-robot path planning problems in a grid-based setting mainly targeting warehouse like environments. In particular, we exploit two novel and effective heuristics: path diversification and optimal sub-problem solution databases. Preliminary evaluation efforts demonstrate that our method achieves promising scalability and good solution optimality.**

I. INTRODUCTION

Labeled optimal multi-robot path planning (MPP) has been actively studied for many decades [1]–[4], which finds applications in a wide range of domains including assembly [5], evacuation [6], formation [7], [8], localization [9], microdroplet manipulation [10], object transportation [11], search and rescue [12], human robot interaction [13], and large-scale warehouse automation [14], to list a few. Optimal solvers for MPP are realized through reduction to other problems, e.g., answer set programming [15], SAT [16], and multi-commodity flow [17]. Popular decoupled approaches [2] first compute independent paths then schedule them. Commonly found discrete decoupled approaches span sub-dimensional expansion [18], conflict-based search [19], [20], independence detection [21], among others. There also exists prioritized methods [22]–[25] and global decoupling based approach [26] which achieve superior scalability at the cost of completeness or optimality. MPP is examined from many other angles. As a fairly incomplete list, readers may refer to [27]–[34] for additional algorithmic coverage for MPP under partially labeled and continuous settings.

In this extended abstract, we perform a preliminary study of two novel heuristics: path-diversification and pre-computed solution database. Adapting effective decoupled planning paradigm [2], [22], [23], [25], [35], our algorithm first compute initial shortest paths independently for each robot and then resolves local conflicts between paths. In computing initial paths, a diversification heuristic makes the paths use the workspace in a balanced manner to robot aggregation. In resolving local path conflicts, they can be resolved in a small 3×3 area. A second heuristic is introduced that constructs a solution database for 3×3 sub-problems. Together, these improve computational efficiency and solution optimality in terms of computing near-optimal solutions under practical settings.

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II. PRELIMINARIES

Consider n robots in a square grid $G(V, E)$. Following the traditional 4-way connectivity rule, for each vertex $(i, j) \in V$, its neighborhood is $N(i) = \{(i+1, j), (i-1, j), (i, j+1), (i, j-1)\} \cap V$. For a robot i with initial and goal vertices $x_i^I, x_i^G \in V$, a *path* is defined as a sequence of vertices $P_i = (p_i^0, \ldots, p_i^T)$ satisfying: *(i)* $p_i^0 = x_i^I$; *(ii)* $p_i^T = x_i^G$; *(iii)* $\forall 1 \leq t \leq T$, $p_i^{t-1} = p_i^t$ or $p_i^{t-1} \in N(p_i^t)$. Denoting the joint initial and goal configurations of the robots as $X^I = \{x_1^I, \ldots, x_n^I\} \subseteq V$ and $X^G = \{x_1^G, \ldots, x_n^G\} \subseteq V$, the *solution paths* of all the robots is then $\mathcal{P} = \{P_1, \ldots, P_n\}$. For $\mathcal{P}$ to be collision-free, $\forall 1 \leq t \leq T$, $P_i, P_j \in \mathcal{P}$ must satisfy: *(i)* $p_i^t \neq p_j^t$ (no conflicts on vertices); *(ii)* $(p_i^{t-1}, p_i^t) \neq (p_j^t, p_j^{t-1})$ (no "head-to-head" collisions on edges). An optimal solution minimizes the *makespan* T, which is the time for all the robots to reach X^G.

Problem 1. Time-optimal Multi-robot Path Planning (MPP). *Given $\langle G, X^I, X^G \rangle$, find a collision-free path set $\mathcal{P}$ that routes the robots from X^I to X^G and minimizes T.*

We assume that G is a *low-resolution graph* which assumes the width of every passage in G is at least 3. The restriction on low-resolution graphs effectively prevents environments with narrow passages and mimics typical warehouse environments [14]. As previously stated, our main goal in developing this work is to tackle structured warehouse-like environments. In particular, narrow passages are not addressed by this current preliminary study.

III. ALGORITHM OVERVIEW

The is described in Algorithm 1. It first creates initial independent paths (line 2), which is done by computing paths for individual robots ignoring other robots. Note that the path diversification heuristics is embedded here. Then, a simulated execution (line 3-8) is carried out and as local conflicts are detected, they are resolved within *local* sub-graphs (line 7).

Algorithm 1: Our Method for One-shot MPP

```
1 X^C ← X^I
2 for i ∈ R do P_i ← GetPaths(G, x_i^I, x_i^G)
3 while X^C ≠ X^G do
4     X^N ← GetNextStep(P_1, ..., P_n)
5     if HasCollision(X^C, X^N) then
6         for each colliding pair of robots i, j do
7             P_1, ..., P_n ← CheckDatabase(G, X^C, i, j)
8     X^C ← ExecutePaths(X^C, P_1, ..., P_n)
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A. Path Diversification

Briefly, path diversification is achieved using a heuristic during the path planning phase. As the priority queue of A* or uniform-cost search is maintained, we perform some fine sort of the items with the same value so that vertices or edges that are have so far been used more frequently will be put toward the end of the queue. This has the effect to cause the path footprint use the workspace more evenly. In particular, this makes paths less likely aggregate at corners of obstacles in our preliminary evaluation.

B. Solution Database

To realize the solution database functionality, we exhaustively compute solutions to all possible 3×3 sub-problems. A technical challenge here is how to store all the resulting entries in memory for fast look-up. This is achieved by exploring the (mirror and rotation) symmetries that exist in the 3×3 problems. In the end, we were able to successfully store solutions to all 3×3 problems in the memory of a commodity PC.

IV. Preliminary Experimental Result

In this section, we compare our algorithm with integer linear programming (ILP) and ILP with split heuristic [17], which appears to be one of the fastest (near-)optimal solvers available for our target problem. ILP is an exact algorithm, while the split heuristic reduces ILP's computation time but makes it sub-optimal. The results indicate that our method has superior scalability as well as competitive optimality.

Fig. 1 shows the tested algorithms' performance on a 24×18 grid without obstacles. We observe that our method is at least 25 times quicker than the other approaches, while generates better solutions than ILP with split heuristic when the graph is not too crowded; the optimality of our method remains competitive when n gets larger. We also observe a noticeable benefit of using the path diversification heuristics.

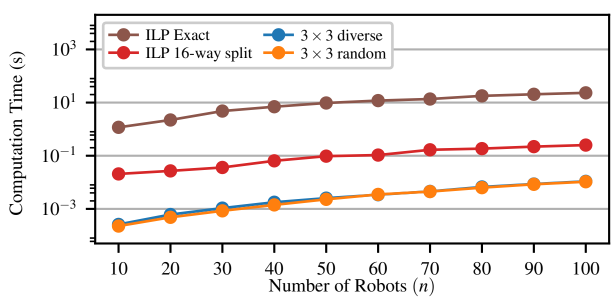

Fig. 1. Evaluation results of one-shot MPP on a 24×18 obstacle-free grid. We use notation *diverse* and *random* to indicate whether the path diversification heuristic is used.

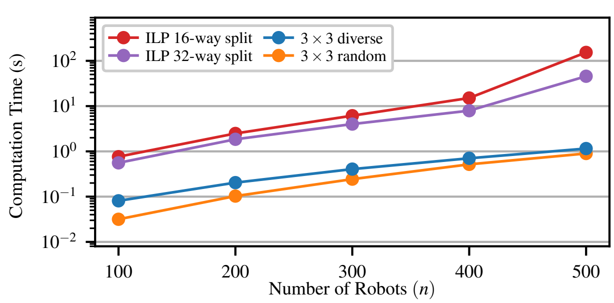

Fig. 2. Evaluation results of one-shot MPP in the warehouse-style workspace. The figure style follows the style in Fig. 1.

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