Received: 27 June 2016 | Accepted: 1 January 2017
DOI: 10.1111/nrm.12124
WILEY Natural Resource Modeling

Optimal transport to cold chain in perishable hand-picked agriculture

Devasia Manuel[1] | **Richard B. Sowers**[2]

[1]Department of Electrical and Computer Engineering, University of Illinois at Urbana–Champaign, Urbana, IL, USA (Email: devasia1000@gmail.com)

[2]Department of Industrial and Enterprise Systems Engineering, Department of Mathematics, University of Illinois at Urbana–Champaign, Urbana, IL, USA

Correspondence
Richard Sowers, Department of Industrial and Enterprise Systems Engineering, Department of Mathematics, University of Illinois at Urbana–Champaign, Urbana, IL 61801.
Email: r-sowers@illinois.edu

Thanks to Nathan Dorn for making us aware of this problem.

Abstract
We develop and analyze a model for scheduling transport for perishable products. Once the harvest is picked, its quality starts to deteriorate, eventually having no value. In hand-picked crops, the rate of picking often varies during the day. One would like to transport the harvest to a cooling station (cold chain), but to do so according to an optimal policy. This optimal policy should reflect a trade-off between the loss of quality and the rate of harvest, and the fact that only a finite number of transports can be scheduled. We model the harvest and loss of quality and arrive at a computationally solvable optimization problem.

KEYWORDS
cold chain, hand-picked agriculture, optimization, supply chain

1 | INTRODUCTION

Optimization is an important part of the food system. Interacting demands on quality, quantity, and sustainability often can have decisive influence on a product's viability. With the advent of precision agriculture, these decisions can be made at more and more granular levels. We are interested in one specific problem; *optimal transportation scheduling for hand-picked crops*.

In practice, various hand-picked crops are picked, loaded onto a truck, and then transported to a cooling facility. Cooling slows the degradation of the product, but ambient temperatures may lead to noticeable loss of quality while the crop is being aggregated and loaded onto a truck for transport. This creates a number of challenges. Hand harvesting is, almost by definition, subject to much more variability than machine harvesting. There are variations from person to person, and also from moment to moment. Our interest is to start to understand how to think of some of these trade-offs.

There are a number of works which lay foundations in this area. Some survey articles are Amorim, Meyr, Almeder, and Almada-Lobo (2013) and Akkerman, Farahani, and Grunow (2010). Vehicle routing problems are well-understood. Vehicle-routing problems have been applied to food supply chains with a number of case studies providing focus to particular issues. A Portuguese food network was studied in Amorim, Parragh, Sperandio, and Almada-Lobo (2014), and Osvald and Stirn (2008) consider a model of a Slovenian system. Rong, Akkerman, and Grunow (2011) consider optimization at a large system level, and then focuses on a study using bell peppers. Some of these take an even more granular perspective and look at multitemperature systems; see Hsu and Chen (2014) (cf. Kuo and Chen, 2010). Both Garcia and Lozano (2005) and Hsu, Hung, and Li (2007) look at the effect of time windows. Similarly, Arbib, Pacciarelli, and Smriglio (1999) seek to optimize when "launch times" can be varies.

At a farm level, Ferguson and Koenigsberg (2007) consider perishability at the farm level, seeking to optimize revenue from deteriorating inventory which can be sold, but which then competes with fresher inventory. Similarly, Lin and Chen (2003) consider optimal order placement in the face of perishability.

Our interest seems to be unique, in that we are interested particularly in the time spent *before* the harvest enters the system. In hot ambient temperatures, some produce can suffer a loss of over 10% per hour while waiting to be cooled. We are interested in scheduling optimal loading times in the face of empirically observed *time-varying harvest rates* (see also Bandiera (2004) for some mathematical modeling of worker incentives).

Precision agriculture is opening up new challenges. The current literature has focused primarily on optimization once a harvest has entered the transportation and distribution system. With more and better sensing and extremely granular data, estimation of the instantaneous rate of harvesting is technically feasible. We believe that optimization at the actual level of individual harvesting can provide meaningful improvements.

Our model is continuous both in time and harvest quality. Harvesting occurs on an interval $[0, T]$. When the fruit is harvested, it decays at a fixed rate, at some point becoming worthless. Storage in cold chain retards the decay, so we want to carefully plan for transport to cold chain. We pose a problem involving quality decay and transport to cold chain, reframe it as a constrained optimization problem on a convex feasible region, and then develop a computationally feasible simulated annealing algorithm to find the optimum.

2 | THE MODEL

Suppose that produce is picked at time-varying rate $\{r(t);\ 0 \le t \le T\}$ during the day.[1] In other words, the amount of produce picked between times s and t (with $0 \le s \le t \le T$) is

$$\int_{u=s}^{t} r(u)du. \tag{1}$$

They immediately start to spoil once they are picked. Let us quantify this by assuming that they have "value" 1 the instant they are picked, and they have value $\mathsf{v}_k(\tau) \stackrel{\text{def}}{=} (1 - k\tau)^+$ after τ minutes (i.e., they lose $k\%$ of their value, but cannot have negative value[2,3]). This is similar to the perishability function of Zhang, Hu, and Wang (2011). The challenge is when to collect the berries and transport them to a cooling area. We have N trucks, each of which has capacity C, and we want to identify instants $\{t_n\}_{n=1}^{N}$ at which these trucks should collect the picked berries.

Define $t_0 \stackrel{\text{def}}{=} 0$; then the nth truck will collect fruit picked in $(t_{n-1}, t_n]$. Mathematically, we want to maximize

$$P(t_1, t_2 \dots t_N) \stackrel{\text{def}}{=} \sum_{n=1}^{N} \int_{u=t_{n-1}}^{t_n} \mathsf{v}_k(t_n - u) r(u) du \tag{2}$$

over all nondecreasing sequences $(t_n)_{n=0}^N$ with $t_0 = 0$ such that

$$\max_{1\le n\le N} \int_{u=t_{n-1}}^{t_n} r(u)du \le C. \tag{3}$$

The inequality (3) means (recall (1)) that the amount in each truck is less than the capacity C, and (2) means that we maximize the value of the day's harvest (when spoilage is accounted for).[4]

We numerically solve this problem, and show that various "naive" scheduling problems are noticeably suboptimal. Note that the optimal scheduling decisions *propagate*; if we modify the load of any given truck, it may affects how the remaining trucks should be loaded.

3 | RESCALING TIME

Let us recalibrate our problem in terms of amount harvested; this allows us to more directly impose the constraints of (3). For a sequence $(\ell_n)_{n=1}^N$ of elements in $\mathbb{R}_+$, define

$$\hat{P}(\ell_1, \ell_2 \dots \ell_N) \stackrel{\text{def}}{=} \sup\left\{ P(t_1, t_2 \dots t_N) : 0 = t_0 < t_1 \dots t_N \le T, \int_{u=t_{n-1}}^{t_n} r(u)du = \ell_n \right\}. \tag{4}$$

Instead of trying to maximize a function of times (i.e., P of (2)), we can now maximize function of harvested amounts. This will simplify how we search through configuration space (namely, we will end up with a *convex* configuration space in (10)). We then have that (2)–(3) is equivalent to

$$\inf\left\{ \hat{P}(\ell_1, \ell_2 \dots \ell_N) : \ell_n \le C \right\}. \tag{5}$$

Let us now define

$$R_0(t) \stackrel{\text{def}}{=} \int_{u=0}^{t} r(u)du \qquad R_1(t) \stackrel{\text{def}}{=} \int_{u=0}^{t} ur(u)du$$

for $t \in [0, T]$. The increments of R_0 directly connect to the capacity constraints of (3). Since v_k is a combination of a constant and a linear function, the value function (2) can at least locally be written as a combination of R_0 and R_1; doing so will help simplify things a bit further. Note that $1 - k(t-u) \ge 0$ if and only if $u > t - 1/k$. Thus[5]

$$\begin{aligned} P(t_1, t_2 \dots t_N) &= \sum_{n=1}^{N} \int_{u=t_{n-1}}^{t_n} \{1 - k(t_n - u)\} \chi_{\{u > t - 1/k\}} r(u)du \\ &= \sum_{n=1}^{N} \int_{u=t_{n-1} \vee (t_n - 1/k)}^{t_n} \{1 - k(t_n - u)\} r(u)du \\ &= \sum_{n=1}^{N} \left\{ (1 - kt_n) \left\{ R_0(t_n) - R_0\left(t_{n-1} \vee (t_n - 1/k)\right) \right\} \right. \\ &\quad \left. + k \left\{ R_1(t_n) - R_1\left(t_{n-1} \vee (t_n - 1/k)\right) \right\} \right\}. \end{aligned} \tag{6}$$

This almost allows us to write P (and $\hat{P}$) directly in terms of R_0 and R_1.

Let us now define *harvest time*. For $\ell > 0$, define

$$\tilde{R}(\ell) \stackrel{\text{def}}{=} \inf \left\{ t > 0 : \ R_0(t) \geq \ell \right\};$$

$\tilde{R}(\ell)$ is the first time that at least ℓ units have been harvested. In fact, since R_0 is defined as the running Lebesgue integral,

$$\tilde{R}(\ell) = \inf \left\{ t \geq 0 : \ R_0(t) = \ell \right\}.$$

We also have that $\tilde{R}$ is the left-continuous inverse of R_0. Note that if $r = 0$ on an interval (a, b) (e.g., during a work break), R_0 is flat on (a, b) and thus not invertible. The left-inverse is well-defined, and turns out to lead to the correct calculations (see below). We would like to use $\tilde{R}$ to rewrite $\hat{P}$.

Comparing (4) and (6), there are several possibilities. First of all, we say that R_0 is invertible at level ℓ if

$$R_0^{-1}(\ell) \stackrel{\text{def}}{=} \{t' \in [0, T] : \ R_0(t') = \ell\}$$

is a singleton. If R_0 is not invertible at level ℓ, there is an interval $[a, b]$ (with $a < b$) such that $R_0(s) = \ell$ for all $s \in [a, b]$. We note that then $a = \tilde{R}(\ell)$.

Let us take a closer look at (4). Fix $(\ell_n)_{n=1}^N$. Define

$$L_n \stackrel{\text{def}}{=} \sum_{1 \leq n' \leq n} \ell_{n'} \tag{7}$$

for $n \in \{1, 2 \ldots N\}$. Fix $n \in \{1, 2 \ldots N\}$ and note that

$$R_0(t_n) = L_n.$$

If R_0 is invertible at L_n, then

$$t_n = \tilde{R}(L_n).$$

Assume next that $n < N$ and R_0 is not invertible at L_n. Thus there is an interval $[a, b]$ (with $a < b$) such that

$$R_0(s) = L_n$$

for all $s \in [a, b]$. Let us also fix t_{n+1} and t_{n-1} such that

$$R_0(t_{n-1}) = L_{n-1} \qquad \text{and} \qquad R_0(t_{n+1}) = L_{n+1}.$$

Note that

$$\sup_{s \in [a,b]} \left\{ \int_{u=t_{n-1}}^{s} \{1 - k(s-u)\}^+ r(u) du + \int_{u=s}^{t_{n+1}} \left\{1 - k(t_{n+1} - u)\right\}^+ r(u) du \right\}$$
$$= \sup_{s \in [a,b]} \left\{ \int_{u=t_{n-1}}^{a} \{1 - k(s-u)\}^+ r(u) du + \int_{u=a}^{t_{n+1}} \left\{1 - k(t_{n+1} - u)\right\}^+ r(u) du \right\}$$

$$= \left\{ \int_{u=t_{n-1}}^{a} \{1 - k(a-u)\}^+ r(u)du + \int_{u=a}^{t_{n+1}} \left\{1 - k(t_{n+1} - u)\right\}^+ r(u)du \right\} \tag{8}$$

(the first equality holds since $r(u)du$ integrals are zero over $[a, b]$, and the second equality holds since $s \mapsto (1 - k(s-u))^+$ is nonincreasing in s). We can now use the fact that $a = \tilde{R}(L_n)$.

Assume next that $n = N$ and R_0 is not invertible at L_n. Thus there is an interval $[a, b]$ (with $a < b$) such that

$$R_0(s) = L_n$$

for all $s \in [a, b]$. Let us also fix t_{n-1} such that

$$R_0(t_{n-1}) = L_{n-1}.$$

Similar to (8), we have that

$$\begin{aligned} &\sup_{s\in[a,b]} \left\{ \int_{u=t_{n-1}}^{s} \{1 - k(s-u)\}^+ r(u)du \right\} \\ &= \sup_{s\in[a,b]} \left\{ \int_{u=t_{n-1}}^{a} \{1 - k(s-u)\}^+ r(u)du \right\} \\ &= \left\{ \int_{u=t_{n-1}}^{a} \{1 - k(a-u)\}^+ r(u)du \right\} \end{aligned}$$

and $a = \tilde{R}(L_n)$.

With these calculations in mind, we can replace t_n in (6) with $\tilde{R}(L_n)$, giving us that

$$\begin{aligned} \hat{P}(\ell_1, \ell_2 \dots \ell_N) = \textstyle\sum_{n=1}^{N} \Big\{ &(1 - k\tilde{R}(L_n)) \left\{ R_0(\tilde{R}(L_n)) - R_0\left(\tilde{R}(L_{n-1}) \vee (\tilde{R}(L_n) - 1/k)\right) \right\} \\ &+ k \left\{ R_1(\tilde{R}(L_n)) - R_1\left(\tilde{R}(L_{n-1}) \vee (\tilde{R}(L_n) - 1/k)\right) \right\} \Big\}. \end{aligned} \tag{9}$$

Finally, let us take a closer look at the allowable ℓ_n's. We must of course have that

$$\sum_{n=1}^{N} \ell_n \leq R_0(T)$$

(if not, $\tilde{P}(\ell_1, \ell_2 \dots \ell_N) = -\infty$). Thus the set of ℓ_n's for which $\hat{P}$ will be positive will be

$$\mathcal{C} = \left\{ (\ell_1, \ell_2 \dots \ell_N) \in \mathbb{R}_+^N : \sum_{n=1}^{N} \ell_n \leq R_0(T),\ \max_{1\leq n\leq N} \ell_n \leq C \right\}. \tag{10}$$

Clearly, $\mathcal{C}$ is a convex set. For $\mathcal{C}$ itself to be nonempty, we must in turn have that $R_0(T) \leq NC$. In this case, the vector $(\ell_n)_{n=1}^N$ given by

$$\ell_n \stackrel{\text{def}}{=} \frac{R_0(T)}{N}$$

is admissible.

If $(\ell_1^*, \ell_2^* \ldots \ell_N^*)$ is the optimizer of (5), then the optimizer of (2)–(3) is given as

$$t_n^* = \tilde{R}\left(\sum_{n'=1}^{N} \ell_n^*\right).$$

We now have rewritten our original problem as an optimization problem (of a nonconvex function, namely, $\hat{P}$) over a convex set (e.g., $\mathcal{C}$).

4 | OPTIMIZATION SCHEME

At this point, our task is to maximize $\hat{P}$ of (9) over the convex set $\mathcal{C}$ of (10) (where we use (7) to define the L_n's in (9)). Convex sets have appealing geometry, leading to numerical methods which are simpler than directly optimizing (2) subject to the constraints of (3). In particular, if we numerically test a point in (10), the structure of (10) naturally suggests ways to find a "nearby" candidate next point also in (10).

Definition 1 (Projection to $\mathcal{C}$). *Fix $(\tilde{\ell}_1, \tilde{\ell}_2 \ldots \tilde{\ell}_N) \in \mathbb{R}_+^N$. If $\sum_{n=1}^N \tilde{\ell}_n > R_0(T)$, define*

$$\tilde{\ell}_n^A \stackrel{\text{def}}{=} \tilde{\ell}_n \frac{R_0(T)}{\sum_{n'=1}^N \tilde{\ell}_{n'}}$$

(thus $\sum_{n=1}^N \tilde{\ell}_n^A = R_0(T)$). If $\sup_{1\le n\le N} \ell_n^A \le C$, define $\tilde{\ell}_n^B \stackrel{\text{def}}{=} \tilde{\ell}_n^A$ for $n \in \{1, 2 \ldots N\}$. Otherwise, define

$$\lambda \stackrel{\text{def}}{=} \max\left\{\frac{\tilde{\ell}_n^A - C}{\tilde{\ell}_n^A - R_0(T)/N} \;:\; \tilde{\ell}_n^A > C\right\} \tag{11}$$

and then define

$$\tilde{\ell}_n^B \stackrel{\text{def}}{=} (1-\lambda)\tilde{\ell}_n^A + \lambda \frac{R_0(T)}{N}.$$

We write $(\tilde{\ell}_1^B, \tilde{\ell}_2^B \ldots \tilde{\ell}_N^B) = \mathbf{P}_{\mathcal{C}}(\ell_1, \ell_2 \ldots \ell_N)$ ($(\tilde{\ell}_1^B, \tilde{\ell}_2^B \ldots \tilde{\ell}_N^B)$ is the projection *of $(\ell_1, \ell_2 \ldots \ell_N)$ onto $\mathcal{C}$).*

We have defined (11) as the smallest λ such that

$$\sup_{1\le n\le N}\left\{(1-\lambda)\tilde{\ell}_n^A + \lambda\frac{R_0(T)}{N}\right\} \le C.$$

Definition 2 (Random Admissible Point). *Let $(\ell_1, \ell_2 \ldots \ell_N)$ in $\mathbb{R}_+^N$ be such that ℓ_n is randomly chosen in $[0, R_0(T)]$. Set $(\tilde{\ell}_1, \tilde{\ell}_2 \ldots \tilde{\ell}_N) = \mathbf{P}_{\mathcal{C}}(\ell_1, \ell_2 \ldots \ell_N)$ (in the sense of Definition 1). We say that $(\tilde{\ell}_1, \tilde{\ell}_2 \ldots \tilde{\ell}_N)$ is a* random admissible point.

Definition 3 (Iteration). *Fix an admissible $(\ell_1, \ell_2 \ldots \ell_N) \in \mathbb{R}_+^N$ and $\delta > 0$. For each $n \in \{1, 2 \ldots N\}$, let $\tilde{\ell}_n^A$ be randomly chosen in $[0, R_0(T)] \cap (\ell_n - \delta, \ell_n + \delta)$. Set $(\tilde{\ell}_1^B, \tilde{\ell}_2^B \ldots \tilde{\ell}_N^B) = \mathbf{P}_{\mathcal{C}}(\tilde{\ell}_1^A, \tilde{\ell}_2^A \ldots \tilde{\ell}_N^A)$ (in the sense of Definition 1). We say that $(\tilde{\ell}_1^B, \tilde{\ell}_2^B \ldots \tilde{\ell}_N^B)$ is a δ-iterate of $(\ell_1, \ell_2 \ldots \ell_N)$.*

Let us construct a simulated annealing scheme. Starting with an initial (admissible) $\ell \stackrel{\text{def}}{=} (\ell_1, \ell_2 \dots \ell_N) \in \mathbb{R}_+^N$, we want to test a new $\ell' \stackrel{\text{def}}{=} (\ell'_1, \ell'_2 \dots \ell'_N) \in \mathbb{R}_+^N$ and update to that point with a probability which is higher if $\hat{P}(\ell') > \hat{P}(\ell)$. As our iterative scheme progresses, we want more and more certainty. By tuning various coefficients, we want to explore all of C and find the global maximum.

Define

$$T_n \stackrel{\text{def}}{=} 1/\ln(n+e)$$

and set

$$\delta_n \stackrel{\text{def}}{=} C/T_n.$$

Assume that we have an admissible point $(\ell_1^{(m)}, \ell_2^{(m)} \dots \ell_N^{(m)})$ (let $(\ell_1^{(1)}, \ell_2^{(1)} \dots \ell_N^{(1)})$ be a random admissible point). Let $(\hat{\ell}_1, \hat{\ell}_2 \dots \hat{\ell}_N)$ be a δ_m-iterate of $(\ell_1^{(m)}, \ell_2^{(m)} \dots \ell_N^{(m)})$. Define

$$p \stackrel{\text{def}}{=} \min\left\{ \exp\left[\frac{1}{T_m} \left\{ \hat{P}\left(\hat{\ell}_1^{(m)}, \hat{\ell}_2^{(m)} \dots \hat{\ell}_N^{(m)}\right) - \hat{P}\left(\ell_1^{(m)}, \ell_2^{(m)} \dots \ell_N^{(m)}\right) \right\} \right], 1 \right\}.$$

Flip a coin with bias p. If the coin comes up heads, define $\ell_n^{(m+1)} \stackrel{\text{def}}{=} \hat{\ell}_n$, and if the coin comes up tails, define $\ell_n^{(m)} \stackrel{\text{def}}{=} \ell_n^{(m)}$.

5 | EXAMPLES

5.1 | Example 1

Let us see what happens with a somewhat realistic example. Let us say that the day is divided into four intervals defined by time $\{\tau_n\}_{n=0}^4$. Let us furthermore say that

$$r(t) = b_j + m(t - \tau_j)$$

for $\tau_j \le t < \tau_{j+1}$ for $j \in \{0, 1, \dots 7\}$ and $r \equiv 0$ outside of $[\tau_0, \tau_5]$. The period $[\tau_0, \tau_1)$ corresponds to the beginning of the day, and workers are warming up. The workers work at peak efficiency, a constant rate, on $[\tau_1, \tau_2)$. In the later morning, on $[\tau_2, \tau_3)$, efficiency declines. Lunch break is $[\tau_3, \tau_4)$. The workers resume at constant rate on $[\tau_4, \tau_5)$, and slow again on $[\tau_5, \tau_6)$ as the day draws to a close. For specificity, let us assume that

$$\tau_0 = 0$$

$$\tau_1 = 1$$

$$\tau_2 = 3$$

$$\tau_3 = 4$$

$$\tau_4 = 4.5$$

$$\tau_5 = 6$$

$$\tau_6 = 7$$

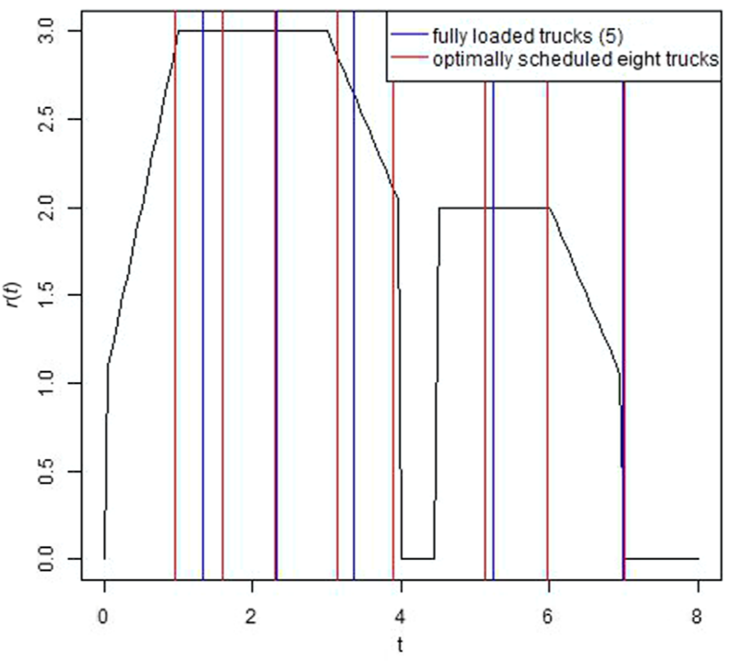

FIGURE 1 Rates for Example 1

and we assume that

$$r(t) = \begin{cases} 1 + 2t & \text{if} \quad 0 \le t < 1 \\ 3 & \text{if} \quad 1 < t \le 3 \\ 3 - (t - 3) & \text{if} \quad 3 \le t < 4 \\ 0 & \text{if} \quad 4 \le t < 4.5 \\ 2 & \text{if} \quad 4.5 \le t < 6 \\ 2 - (t - 6) & \text{if} \quad 6 \le t < 7. \end{cases} \tag{12}$$

The total harvest is 15 units. Let us assume that truck capacity is 3 units, so five trucks are needed. Let us in fact assume that eight trucks are available. See Figure 1. Let us also assume a coefficient of spoilage of $k = 0.2$.

A naive scheduling policy would be to pack all trucks to capacity (except perhaps the last one); i.e., $\ell_n = 3$ for $n \in \{1, 2 \ldots 5\}$. This would give us a value of 21.8, and the trucks would be scheduled for times 1.33, 2.33, 3.35, 5.24, and 6.98. However, if eight trucks are available, the optimal loads are 1.83, 1.96, 2.13, 2.50, 1.87, 1.51, 1.68, and 1.52, occurring at times 0.94, 1.59, 2.30, 3.14, 3.89, 5.14, 5.98, and 6.99. The value of this scheduling policy is 22.4, which is a 2.7% increase over the value of the equally spaced loads.

One might also space the trucks so that they all have equal loads of $15/8 = 1.875$. This would have value 22.29; the optimal policy is a 0.2% increase over equally-spaced loads. In this case, equally-spaced loads are almost optimal.

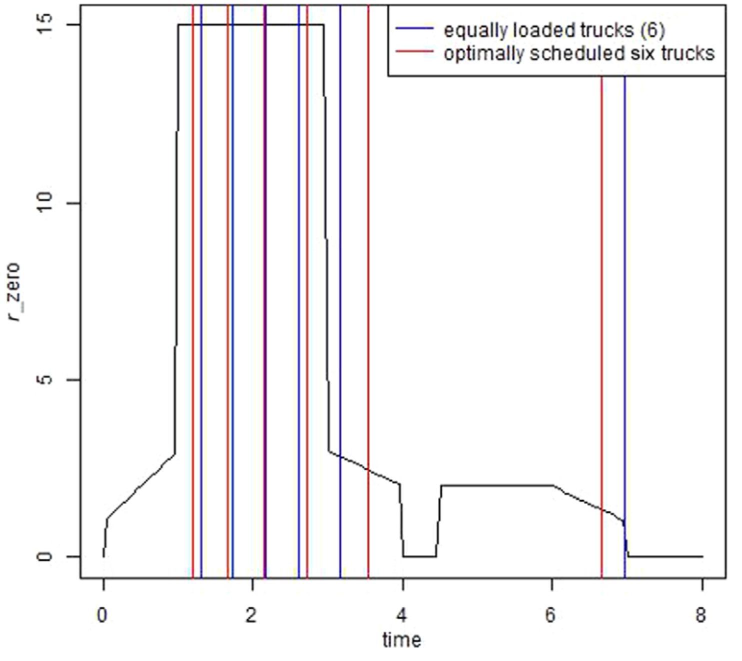

FIGURE 2 Rates for Example 2

5.2 | Example 2

Let us now change r of (12) to reflect a burst of activity in the interval $[1, 3]$. Namely, let us set $r(t) = 15$ if $1 \le t < 3$. Let us also set the coefficient of spoilage to be $k = 0.3$. If we wait much longer after time 3, the harvest just picked will spoil. Let us increase the truck capacity to 10.

Numerically, 39 units are harvested, so we need at least four trucks. Let us assume that we use six trucks. Equally spaced loads would give $39/6 = 6.5$ units to each truck, with a resulting harvest value of 53.6. On the other hand, optimal loading would give a value of 55.6, a 3.8% increase. See Figure 2.

Both of these examples admittedly show modest gains in value. More extreme fluctuations would probably reveal scenarios where the gains are more significant.

6 | CONCLUSIONS AND FUTURE WORK

Real production methods are in fact more complicated than the model developed here. Harvest quality is typically *quantized* into integer scores. We also have of course completely ignored *stochasticity*. We have assumed that the initial value of the harvest and the rate of harvesting are both deterministic. The initial value of the harvest produce in fact takes on a distribution, and the rate of harvest is also random, both due to variations from worker to worker and also from day to day for the same worker. A well-motivated stochastic model could be designed to capture these effects. Finally, quality is typically determined (and decisions made) via *sampling*; this adds an extra layer of statistical variability.

[1] Realistically, we will only be able to estimate r at discrete points during the day.

[2] The constant k may be dependent on weather, humidity, and other environmental factors.

[3] We use the standard notation that $x^+ \stackrel{\text{def}}{=} \max\{x, 0\}$ for $x \in \mathbb{R}$.

[4] If $t_n = t_{n-1}$, the nth truck is empty after having picked up an infinitesimally small amount of harvest.

[5] Using the standard notation that $x \vee y \stackrel{\text{def}}{=} \max\{x, y\}$ for x and y in $\mathbb{R}$.

REFERENCES

Akkerman, R., Farahani, P., & Grunow, M. (2010). Quality, safety and sustainability in food distribution: A review of quantitative operations management approaches and challenges. *OR Spectrum*, *32*(4), 863–904.

Amorim, P., Meyr, H., Almeder, C., & Almada-Lobo, B. (2013). Managing perishability in production-distribution planning: A discussion and review. *Flexible Services and Manufacturing Journal*, *25*(3), 389–413.

Amorim, P., Parragh, S., Sperandio, F., & Almada-Lobo, B. (2014). A rich vehicle routing problem dealing with perishable food: A case study. *TOP*, *22*(2), 489–508.

Arbib, C., Pacciarelli, D., & Smriglio, S. (1999). A three-dimensional matching model for perishable production scheduling. *Discrete Applied Mathematics*, *92*(1), 1–15.

Bandiera, O. (2004). *Relative and Absolute Incentives: Evidence on Worker Productivity*. Econometric Society 2004 North American Summer Meetings 277, Econometric Society, August.

Ferguson, M. E., & Koenigsberg, O. (2007). How should a firm manage deteriorating inventory? *Production and Operations Management*, *16*(3), 306–321.

Garcia, J. M., & Lozano, S. (2005). Production and delivery scheduling problem with time windows. *Computers and Industrial Engineering*, *48*(4), 733–742. Selected Papers from the 30th International Conference on Computers and Industrial Engineering.

Hsu, C. I., & Chen, W. T. (2014). Optimizing fleet size and delivery scheduling for multi-temperature food distribution. *Applied Mathematical Modelling*, *38*(3), 1077–1091.

Hsu, C. I., Hung, S. F., & Li, H. C. (2007). Vehicle routing problem with time-windows for perishable food delivery. *Journal of Food Engineering*, *80*(2), 465–475.

Kuo, J. C., & Chen, M. C. (2010). Developing an advanced multi-temperature joint distribution system for the food cold chain. *Food Control*, *21*(4), 559–566.

Lin, C. W. R., & Chen, H. Y. S. (2003). Dynamic allocation of uncertain supply for the perishable commodity supply chain. *International Journal of Production Research*, *41*(13), 3119–3138.

Osvald, A., & Stirn, L. Z. (2008). A vehicle routing algorithm for the distribution of fresh vegetables and similar perishable food. *Journal of Food Engineering*, *85*(2), 285–295.

Rong, A., Akkerman, R., & Grunow, M. (2011). An optimization approach for managing fresh food quality throughout the supply chain. *International Journal of Production Economics*, *131*(1), 421–429.

Zhang, S., Hu, Q., & Wang, D. (2011). Research of fresh agricultural products logistics vehicle optimization. *International Journal of Intelligent Information Processing*, *2*(2), 45–59.

How to cite this article: Manuel D, Sowers RB. Optimal transport to cold chain in perishable hand-picked agriculture. *Natural Resource Modeling*. 2017;30:e12124. https://doi.org/10.1111/nrm.12124