Distributionally Robust Reliability Assessment for Transmission System Hardening Plan Under $N-k$ Security Criterion

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***Abstract*—Increasing the complexity of power transmission networks has led power systems to be more vulnerable to cascading failures. Thus, hardening and reliability assessment of such complex networks have become a must. In addition, the commonly used $N-1$ security criterion does not guarantee the reliability of the system against possible cascading failures. In this paper, given a hardening plan, we develop two models to evaluate the reliability of the power transmission system under $N-k$ security criterion. In the first model, we quantify the probability of no load-shedding in the system to assess the possibility of load curtailment. Then, to have a better insight of the amount of load-shed in the second model, we use the conditional value-at-risk as a risk measure to evaluate the reliability of the system. To perform reliability assessment, the information of contingency probabilities is required. However, such probability information is usually unknown and cannot be estimated precisely. Therefore, in this paper, we assume the probability of contingencies as unknown and ambiguous. Then, we construct an ambiguity set for the unknown probability distribution of contingencies. Our approaches are robust because they analyze the reliability of the transmission system with respect to the worst-case distribution in the ambiguity set. We formulate both models as bilevel programs and solve them using the Bender's decomposition technique. Finally, we conduct numerical experiments on 6-bus and IEEE 118-bus test systems to show the effectiveness of the proposed approaches.**

***Index Terms*—Conditional value-at-risk (CVaR), distributionally robust optimization (DRO), N − k security criterion, transmission system reliability analysis.**

Nomenclature

A. Sets

$\mathcal{B}$	Index set of all buses.
$\mathcal{B}_i$	Index set of all buses directly connected to bus i.
$\mathcal{B}_i(.,i)$	Index set of all incoming transmission lines to bus i.
$\mathcal{B}_i(i,j)$	Index set of all outgoing transmission lines from bus i.
$\mathcal{E}$	Index set of transmission lines.
$\mathcal{E}_H$	Index set of hardened transmission lines.
$\mathcal{T}$	Index set of load blocks.

B. Parameters

F_{ij}	Flow capacity of transmission line (i,j).
C_i	Generation capacity at bus i.
X_{ij}	Reactance of transmission line (i,j).
$\theta_i^{\min}$	Phase angle lower limit at bus i.
$\theta_i^{\max}$	Phase angle upper limit at bus i.
d_{it}	Demand at bus i for load block t.

C. Decision Variables

x_{it}	Power generation at bus i for load block t.
$f_{ij,t}$	Power flow from bus i to bus j on transmission line (i,j) for load block t.
θ_{it}	Phase angle at bus i for load block t.
s_{it}	Load shedding at bus i for load block t.

D. Random Variables

v_{ij}	Binary variable indicating whether transmission line (i,j) is under contingency ($v_{ij}=0$) or not ($v_{ij}=1$).

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I. Introduction

The rapid growth of electric power systems and the creation of very complex interconnected power transmission networks have made power systems more vulnerable to cascading failures and large blackouts than in the past [1]. Large blackouts are among the most catastrophic disasters that threaten the U.S. economy through massive economic damage of tens of billions of dollars per year [2]. Transmission system outages (mostly caused by severe weather conditions [3], [4], aging [5], [6], and terrorist attacks [7], [8]) are among the major causes of large blackouts [9]. As a matter of fact, in the deregulated electricity market, the transmission system is utilized such that it operates near its limits, i.e., as economically as possible [10], in which case an initial line outage may affect other system components and result in cascading failures and large blackouts (for example, blackouts in February 2008 in Florida [11] and September 2011 in North America [12]). Therefore, because of the criticality of the electric power industry to the national economy and society in general, hardening planning and reliability evaluation of power transmission systems is of significant importance.

According to the North American Electric Reliability Council (NERC), reliability is the degree of power system performance under which customers' electricity demand is supplied and delivered under accepted standards [13]. This definition of reliability contains two concepts: 1) adequacy: the ability of a power system to supply the customers' electricity demand via available generation units and transmission systems and reserves; and 2) security: the ability of a power system to keep working after some contingencies such as transmission line outages or equipment failures. In the literature of power system, the most popular reliability indices are loss of load probability, loss of load expectation, and expected energy not supplied [14]. Moreover, two frequently used techniques for power system's reliability analysis are Monte Carlo simulations and contingency analysis.

In the techniques used in Monte Carlo simulations, the assessment of system reliability is carried out by sampling system component states. Component state samples are generated randomly either from an estimated distribution of failures (using historical data of component failures), i.e., random sampling (e.g., see [15]), or by considering component state transition probabilities, i.e., sequential sampling (e.g., see [16]). Moreover, to improve the efficiency of Monte Carlo simulation algorithms, several variations of this technique have recently been proposed, such as variance reduction techniques [17], [18], least square support vector classifier [19], artificial neural network [20], fuzzy Monte Carlo technique [21], cross-entropy methods [22], Latin hypercube sampling [23].

In contingency analysis, a set of contingency states is used to examine the power system reliability. The contingency set is created by taking $N-1,\dots,N-k, k=1,2,3,\dots$ security criteria into account, where k denotes the number of simultaneous component (generation units, transmission lines, etc.) outages. In industry practice, $N-1$ is the most widely used security criterion by most power systems around the world [24], [25]. However, it just guarantees the normal operations of the system under only a single component failure. Although the probability of two or more simultaneous component failures is very small, yet it may lead to very severe cascading failures and blackouts if simultaneous failures happen. Therefore, to establish more reliable system operations against multiple simultaneous contingencies, revised NERC reliability standards [26] require system operators to apply $N-k, k \geq 2$ security criterion in their analyses. However, for $k \geq 2$, the combinatorial nature of contingency states makes the full contingency enumeration almost impossible even for medium size systems and moderate values of k. For more discussions on the complexity of full contingency enumeration, readers are referred to [27].

To mitigate the computational burden of $N-k$ contingency analysis, contingency selection procedures have been proposed. Briefly, contingency selection is the process of identifying the critical components and constructing a contingency list that includes very serious single and multiple contingencies. Some previously conducted studies (see, e.g., [1], [28]–[30]) estimate the probability of system component failures using the historical data of component failures. Then, contingencies with higher probabilities form the contingency list. A statistical method to estimate the probability of transmission line failures and to identify vulnerable lines in a transmission system is presented in [1]. A method based on sub-station configuration and probability analysis of protection system failures to form a $N-k$ contingency list is proposed in [28]. Three probabilistic models to estimate the probabilities of multiple transmission line contingencies are used in [29]. Also, a data mining based method for contingency analysis is developed in [30]. However, some other works (see, e.g., [10], [31], [32]) consider the consequence (e.g., load shedding) of contingency scenarios to form the contingency list. A fast and reliable heuristic algorithm based on iterative pruning to identify critical $N-2$ contingencies is presented in [31]. Two contingency screening algorithms to determine the threatening $N-2$ contingencies without solving the full contingency set are developed in [10]. A method that uses a small number of representative constraints instead of enumerating exponentially many constraints for $N-k$ contingency analysis is developed in [32]. In addition, many optimization-based approaches have also been utilized for contingency selection procedures and protective resources allocation, i.e., hardening planning. For example, [33]–[36], among others, develop bilevel programs to identify the most critical transmission system components under $N-k$ contingency criterion. Also, [7], [8], and [37], among others, develop tri-level optimization models to simultaneously identify the most critical system components and the optimal hardening strategy to mitigate (enhance) the transmission system vulnerability (reliability) against intentional attacks. Moreover, other approaches such as a random chemistry algorithm [38] and a combined neural network and evolutionary algorithm [39] have recently been developed to study $N-k$ contingency analysis.

All the above-mentioned reliability analysis approaches either estimate the probabilities of component failures or assume the same probability for each component and deal with the consequence of failures. However, historical data for equipment failures are usually rare; also, the estimated equipment failure rates, which rely on expert information, are subject to errors. Even with sufficient historical data, the accurate estimation of failure rate is extremely difficult. On the contrary, in this paper, we allow the probability of contingencies to be unknown and ambiguous. We apply the distributionally robust optimization (DRO) concept (see, e.g., [40]–[42]) to perform the reliability analysis of power transmission systems. DRO-based approaches have recently received attention from power system specialists for various problems such as unit commitment [43], [44], transmission expansion planning and hardening [45], [46] and reserve scheduling [47], [48]. In this approach, instead of fixing the probabilities of component failures, we allow them to vary within a set of probability distributions, which is the ambiguity set. We develop two DRO models to determine the reliability of the system, for a given hardening plan, based on two reliability indices. In the first model, we develop a model to quantify the worst-case probability under which both system security and system adequacy are satisfied [we call it worst-case no load-shed probability (WNLP)]. In the second model, we use the conditional value-at-risk (CVaR) as its risk measure. CVaR, which is a well-known risk measure, has gained considerable attention in finance and insurance industries [49]. We propose a model to evaluate the worst-case CVaR associated with the worst-case contingency distribution in the ambiguity set. The contributions of this paper can be listed as follows.

1) We propose two reliability analysis models for transmission system hardening plans that hedge against the uncertainty (inaccuracy) associated with the estimation of probabilities of component failures. Instead of relying on estimates, our models consider the worst-case reliability with the worst-case failure distribution in the ambiguity set. Moreover, the conservatism of the proposed models can be adjusted by system operators.
2) Both of our models are formulated in such a way that decomposition techniques can be easily employed to solve them using commercial solvers. Here, we apply the Bender's decomposition technique to solve both the models.
3) We conduct expensive numerical experiments on a modified 6-bus and IEEE 118-bus test systems. We compare both the models for various contingency levels and different hardening plans to test the performance of our models.

The rest of this paper is organized as follows. In Section II, we first discuss how to construct the ambiguity set for the unknown contingency distribution. Then, we develop a DRO-based model to quantify the WNLP of the system. Afterward, we formulate a DRO-based model to evaluate the reliability of the transmission system on the basis of the worst-case CVaR risk measure. In Section III, we describe the proposed decomposition framework and the solution algorithms. In Section IV, we present and discuss numerical results. Finally, we conclude this paper in Section V.

II. Problem Formulation

In this paper, we aim to develop risk assessment models for the transmission system hardening decisions under distributional uncertainty of $N-k$ contingencies. Basically, we consider the following constraints to analyze the reliability of a power transmission system:

$$x_{\text{it}} + \sum_{j \in \mathcal{B}_i(.,i)} f_{ji,t} - \sum_{j \in \mathcal{B}_i(i,.)} f_{ij,t} + s_{\text{it}} = d_{\text{it}} \; \forall i \quad \forall t \tag{1}$$

$$(\theta_{\text{it}} - \theta_{jt}) - X_{ij} f_{ij,t} \geq 0 \; \forall t \in \mathcal{T} \quad \forall (i,j) \in \mathcal{E}_H \tag{2}$$

$$(\theta_{\text{it}} - \theta_{jt}) - X_{ij} f_{ij,t} \leq 0 \; \forall t \in \mathcal{T} \quad \forall (i,j) \in \mathcal{E}_H \tag{3}$$

$$(\theta_{\text{it}} - \theta_{jt}) - X_{ij} f_{ij,t} + M(1 - v_{ij}) \geq 0 \quad \forall t \in \mathcal{T} \quad \forall (i,j) \in \mathcal{E} \backslash \mathcal{E}_H \tag{4}$$

$$(\theta_{\text{it}} - \theta_{jt}) - X_{ij} f_{ij,t} - M(1 - v_{ij}) \leq 0 \quad \forall t \in \mathcal{T} \quad \forall (i,j) \in \mathcal{E} \backslash \mathcal{E}_H \tag{5}$$

$$-F_{ij} \leq f_{ij,t} \leq F_{ij} \; \forall t \in \mathcal{T} \quad \forall (i,j) \in \mathcal{E}_H \tag{6}$$

$$-F_{ij} v_{ij} \leq f_{ij,t} \leq F_{ij} v_{ij} \quad \forall t \in \mathcal{T} \quad \forall (i,j) \in \mathcal{E} \backslash \mathcal{E}_H \tag{7}$$

$$x_{\text{it}} \leq C_i \; \forall t \in \mathcal{T} \quad \forall i \in \mathcal{B} \tag{8}$$

$$\theta_i^{\min} \leq \theta_{it} \leq \theta_i^{\max} \; \forall t \in \mathcal{T} \quad \forall i \in \mathcal{B} \tag{9}$$

$$x_{\text{it}}, s_{\text{it}} \geq 0 \; \forall t \in \mathcal{T} \quad \forall i \in \mathcal{B} \tag{10}$$

where constraints (1) observe the power supply adequacy at each bus. Constraints (2) to (3) represent the relationship between dc power flow and phase angle for hardened lines. Constraints (4) to (5) represent the relationship between dc power flow and phase angle for unhardened lines. That is, if the unhardened lines are under contingency, i.e., $v_{ij} = 0$, the relationship of power flow and phase angle as suggested in (2) to (3) does not necessarily hold. Similarly, constraints (6) and (7) observe power flow capacities for hardened and unhardened lines, respectively. Constraints (8) and (9) impose power generation capacities and phase angle limits, respectively.

A. Ambiguity Set

Because of the fact that there are exponentially many $N-k$ contingency scenarios, and enumerating all of them is almost impossible (it takes exponential time), in this paper, we create a list of $\mathcal{N}$ contingencies. To this end, two groups of contingencies can be considered: 1) the most probable contingencies; and 2) the highest impact contingencies (contingencies with the worst consequences). Then, on the basis of the system operator's preferences, either of these or a mixture of them can be used to create the contingency list. The most probable contingencies and their probabilities can be identified by utilizing the existing methods in the literature (see, e.g., [1], [28]–[30] among others). The high impact contingency scenarios can also be identified by optimization-based methods (see, e.g., [7], [33]–[36], among others). These contingency scenarios and their assigned probabilities form a discrete distribution, $\hat{P}$, which we call the reference distribution. Considering the fact that the true probability of $N-k$ contingencies is unknown, using the reference distribution, $\hat{P}$, we construct an ambiguity set for the ambiguous probability distribution of $N-k$ contingencies. We define the following ambiguity set:

$$\mathcal{D} := \Big\{ P \in \mathcal{M}_+ : d(P, \hat{P}) \leq \varphi \tag{11}$$

$$\underline{P} \leq P \leq \overline{P} \Big\} \tag{12}$$

where $\mathcal{M}_+$ represents the set of all probability distributions. In (11), $d(P, \hat{P})$ denotes the probability distance between any arbitrary distribution, $P \in \mathcal{D}$, and the reference distribution, $\hat{P}$. Also, φ denotes the tolerance level for the probability distance. To measure $d(P, \hat{P})$, several probability metrics such as L_1, L_∞, Wasserstein metric can be utilized. For more detail, interested readers are referred to [50]. In this paper, we use L_1 norm. According to [43, Proposition 1], given a set of historical data, the following equation defines the relationship between the size of data and the value of φ under L_1 norm:

$$\varphi = \frac{\mathcal{N}}{2S} \log \frac{2(\mathcal{N})}{1-\gamma} \tag{13}$$

where $S, \mathcal{N}$, and γ denote the size of historical data, the number of contingency scenarios, and the confidence level, respectively. However, if in practice, the data information for contingencies is very limited and enough data are not available to learn a practical value of φ, then φ can also be decided by the system operators on the basis of their judgment or preference on the conservativeness level. Since the worst-case probability distribution is considered

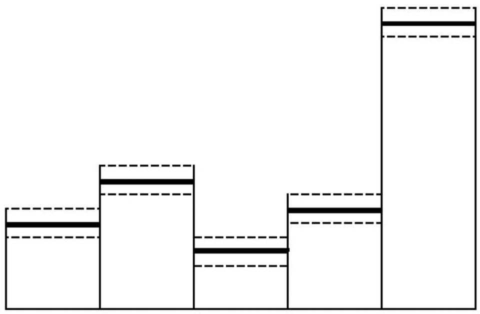

Fig. 1. Example of the ambiguity set.

in $\mathcal{D}$ in the proposed models, to control the conservativeness, we limit the probability of each contingency occurring with inequalities (12), where $\underline{P}$ and $\overline{P}$ are the lower and the upper probability limit matrices. Then, the ambiguity set, $\mathcal{D}$, under L_1 norm can be represented as follows:

$$\mathcal{D} := \left\{ P \in \mathbb{R}_+^{\mathcal{N}} : \sum_{n=1}^{\mathcal{N}} |p_n - \hat{p}_n| \leq \varphi \right. \tag{14}$$

$$\left. \underline{p}_n \leq p_n \leq \overline{p}_n \forall n \right\} \tag{15}$$

where p_n, $\underline{p}_n$, and $\overline{p}_n$ denote the unknown probability, the lower probability limit, and the upper probability limit for contingency scenario n, respectively. By setting $\underline{p}_n = \hat{p}_n - \delta$ and $\overline{p}_n = \hat{p}_n + \delta$, we can adjust the conservativeness of our models by changing parameter δ. Note here this ambiguity set also implicitly contains the correlation information of the contingencies, since it is constructed on the basis of the historical data, which capture the correlation of contingencies automatically. Fig. 1 illustrates one sample of the ambiguity set, $\mathcal{D}$, with five contingency scenarios. The black solid lines show the reference probabilities of contingency scenarios, and the dash lines show the ambiguity of the true probabilities from the reference ones.

B. Worst-Case No Load-Shed Probability

In this section, given a hardening plan, we intend to quantify the worst-case probability of a no load-shedding occurrence over the entire system. For notational brevity, we let the following:

$$g_{i,t}(x,f) = x_{\text{it}} + \sum_{j \in \mathcal{B}_i(.,i)} f_{ji,t} - \sum_{j \in \mathcal{B}_i(i,.)} f_{ij,t} - d_{\text{it}} \quad \forall i \quad \forall t. \tag{16}$$

Then, to satisfy the adequacy requirement, we need to determine decision variables x_{it}, $f_{ij,t}$, and θ_{it} such that the probability of no load-shed over the entire network is maximized. So, we consider the following program:

$$\max_{x,f,\theta \in \mathbf{X}} \; Pr\left(g_{i,t}(x,f) \geq 0 \quad \forall i \quad \forall t\right) \tag{17}$$

where $\mathbf{X}$ denotes the set of all feasible solutions satisfying security requirements, i.e., system constraints (2)–(10). In addition, notice that we have a joint probabilistic objective function, in (17), to quantify the no load-shed probability for the whole system. We can reformulate the objective function in (17) as follows:

$$\max_{x,f,\theta \in \mathbf{X}} \; Pr\left(\min_{i,t} g_{i,t}(x,f) \geq 0\right). \tag{18}$$

The probabilistic objective function above can be expressed as follows:

$$\max_{x,f,\theta \in \mathbf{X}} \; E_P\left[\mathbf{1}_{[0,\infty)}\left(\min_{i,t} g_{i,t}(x,f)\right)\right] \tag{19}$$

where indicator function $\mathbf{1}_{[0,\infty)}(x)$ equals 1 if $x \in [0,\infty)$ and zero otherwise. Note that, in this paper, instead of making an assumption (by estimation or expert information) for contingency scenario distribution, P, and taking expectation over P, we let P be unknown and ambiguous, but belong to the ambiguity set, $\mathcal{D}$. Then, the strategy here is to allow P to act adversely against the maximization of the expected value in (19), i.e., bringing robustness into the model. Hereby, we compute the WNLP by the following distributionally robust bilevel min–max problem:

$$\min_{P \in \mathcal{D}} \max_{x,f,\theta \in \mathbf{X}} \; E_P\left[\mathbf{1}_{[0,\infty)}\left(\min_{i,t} g_{i,t}(x,f)\right)\right]. \tag{20}$$

By defining new variable, $y_n = \mathbf{1}_{[0,\infty)}\left(\min_{i,t} g_{i,t}^n(x,f)\right)$, for each contingency scenario, n, and using the big-M method, we can obtain the following equivalent reformulation of (20):

$$\min_{P \in \mathcal{D}} \max_{x,f,\theta \in \mathbf{X}} \sum_{n=1}^{\mathcal{N}} p_n y_n \tag{21}$$

$$g_{i,t}^n(x,f) \geq -M(1-y_n) \quad \forall i \quad \forall t \quad \forall n \tag{22}$$

$$y_n \in \{0,1\} \quad \forall n. \tag{23}$$

C. Worst-Case CVatR (WCVaR)

In this section, given a hardening plan, we propose a new reliability analysis scheme for power transmission systems by considering the CVaR risk measure, which is also known as mean excess loss, mean shortfall, or tail VaR [49]. CVaR is closely related to the popular measure of risk VaR (an upper percentile of the loss distribution) and is defined as the weighted average of VaR and losses strictly exceeding VaR. CVaR, in comparison with VaR, has nice mathematical properties such as translational invariance, sub-additivity, convexity, and homogeneity, all of which make this risk measure coherent and practical to use in optimization problems. Moreover, from the above-mentioned definition, VaR can never be more than CVaR. For more details, readers are referred to [49]. We let the following:

$$L_{i,t}(x,f) = (-g_{i,t}(x,f))^+ \quad \forall i \quad \forall t \tag{24}$$

be the loss (load shedding) associated with decision variables x_{it}, $f_{ij,t}$, and θ_{it}, where $(x)^+ = \max\{0,x\}$. Then, since we aim to evaluate the reliability of the whole transmission system, we define a joint loss function as $L(x,f) = \sum_i \sum_t L_{i,t}(x,f)$. Therefore, β–CVaR for the joint loss function $L(x,f)$ and for the probability level $\beta \in (0,1)$ can be presented as [49] follows:

$$\beta\text{–CVaR} = \min_{\alpha} \alpha + \frac{1}{1-\beta} E_P\left[(L(x,f)-\alpha)^+\right] \tag{25}$$

where P denotes the probability distribution of random contingencies. To ensure adequacy, we need to determine decision variables x_{it}, $f_{ij,t}$, and θ_{it} such that β–CVaR is minimized. Accordingly, we develop the following program:

$$\min_{x,f,\theta\in\mathbf{X},\alpha} \beta\text{–CVaR} \tag{26}$$

where $\mathbf{X}$ denotes the same set as that in Section II-B. Following the procedure in Section II-B, we propose a DRO-based approach to minimize the worst-case β–CVaR (β-WCVaR), based on the worst-case probability distribution of random contingencies in the ambiguity set, $\mathcal{D}$. Accordingly, we have the following bilevel max–min program:

$$\max_{P\in\mathcal{D}} \min_{x,f,\theta\in\mathbf{X},\alpha} \alpha + \frac{1}{1-\beta} E_P\left[(L(x,f)-\alpha)^+\right]. \tag{27}$$

Here, notice that there exist two $(.)^+$ terms in the objective function of β-WCVaR model (27), i.e., $(L(x,f)-\alpha)^+$ and $L(x,f)$, according to (24). To linearize $L_{i,t}(x,f)$, we use auxiliary variables, z_n, for each contingency scenario, n, and add the following constraints to β-WCVaR model (27):

$$z_{\text{it}}^n \geq -g_{i,t}^n(x,f) \quad \forall i \quad \forall t \quad \forall n \tag{28}$$

$$z_{\text{it}}^n \geq 0 \quad \forall i \quad \forall t \quad \forall n. \tag{29}$$

Also, to linearize $(L(x,f)-\alpha)^+$ in (27), we use auxiliary variables, u_n, for each contingency scenario, n, and add the following constraints to β-WCVaR problem (27):

$$u_n \geq \sum_i \sum_t z_{\text{it}}^n - \alpha \quad \forall n \tag{30}$$

$$u_n \geq 0 \quad \forall i \quad \forall t \quad \forall n. \tag{31}$$

We eventually attain the following reformulation of β-WCVaR problem (27):

$$\max_{P\in\mathcal{D}} \min_{x,f,\theta\in\mathbf{X},\alpha} \alpha + \frac{1}{1-\beta}\sum_{n=1}^{\mathcal{N}} p_n u_n \tag{32}$$

$$\text{s.t.} \quad \text{Constraints (28)–(31).} \tag{33}$$

III. Solution Methodology

In this section, we describe our solution approaches for solving both the proposed DRO-based models: WNLP [i.e., (21)–(23)] and β-WCVaR [i.e., (32) and (33)]. We employ the Bender's decomposition algorithm to solve both the models. We explain the solution algorithms in detail in the following sections.

A. Solution Approach for WNLP Model

Based on WNLP problem formulation, (21)–(23), we utilize the following Bender's decomposition framework to solve this problem. We define the master problem and the sub-problem as follows:

1) Master Problem: We first represent the linear reformulation of (14) in the ambiguity set, $\mathcal{D}$, (14) and (15), as the following inequalities:

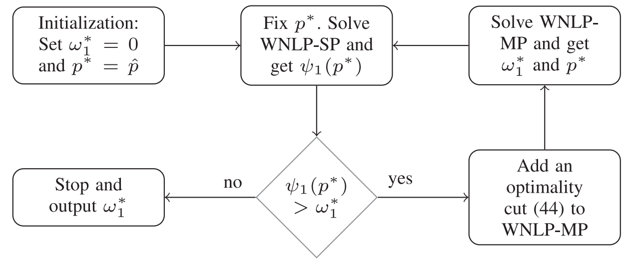

Fig. 2. Solution algorithm for WNLP model.

$$\sum_{n=1}^{\mathcal{N}} k_n \leq \varphi \tag{34}$$

$$k_n \geq p_n - \hat{p}_n \quad \forall n \tag{35}$$

$$k_n \geq \hat{p}_n - p_n \quad \forall n \tag{36}$$

where variables k_n represent $|p_n - \hat{p}_n|$. The master problem for WNLP model is represented as follows:

$$\text{(WNLP-MP)} \quad \min_p \omega_1 \tag{37}$$

$$\text{s.t.} \quad \sum_{n=1}^{\mathcal{N}} p_n = 1 \tag{38}$$

$$\text{Constraints (15) and (34)–(36)} \tag{39}$$

$$\text{Cutting planes} \tag{40}$$

where ω_1 denotes the objective value of the sub-problem that we will discuss in the next section. Also, constraint (38) ensures that variables p_n represent scenario probabilities.

2) Sub-Problem: In each iteration of the algorithm, given the solution, p^*, obtained in the master problem, we solve sub-problem $\psi_1(p^*)$ as follows:

$$\text{(WNLP-SP)} \quad \psi_1(p^*) = \max_{x,f,\theta} \sum_{n=1}^{\mathcal{N}} p_n^* y_n \tag{41}$$

$$\text{s.t.} \quad \text{Constraints (22)–(23)} \tag{42}$$

$$x, f, \theta \in \mathbf{X}. \tag{43}$$

3) Solution Algorithm: Since we allow load-shedding in the system, the sub-problem is always feasible. In addition, the master problem solution, p^*, only appears in the sub-problem objective function and does not affect the feasibility of the sub-problem. So, we do not need to check the feasibility of the solution of the master problem. Therefore, to generate optimality cuts in each iteration, for fixed WNLP-MP solutions p^* and ω_1^*, WNLP-SP is solved to obtain $\psi_1(p^*)$. Then, for the case where $\psi_1(p^*) > \omega_1^*$, the following optimality cut is added to WNLP-MP:

$$\omega_1 \geq \sum_{n=1}^{\mathcal{N}} p_n y_n^*. \tag{44}$$

The solution algorithm is summarized in the flowchart shown in Fig. 2.

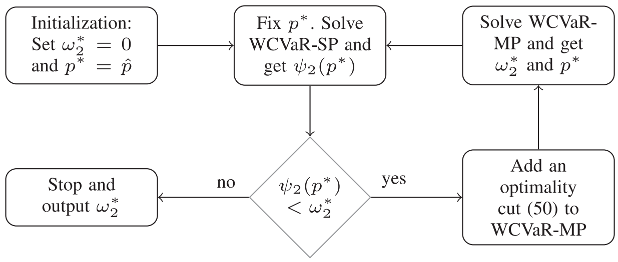

Fig. 3. Solution algorithm for WCVaR model.

B. Solution Approach for WCVaR Model

For WCVaR model, the problem formulation [i.e., (32) to (33)] has a similar structure to that of WNLP model. Hence, a similar solution procedure based on the Bender's decomposition is applied to solve the proposed model.

1) Master Problem: For WCVaR model, we can represent the master problem (WCVaR-MP) as follows:

$$\text{(WCVaR-MP)} \quad \max_{p} \ \omega_2 \tag{45}$$

$$\text{s.t.} \quad \text{Constraints (38)–(40)} \tag{46}$$

where ω_2 denotes the objective value of the sub-problem presented in the next section.

2) Sub-Problem: For the solution, p^*, obtained by solving WCVaR-MP, we have the following subproblem:

$$\text{(WCVaR-SP)} \quad \psi_2(p^*) = \min \ \alpha + \frac{1}{1-\beta} \sum_{n=1}^{\mathcal{N}} p_n^* u_n \tag{47}$$

$$\text{s.t.} \quad \text{Constraints (28)–(31)} \tag{48}$$

$$x, f, \theta \in \mathbf{X}. \tag{49}$$

3) Solution Algorithm: Similarly, the feasibility of the master problem solution, p^*, to WCVaR-SP is always guaranteed. Thus, only optimality cuts are needed. In each iteration, for the fixed WCVaR-MP solutions, p^* and ω_2^*, we obtain $\psi_2(p^*)$. Then, we check whether $\psi_2(p^*) < \omega_2^*$. If so, we add an optimality cut to WCVaR-MP as follows:

$$\omega_2 \le \alpha^* + \frac{1}{1-\beta} \sum_{n=1}^{\mathcal{N}} p_n u_n^*. \tag{50}$$

We can illustrate the solution algorithm for WCVaR model by the flowchart in Fig. 3.

IV. Numerical Results

In this section, to show the effectiveness and efficiency of the proposed approaches, we present numerical experiments on a modified 6-bus test system and IEEE 300-bus test system (available at http://www.maths.ed.ac.uk/optenergy/LocalOpt/300busnetwork_other.html). We use C++ and CPLEX 12.6 on a computer with Intel(R) Xeon(R) 3.2 GHz and 8 GB memory to implement all the experiments.

For computational simplicity, we set the probability of each component failure/contingency to be 0.01, and we consider the critical contingencies with high impact in the contingency list to increase the robustness of the proposed approaches. Therefore, our contingency list includes the worst-consequence contingency scenarios and the scenario of no line outage occurrence. We use the robust optimization approach proposed in [7] to identify the worst-consequence contingency scenarios.

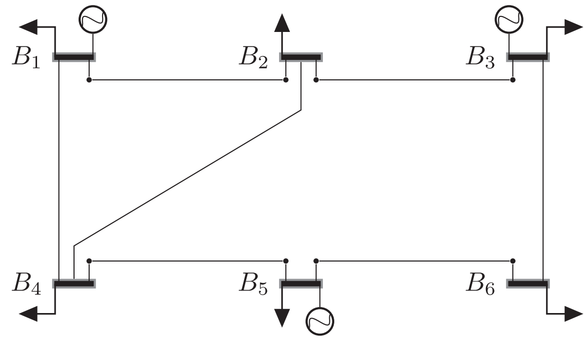

Fig. 4. Modified 6-bus system.

TABLE I
Bus Data for 6-Bus System

Bus No.	Generation capacity (MW)	Load (MW)
1	270	100
2	0	100
3	200	100
4	0	100
5	300	100
6	0	100

TABLE II
Line Data for 6-Bus System

Line no.	From bus	To bus	Reactance (pu)	Line flow (MW)
1	1	2	0.037	200
2	1	4	0.016	200
3	2	3	0.1015	175
4	2	4	0.117	175
5	3	6	0.0355	175
6	4	5	0.037	200
7	5	6	0.127	200

A. 6-Bus System

We test a modified 6-bus system consisting of three generation units, seven transmission lines, and six load nodes. The 6-bus system specifications are presented in Fig. 4 and Tables I and II.

We test this system for various contingency levels. We construct three contingency lists for $k = 1, 2, 3$. For $k = 1$, the contingency list includes all single transmission line contingency scenarios and a no-contingency scenario. For $k = 2$, we consider a contingency list of 11 scenarios (10 critical contingency states out of all contingencies with $k = 1, 2$ and a no-contingency state). Furthermore, for $k = 3$, we construct a contingency list of 16 scenarios (15 critical contingency states out of all contingencies with $k = 1, 2, 3$ and a no-contingency state). We report the WNLP and the worst-case CVaR (WCVaR) (for $\beta = 0.95$) in Tables III and IV, respectively. Here, we

TABLE III
WNLP FOR 6-BUS SYSTEM

Hardening plan	Security criterion		
	N-1	N-2	N-3
NH	1.000	0.9910	0.9905
(2)	1.000	0.9920	0.9915
(6)	1.000	0.9930	0.9926
(2,4)	1.000	0.9940	0.9936
(4,6)	1.000	1.0000	0.9948
(2,4,5)	1.000	1.0000	0.9949
(4,5,6)	1.000	1.0000	1.0000

TABLE IV
WCVAR FOR 6-BUS SYSTEM

Hardening plan	Security criterion		
	N-1	N-2	N-3
NH	0.00	447.12	542.64
(2)	0.00	378.48	470.64
(6)	0.00	268.08	327.84
(2,4)	0.00	314.14	327.36
(4,6)	0.00	0.00	67.68
(2,4,5)	0.00	0.00	73.44
(4,5,6)	0.00	0.00	0.00

also use several hardening plans with different budgets (i.e., the number of hardened lines) to show our model's capability for examining the performance of different hardening decisions. For this test system, we observe that the system reliability is guaranteed under $N-1$ security criterion both under WNLP and WCVaR reliability indices. However, this is not the case for $k = 2, 3$. Nevertheless, by putting hardening plans into action, we are able to bring higher levels of reliability into the system. In Tables III and IV, the first column represents hardening decisions, where NH denotes no transmission line has been hardened. For example, plan (4, 6) indicates that transmission lines 4 and 6 are hardened. However, different hardening plans lead to different levels of reliability. For example, plan (4, 6), compared with plan (2, 4) in $N-2$ case, or plan (4, 5, 6), compared with plan (2, 4, 5) in $N-3$ case, show higher WNLP but less WCVaR (better reliability indices).

B. 300-Bus System

In this section, we test our reliability analysis models by conducting experiments on a larger test system, i.e., IEEE 300-bus test system, which consists of 300 buses, 69 generators, and 411 transmission lines. We test our models based on $N-3$ transmission line contingencies. Accordingly, we create a contingency list of 46 scenarios, including 45 worst-consequence contingency scenarios among all contingencies with $k = 1, 2, 3$ and a no-line-outage scenario. In our experiments, we evaluate our model's performance by using different hardening plans with various hardening budgets.

1) Sensitivity Analysis of φ: We conduct experiments for different values of φ (the distribution distance tolerance level) to see how the value of φ affects our model's performance. Here, we set the value of δ (the scenario probability tolerance level) to be 0.005. Tables V and VI represent the values of WNLP and WCVaR (for $\beta = 0.95$), respectively. In Table V, we observe that as the number of hardened lines increases (more hardening budget), the worst-case probability of no load-shed increases. From Table VI, we see that with more hardened lines, the worst-case CVaR value decreases. In other words, with more hardening budget, the transmission system reliability increases. We also observe that as the value of φ decreases, the value of WNLP increases (WCVaR decreases). In fact, with smaller values of φ, the ambiguity set of probability distributions gets tighter and both WNLP and WCVaR models become less conservative. Therefore, the objective value of WNLP increases (WCVaR decreases). In addition, by comparing the results in Tables V and VI, another important observation is that although the values of WNLP for plans 208 and 316 are the same, there is a significant difference between the values of WCVaR for these plans. This observation admits that the worst-case CVaR can bring about a better insight of system reliability than that by WNLP. The computational times (denoted by *T*), for all the settings, are also presented in Tables V and VI.

TABLE V
EFFECTS OF φ ON WNLP

Hardening plan	$\varphi = 0.001$		$\varphi = 0.005$		$\varphi = 0.01$	
	WNLP	T(s)	WNLP	T(s)	WNLP	T(s)
NH	0.8330	446	0.8310	473	0.8285	457
(316)	0.8430	507	0.8410	498	0.8385	510
(208)	0.8430	514	0.8410	468	0.8385	494
(208, 316)	0.8540	592	0.8520	511	0.8495	503
(208, 316, 118)	0.8651	518	0.8631	521	0.8606	530
(208, 316, 118, 311)	0.8762	555	0.8742	547	0.8717	542
(208, 316, 118, 311, 342)	0.8874	605	0.8854	567	0.8829	586

TABLE VI
EFFECTS OF φ ON WCVAR

Hardening plan	$\varphi = 0.001$		$\varphi = 0.005$		$\varphi = 0.01$	
	WCVaR	T(s)	WCVaR	T(s)	WCVaR	T(s)
NH	13152	682	14542	913	16279	662
(316)	11169	644	12562	963	14305	674
(208)	6352	667	7557	997	9045	1081
(208, 316)	4294	655	5360	918	6676	1045
(208, 316, 118)	4035	685	5116	932	6452	1016
(208, 316, 118, 311)	3724	626	4837	848	6196	1028
(208, 316, 118, 311, 342)	3245	573	4378	815	5775	954

TABLE VII
EFFECTS OF δ ON WNLP

Hardening plan	$\delta = 0.0005$		$\delta = 0.001$		$\delta = 0.005$	
	WNLP	T(s)	WNLP	T(s)	WNLP	T(s)
NH	0.8330	451	0.8325	481	0.8285	457
(316)	0.8425	473	0.8415	505	0.8385	510
(208)	0.8425	480	0.8415	490	0.8385	494
(208, 316)	0.8525	488	0.8505	486	0.8495	503
(208, 316, 118)	0.8625	528	0.8606	522	0.8606	530
(208, 316, 118, 311)	0.8725	602	0.8717	529	0.8717	542
(208, 316, 118, 311, 342)	0.8829	575	0.8829	582	0.8829	586

2) Sensitivity Analysis of δ: Here, we assess the effects of the value of δ on the performance of our models. We use the same contingency list and the same hardening plans as Section IV-B1. We set the value of φ to be 0.01. We report WNLP and WCVaR values for different values of δ, in Tables VII and VIII, respectively. We see that as the value of δ increases, the value of WNLP

TABLE VIII
EFFECTS OF δ ON WCVAR

Hardening plan	$\delta = 0.0005$		$\delta = 0.001$		$\delta = 0.005$	
	WCVaR	T(s)	WCVaR	T(s)	WCVaR	T(s)
NH	15624	671	15799	947	16279	662
(316)	13035	645	13575	929	14305	674
(208)	7924	1065	8477	1022	9045	1081
(208, 316)	5505	1014	6061	902	6676	1045
(208, 316, 118)	5275	1035	5836	917	6452	1016
(208, 316, 118, 311)	5018	1018	5580	878	6196	1028
(208, 316, 118, 311, 342)	4482	956	5178	892	5775	954

TABLE IX
LOAD-SHEDDING (MW)

Hardening plan	Sim	WNLP	WCVaR
NH	2136.9	1373.27	1421.4
(208)	1444.4	1027.0	729.0
(208, 316)	972.8	555.4	493.2
(208, 316, 118)	896.9	479.6	417.4
(208, 316, 118, 311)	801.0	423.8	377.2
(208, 316, 118, 311, 342)	735.7	376.8	358.9

decreases (WCVaR increases). That is because as the value of δ increases, the ambiguity set becomes larger, and our models get more conservative. Notice again, infrequency of component failures and the scarcity of failure historical data lead to relatively large values of φ and over conservatism. In such cases, as shown in Tables VII and VIII, our model's conservatism can be adjusted by changing the value of δ.

3) Comparison With Simulation Method: In this section, we compare the proposed approaches with the traditional simulation method (denoted by Sim). We numerically show that the proposed WNLP and WCVaR approaches are more reliable than Sim. First, we generate 1000 sample contingency scenarios as the historical record of contingencies, and obtain the reference distribution, $\hat{P}$, and the ambiguity set, $\mathcal{D}$, (see Section II-A) based on the historical data. Then, we use Sim to get the optimal generation level, power flow, and phase angle $(x^*, f^*, \theta^*)_{\text{Sim}}$ with the reference distribution, $\hat{P}$; we then solve WNLP to get the optimal generation level, power flow, and phase angle, all denoted by $(x^*, f^*, \theta^*)_{\text{WNLP}}$, and worst-case distribution, P-WNLP; we then solve WCVaR to get the optimal generation level, power flow, and phase angle, all denoted by $(x^*, f^*, \theta^*)_{\text{WCVaR}}$, and worst-case distribution, P-WNLP. Notice, to get $(x^*, f^*, \theta^*)_{\text{Sim}}$, we use constraints (1)–(10), for each scenario n, with the objective of $\min_{x,f,\theta} \sum_{n=1}^{\mathcal{N}} \hat{p}_n s_n$. We then fix the decisions, (x^*, f^*, θ^*), generated by Sim, WNLP, and WCVaR accordingly and test them with 100 000 new generated contingency scenarios. We compare the load-shedding amount using the obtained (x^*, f^*, θ^*) for Sim, WNLP, and WCVaR, and we report the results in Table IX for variety of hardening plans. We observe that the operation decisions obtained using both WNLP and WCVaR lead to less amount of load-shedding as compared to the one by Sim. That is because the decisions obtained using WNLP and WCVaR are based on the worst-case probability distribution of contingencies, which are more robust and reliable than the traditional simulation approach.

TABLE X
AMOUNT OF ADJUSTMENT

Hardening plan	Sim	WNLP	WCVaR
NH	1322.8	559.2	607.4
(208)	1013.4	596.0	297.9
(208, 316)	672.0	264.6	192.4
(208, 316, 118)	635.2	217.8	155.6
(208, 316, 118, 311)	574.9	197.7	151.1
(208, 316, 118, 311, 342)	547.1	188.3	170.3

TABLE XI
95% CONFIDENCE INTERVAL OF NO LOAD-SHED PROBABILITY USING SIM

Hardening plan	Sim		WNLP	
	CI	T(s)	$\varphi = 0.005$	T(s)
NH	0.8307	>3600	0.8324	458
(208)	0.8415	>3600	0.8415	481
(208, 316)	0.8530	>3600	0.8525	487
(208, 316, 118)	0.8635	>3600	0.8645	504
(208, 316, 118, 311)	0.8745	>3600	0.8744	543
(208, 316, 118, 311, 342)	0.8854	>3600	0.8846	568

Then, we test the case in which the operation decisions, (x, f, θ), are adjustable in real-time operations. In this case, we compare the amount of necessary adjustments in (x, f, θ) to minimize the load-shedding for three approaches, respectively. We report the results in Table X for different hardening plans. We observe that the operations decisions of both WNLP and WCVaR require less adjustments than the ones of Sim, which means that the decisions obtained by WNLP and WCVaR are more robust than the ones obtained by Sim, and, therefore, lead to less real-time operational costs.

In addition, we show that the proposed method is computationally efficient than the transitional simulation approach. First, we set $\gamma = 0.95$ and get the worst-case no load-shed probabilities via WNLP model. Then, we generate 100 samples of 1000 contingency scenarios and obtain the no load-shed probability for each sample, and then obtain the 95% lower tailed confidence interval of no load-shed probability by Sim. We report the bounds of the 95% confidence interval and their corresponding CPU times in Table XI for different hardening plans. We see that the bounds of the confidence intervals (i.e., worst-case no load-shed probabilities) are very close to the ones of WNLP with $\gamma = 0.95$ (i.e., the case of ambiguity set with 95% confidence level). However, we observe that the CPU times of Sim are significantly higher than those of WNLP. These results show that, by using WNLP model, we are able to obtain a lower bound for no load-shed probability without conducting a huge number of experiments and simulations, and it is more computationally efficient.

V. CONCLUSION

In this paper, we proposed two reliability analysis schemes for power transmission system hardening under distributional uncertainty of random contingencies and under $N - k$ security criterion. We used the DRO-based concept to develop two optimization models to assess transmission network reliability for a given transmission hardening plan. First, we developed a model to quantify the worst-case probability of no load-shedding occurrence over the entire system. In the second model, we utilized the CVaR as a risk measure and computed the worst-case CVaR to evaluate the transmission system reliability. In both models, instead of assuming a fixed probability estimate (by historical data or expert information) for contingency scenarios, we let the ambiguous probability distribution of contingencies belong to an ambiguity set. Then, we considered the worst-case probability distribution in the ambiguity set to make a robust reliability analysis of the system. We formulated both models in bilevel programs and employed the Bender's decomposition algorithm to solve them. Using numerical experiments, we showed that our models are capable of distinguishing more effective hardening plans from others. Besides, we observed that as the size of historical data increases, the conservatism of our models decreases. In addition, we showed that, in case of scarcity of historical data, the conservativeness of the proposed models can be adjusted by the system operators.

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