

A Hierarchical Decision-making Process in Social Networks for Disaster Management

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Abstract

The social media have been increasingly used for disaster management (DM) via providing real time data on a broad scale. For example, some smartphone applications (e.g. Disaster Alert and Federal Emergency Management Agency (FEMA) App) can be used to increase the efficiency of prepositioning supplies and to enhance the effectiveness of disaster relief efforts. To maximize utilities of these apps, it is imperative to have robust human behavior models in social networks with detailed expressions of individual decision-making processes and of the interactions among people. In this paper, we introduce a hierarchical human behavior model by associating extended Decision Field Theory (e-DFT) with the opinion formation and innovation diffusion models. Particularly, its expressiveness and validity are addressed in three ways. First, we estimate individual's choice patterns in social networks by deriving people's asymptotic choice probabilities within e-DFT. Second, by analyzing opinion formation models and innovation diffusion models in different types of social networks, the effects of neighbor's opinions on people and their interactions are demonstrated. Finally, an agent-based simulation is used to trace agents' dynamic behaviors in different scenarios. The simulated results reveal that the proposed models can be used to establish better disaster management strategies in natural disasters.

Keywords

Disaster Management, Social Media, Decision Field Theory, Opinion Formation, Innovation Diffusion, Agent-based Simulation

1. Introduction

Recently, social media platforms have had an increasing role in mitigating the effects of natural disasters. In order to scale down the damage caused by natural disasters and to provide citizens with as much relief as possible, governments and humanitarian agencies seek to exploit the social media by collecting relevant information as fast as possible. For example, in the wake of the Haitian earthquake and Hurricane Irma, many people uploaded photos to Facebook and Twitter – as opposed to dialing 911 – to share the situations that they face [1]. Thus, social media is replacing the role of the traditional information service system in disaster relief efforts as digital age technologies continue to rapidly develop and become more widespread [2]. In order to track the movement of hurricanes and earthquakes as they occur, governments have developed smart phone apps which would help in their relief process. In this regard, social media now plays a crucial role in all phases of disaster management efforts by sharing information and by eliciting volunteers to the cause.

In the view of government and humanitarian agencies, social media can be used as a system of collecting information. However, due to the tremendous amount of generated information during the disaster (Figure 1), it is hard to identify trustworthy and helpful sources [1, 3]. Figure 1 shows that around one million tweets were published in one day during Hurricane Irene [3]. Therefore, in order to extract important information from social media, it is necessary for governments or agencies to develop a unified framework such as the Disaster Alert or Federal Emergency Management Agency (FEMA) especially in countries where smartphones are common [1, 2]. Another important task is to make all people aware of these apps or frameworks, encouraging them to use the apps to report their situation and ask for social aids in the event of an emergency [2]. Here, in order to maximize the spread of these innovative apps into communities, it becomes necessary to understand the innovation diffusion behavior in social networks [4].

On the other hand, people may use social media to organize volunteer groups to help their communities during the disaster recovery period [2, 4]. This is known as *crowdsourcing*, derived from the words “crowd” and “source” [1, 4].

Many researchers have defined crowdsourcing in different ways, however, the common idea is to outsource needed relief activities such as the delivery of necessity resources (e.g., foods or water) to the public [2, 4]. Thus, the research goal for them is to create a sympathetic atmosphere in which many people will volunteer to provide aids. Therefore, there is a need to study and understand the opinion formation model in social networks in order to maximize the effect of crowdsourcing, policy, or campaign for crisis relief efforts.

Based on two research needs and opportunities stated above, the goal of this research is to understand the nature of innovation diffusion and opinion formation in order to utilize social media in crisis. As shown in Figure 2, these two behaviors can be understood as two hierarchal decision-making models in social networks: *top-down* and *bottom-up*, respectively. The government can establish different support policies to prepare for emergencies and for relief activities. In both cases, accurate and generalized individual decision-making models should be introduced first to support hierarchical models. To this end, the Decision Field Theory (DFT) and its extensions were introduced, which has been widely applied in many areas [5, 6]. By incorporating DFT in hierarchical models, two research goals - understanding the coverage of the innovation diffusion and the relationships between public opinion and disaster policy characteristics - can be accurately modeled and analyzed. Throughout this paper, two models are discussed to resolve the problems, and simulation analysis is conducted to demonstrate the validity of the proposed models.

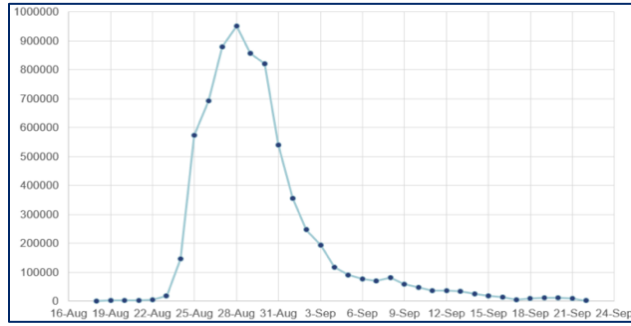


Figure 1 Number of Tweets of Hurricane Harvey

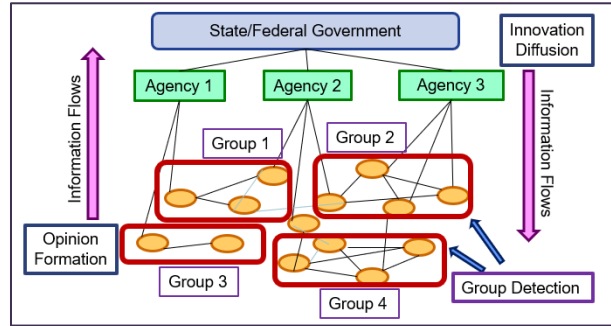


Figure 2 Overview of Hierarchical Decision-making

The structure of this paper is as follows. In Section 2, the basic concept of Decision Field Theory (DFT) and its extensions (i.e. eDFT) are introduced, and the asymptotic nature with choice probability under eDFT are also explained. In Section 3, the extension of innovation diffusion and opinion formation models within the eDFT framework are demonstrated. Finally, experimental results for models' validation are provided in Section 4.

2. Extended Decision Field Theory (eDFT) and Choice Probabilities

2.1 Decision Field Theory (DFT) and its extension (eDFT)

Decision Field Theory (DFT) is a well-known cognitive decision-making framework in psychology, and has been employed in various application areas such as engineering, biology, social science [5, 6]. Unlike approaches based on utility function (another widely used model for individual decision-making processes), DFT does not require any specific mathematical assumption such as convexity or continuity on the model [5]. The main idea of DFT is people's decisions are affected by past experience ($SP(t)$) and current evaluation ($V(t+1)$) as shown in equation (1).

$$\mathbf{DFT: } P(t+1) = SP(t) + V(t+1) = SP(t) + \mathbf{CMW}(t+1) \quad (1)$$

Equation (1) represents how individual preference values of all options ($P(t+1)$) are evaluated. To depict memory loss about past preferences ($SP(t)$), matrix S is used to represent memory loss factors. Personal *current* evaluation of options - valence vector $V(t+1)$ - is equal to $\mathbf{CMW}(t+1)$, which means that the evaluation is the product of the personal relative weights of decision criteria ($\mathbf{W}(t+1)$), individual assessment of options on the criteria ($\mathbf{M}$), and scale factors ($\mathbf{C}$). With a relative simple structure, DFT can represent a general individual decision making process. One may extend DFT by incorporating dynamic changes of environments surrounding decision-makers. Thus, extended DFT (eDFT) illustrates that individual assessment of options on the criteria can alter over time depending on environmental changes [6]. For example, in a natural disaster (such as hurricane and earthquake) the cost of necessary resources (e.g., water)

can be more expensive than the regular period. In order to convey the idea, one can update the matrix $\mathbf{M}$ to $\mathbf{M}(t+1)$, because i th row of the matrix $\mathbf{M}$ represents the personal evaluation of i th option regarding the criteria within DFT. Thus, dynamic changes of characteristics of options over time due to environmental changes can be captured within eDFT [6] as in equation (2):

$$\mathbf{eDFT}: \mathbf{P}(t+1) = \mathbf{S}\mathbf{P}(t) + \hat{\mathbf{V}}(t+1) = \mathbf{C}\mathbf{M}(t+1)\mathbf{W}(t+1) \quad (2)$$

According to (2), eDFT expresses that current preference values are cumulative sum of all past historical values as a recurrence relation. In order to derive asymptotic preference value, it is recommended to represent $\mathbf{P}(t+1)$ as the sum of a finite series. The finite sum of current preference values within eDFT is represented as equation (3):

$$\mathbf{P}(t) = \sum_{k=0}^{n-1} \mathbf{S}^k \mathbf{C}\mathbf{M}((n-k)h)\mathbf{W}((n-k)h) + \mathbf{S}^k \mathbf{P}(0) \quad (3)$$

Our main research interests on eDFT are two folds: i) asymptotic preference values and ii) agent's choice probabilities. Thus, in Section 2.2., we show how asymptotic preference values within eDFT can be derived with the basic information of $\mathbf{M}$ and $\mathbf{W}$ with two lemmas (Lemmas 1 and 2). Then, we also show that how the lemmas are used to determine individual's choice probabilities within the eDFT model.

2.2 Choice probabilities of eDTF

The following two lemmas show asymptotic nature of eDFT, and a corollary describes asymptotic choice probabilities.

Lemma 1. Assume that $\mathbf{M}(t)$ and $\mathbf{W}(t)$ are independent. Let $E[\mathbf{W}(t)] = \boldsymbol{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}$, $E[\mathbf{M}(t)] = \mathbf{M} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$. Let the $\text{Cov}(\mathbf{W}(t)) = \Sigma_{\omega} = \begin{bmatrix} \sigma_{\omega_1}^2 & \sigma_{\omega_1\omega_2} \\ \sigma_{\omega_1\omega_2} & \sigma_{\omega_2}^2 \end{bmatrix}$, and $\text{Cov}(\mathbf{m}_i(t)) = \Sigma_{\mathbf{m}_i} = \begin{bmatrix} \sigma_{m_{i1}}^2 & \sigma_{m_{i1}m_{i2}} \\ \sigma_{m_{i1}m_{i2}} & \sigma_{m_{i2}}^2 \end{bmatrix} \forall i \in \{1,2\}$. Then, mean and covariance matrix of $\mathbf{V}(t)$ can be expressed as follows:

$$E[\mathbf{V}(t)] = \mathbf{C}\mathbf{M}\boldsymbol{\omega}, \text{Cov}[\mathbf{V}(t)] = \mathbf{C}\boldsymbol{\Gamma}\mathbf{C}' \text{ where } \boldsymbol{\Gamma} \text{ is:} \quad (4)$$

$$\boldsymbol{\Gamma} = \begin{bmatrix} \sigma_{m_{11}}^2(\omega_1 + \sigma_{\omega_1}^2) + \sigma_{m_{12}}^2(\omega_2 + \sigma_{\omega_2}^2) + m_{11}\sigma_{\omega_1}^2 + m_{12}\sigma_{\omega_2}^2 & m_{11}m_{21}\sigma_{\omega_1}^2 + m_{12}m_{22}\sigma_{\omega_2}^2 \\ m_{11}m_{21}\sigma_{\omega_1}^2 + m_{12}m_{22}\sigma_{\omega_2}^2 & \sigma_{m_{21}}^2(\omega_1 + \sigma_{\omega_1}^2) + \sigma_{m_{22}}^2(\omega_2 + \sigma_{\omega_2}^2) + m_{21}\sigma_{\omega_1}^2 + m_{22}\sigma_{\omega_2}^2 \end{bmatrix} \quad (5)$$

pf) According to (2), $E[\mathbf{V}(t)] = E[\mathbf{C}\mathbf{M}(t)\mathbf{W}(t)] = \mathbf{C}E[\mathbf{M}(t)]E[\mathbf{W}(t)] = \mathbf{C}\mathbf{M}\boldsymbol{\omega}$ hold. The second last equality holds due to the independence between $\mathbf{M}(t)$ and $\mathbf{W}(t)$. Also, $\text{Cov}[\mathbf{V}(t)] = \text{Cov}[\mathbf{C}\mathbf{M}(t)\mathbf{W}(t)] = \mathbf{C}\text{Cov}[\mathbf{M}(t)\mathbf{W}(t)]\mathbf{C}'$
 $= \mathbf{C}\text{cov}\begin{bmatrix} \mathbf{m}_1^T(t)w_1(t) \\ \mathbf{m}_2^T(t)w_2(t) \end{bmatrix} \mathbf{C}' = \mathbf{C} \begin{bmatrix} \text{var}(\mathbf{m}_1^T(t)w_1(t)) & \text{cov}(\mathbf{m}_1^T(t)w_1(t), \mathbf{m}_2^T(t)w_2(t)) \\ \text{cov}(\mathbf{m}_1^T(t)w_1(t), \mathbf{m}_2^T(t)w_2(t)) & \text{var}(\mathbf{m}_2^T(t)w_2(t)) \end{bmatrix} \mathbf{C}' = \mathbf{C}\boldsymbol{\Gamma}\mathbf{C}'$.

Lemma 2. The asymptotic preference values within the eDFT framework follows asymptotic mean and variance:

$$\lim_{t \rightarrow \infty} E[\mathbf{P}(t)] = (\mathbf{I} - \mathbf{S})^{-1} \mathbf{C}\mathbf{M}\boldsymbol{\omega} \quad (6)$$

$$\lim_{t \rightarrow \infty} \text{Cov}[\mathbf{P}(t)] = \sum_{i=1}^{\infty} \mathbf{S}^i \mathbf{C}\boldsymbol{\Gamma}\mathbf{C}' \mathbf{S}^i \quad (7)$$

pf) Take an expectation and limitation on both sides in equation (2), $\lim_{t \rightarrow \infty} E[\mathbf{P}(t+h)] = \lim_{t \rightarrow \infty} \mathbf{S}E[\mathbf{P}(t)] + \lim_{t \rightarrow \infty} \mathbf{C}E[\mathbf{M}(t+h)\mathbf{W}(t+h)]$ holds. This implies $(\mathbf{I} - \mathbf{S})\lim_{t \rightarrow \infty} E[\mathbf{P}(t)] = \mathbf{C}\mathbf{M}\boldsymbol{\omega}$. Thus, under the condition of $|\mathbf{S}| < 1$, $\lim_{t \rightarrow \infty} E[\mathbf{P}(t)] = (\mathbf{I} - \mathbf{S})^{-1} \mathbf{C}\mathbf{M}\boldsymbol{\omega}$. In addition, $\lim_{t \rightarrow \infty} \text{Cov}[\mathbf{P}(t+1)] = \lim_{t \rightarrow \infty} \text{Cov}[\mathbf{P}(t)] + \lim_{t \rightarrow \infty} \text{Cov}[\mathbf{V}(t+h)]$. By the independence assumption between $\mathbf{M}$ and $\mathbf{W}$, the interaction term will disappear. Using the mathematical induction, $\lim_{t \rightarrow \infty} \text{Cov}[\mathbf{P}(t)] = \mathbf{C}\boldsymbol{\Gamma}\mathbf{C}' + \mathbf{S}^1 \mathbf{C}\boldsymbol{\Gamma}\mathbf{C}' \mathbf{S}^{1'} + \mathbf{S}^2 \mathbf{C}\boldsymbol{\Gamma}\mathbf{C}' \mathbf{S}^{2'} + \dots = \sum_{i=1}^{\infty} \mathbf{S}^i \mathbf{C}\boldsymbol{\Gamma}\mathbf{C}' \mathbf{S}^i$.

Corollary 1. Assume that weight vector ($\mathbf{W}(t)$) in eDFT follows iid Gaussian distribution. Then, the choice probability of option X over Y within eDFT is asymptotically represented as follows:

$$Pr[X|X, Y] = \int_{\{p_x > p_y\}} \frac{\exp[-((p_x - p_y) - (\zeta_x - \zeta_y))^2 / (2(\gamma_x^2 + \gamma_y^2 - 2\gamma_{xy}))]}{(2\pi(\gamma_x^2 + \gamma_y^2 - 2\gamma_{xy}))^{0.5}} dp_x dp_y \quad (8)$$

where ζ_x and ζ_y are asymptotic expected values from lemma 2, and σ_x^2 and σ_y^2 are diagonal elements of asymptotic covariance matrix ($\mathbf{\Gamma}$) while σ_{xy} is a non-diagonal element of $\mathbf{\Gamma}$.

Pf) Since each preference value of eDFT is the cumulative sum of all previous iid normal distribution, all elements still remain to follow Gaussian distribution. Thus, in the binary setting, this choice probability becomes the probability that the X normal variable is greater than Y , which is represented as equation (8).

By showing dynamic change of preferences and their ultimate values in Lemma 1 and Lemma 2, one can find the choice probabilities at the end as shown in equation (8). In Section 3, we demonstrate how the eDFT and its mathematical results can be incorporated in the proposed hierarchical decision-making processes such as opinion formation and innovation diffusion models with the consideration of human interactions in social networks.

3. Hierarchical Modeling: Opinion Formation and Innovation Diffusion

While an extended Decision Field Theory (eDFT) has been used more to represent individual decision-making processes, it is essential to generalize the framework by incorporating human interactions in social networks. In a hierarchical structure of social networks, one cannot concern the decision-making processes without the interactions among people. Therefore, we chose to adopt two major hierarchical decision-making models: 1) Bounded Confidence model (BCM) and 2) Latane's diffusion model to represent opinion formation and innovation diffusion processes, respectively [7, 8]. In this section, we discuss why these two model have been chosen and also introduce extensions of both models by incorporating the eDFT for individual decision-making processes.

Among various opinion formation models, DeGroot's (naïve) learning model is widely known for social interactions [9]. This model insists that the individual's opinion is the equally weighted sum of her neighbors' preferences [9]. For example, if one person has ten mutual friends in Facebook, her preference value of an option becomes an equally weighted sum of all her ten friends' values. The model delivers a clear idea on how people's opinions can be formed with a simple structure, but there exists one limitation: different types of human interactions are not fully considered [10]. For example, even unfriendly people are supposed to communicate with each other in forming their preferences in DeGroot's model. As this is not likely to occur in the real situation, it is imperative to convey different types of relations in opinion formation.

Bounded Confidence Model (BCM) was developed to account for different types of human interactions [10]. The main idea is simple: an individual is affected by the similar opinions of her neighbors. The BCM model is as follows:

$$p_i(t+1) = |I(i, x(t))|^{-1} \sum_{j \in I(i, x(t))} p_j(t), \text{ where } I(i, x(t)) = \{1 \leq j \leq n \mid |p_i - p_j| \leq \epsilon_i\} \quad (9)$$

$p_i(t)$ represents the individual i 's opinion at time $t+1$, while $I(i, x(t))$ represents the set of her neighbors who have similar preference values. Only neighbors who have similar opinions to individual i can affect i 's preference. The important parameter in BCM is ϵ_i , which depicts the individual's generosity. For example, if she has a high value of ϵ_i , she is more open to listen to neighbors of different preferences. Under this setting, if p_i is replaced with the preference values within the extended DFT model, the updated BCM can embody not only individual decision-making framework but also human interactions. We call this extension as extended BCM (eBCM) and demonstrate its validity by showing how different epsilon values affect the asymptotic status of social networks in Section 4.

Furthermore, innovation diffusion processes can also be demonstrated under the eDFT framework. The choice of binary options (such as shifting the older smartphone to a newer model) is the primary phenomena to be posed in the innovation diffusion model. In most literatures, the choice behavior of agents in social networks are supposed to follow S -curve diffusion phenomena [11]. Sigmoid function is a typical example with the shape of S -curve [11]. The domain of this function is the whole real line ($\mathbf{R}$), while all return value stays from 0 to 1, corresponding to the choice probability. For example, logistic or hyperbolic tangent functions are widely used sigmoid functions [11]. In this work, the modified logistic function (MLF) is used to convey Latane's extended innovation diffusion model, because this model has been widely utilized and extended to describe human behaviors in social networks [10, 11]. The main

premise of this model is that if many of your friends have adopted a new technology, you are more likely to adopt it as well. Equation (10) shows the choice probability of MLF:

$$Pr_i = f(I_i) = \frac{e^{\frac{I_i}{\eta}}}{(e^{\frac{I_i}{\eta}} + e^{-\frac{I_i}{\eta}})}, \text{ where } I_i = \sum_{j \in N(i)} \frac{s_j \sigma_j}{SE_i} + h \quad (10)$$

I_i shows the social network's impact on individual decision-maker i , which consists of four parameters: structural embeddedness (SE_i), agent j 's influence on the network (s_j), the neighbor's binary choice ($\sigma_j \in \{0, 1\}$), and the external noise or information to the network (h). In order to assign reasonable values, practical definitions of them (especially the first two terms) have to be discussed. Structural embeddedness (SE_i) represents the number of interactions of the individual i within the network (e.g., this value can be degrees of node i in the network topology), while personal influence on the network (s_j) can be expressed as individual j 's social influence within the networks. Thus, the concept of (normalized) degree centrality can be used to demonstrate $s_j = d_j / \max d_k, k \in N(j)$ [10]. If neighbor's binary choice is assigned based on equation (8) by Corollary 1, MLF is fully incorporated with eDFT. The remaining parameter η shows the entry barrier of the new technology, which can serve the elasticity of MLF (see Figure 3). For example, if the new technology is difficult to understand and utilize, the η value becomes larger. In this way, all the information regarding networks, people, and the innovation are well suited in the MLF model. In Section 4, we demonstrate the validity of both eBCM and MLF within eDFT using agent-based simulation.

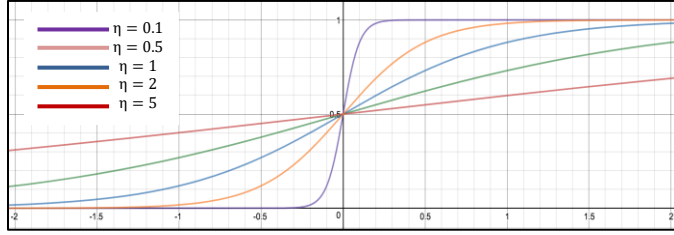


Figure 1 Extended Ellipsoid Function with different η

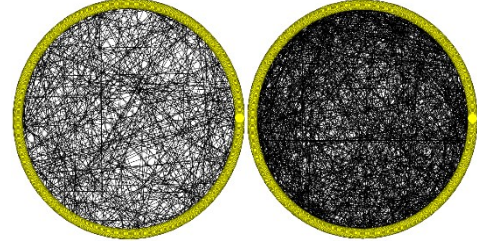


Figure 4 *Sparse* and *Dense* networks

4. Experiments

The goal of this section is to test whether the extended BCM (eBCM) and the Modified Logistic Function (MLF) models can accurately represent decision-making behaviors in social networks through simulation. For the validation of eBCM, we introduce three types of networks (i.e. the “self-centered”, the “neutral”, and the “cooperative”) based on ratios of the populace's willingness to listen to different opinions. In corporative networks, most people are open to listening to another's opinion, which results in a larger value for ϵ . On the other hand, individuals concerned mainly with themselves and have a higher value for ϵ live in the self-centered networks, while properties of the neutral lies between the self-centered and the corporative. A thousand people in each network are forced to decide whether to participate in aid activities based on two sequential reasoning processes: i) eDFT and ii) eBCM. The hypothesis being test is that people in the corporative network participate in aid activities more than in the self-centered. All experimental configurations (i.e. parameters of both models) are depicted demonstrated in Tables 1 and 2.

Table 1 Information of Parameters within eDFT

matrix $\mathbf{M}(t)$		
Participation	Uniform (0.7, 0.9)	Uniform (0.1, 0.3)
No participation	Uniform (0.1, 0.3)	Uniform (0.7, 0.9)
matrix $\mathbf{S}$		
Participation	0.95	-0.01
No participation	-0.01	0.95
matrix $\mathbf{C}$		
Participation	1	-1
No participation	-1	1

Table 2 Network Configurations

Networks for eBCM	ϵ
Self-centered	Uniform (0, 0.1)
Neutral	Uniform (0.1, 0.3)
Cooperative	Uniform (0.3, 0.5)
Networks for MLF	
Sparse	2
Neutral	5
Dense	8

Table 3 Simulation Results

Network Type	Participation Rate	Structures	Time Duration until Stability	Coverage
Self-centered	0.295	Dense	140.7	0.753
Neutral	0.494	Neutral	174.8	0.518
Corporative	0.702	Sparse	201.3	0.443

On the other hand, the similar procedure was used to evaluate MLF. We test the following hypothesis holds within MLF: if people are more connected as in a dense network, the innovation diffusion process spreads wider and faster than the sparse [11]. Similarly, new three types of networks are tested: 1) sparse, 2) neutral, and 3) dense, based on the average number of connections (see Figure 4 and Table 2). In the beginning of the simulation, only “the leader”, determined as the individual with the largest degree in the community, has installed the disaster management application while all the others have not. They update their preferences and the choice probabilities for the decision to download the app by MLF under the eDFT model. We monitored the number of people who have installed the app at the end and the time until networks’ stability is achieved (at the time when nobody changes her decision any more).

We confirmed that our hypotheses, when tested within the framework of eBCM and MLF, are found to be valid. First, the most people in the corporative network volunteered to provide aids for communities and the least in the self-centered network (Table 3), which shows that our extension of BCM accurately mimics social learning through interactions within the community. Second, the diffusion process within MLF spread the slowest in the sparse network and the fastest in the sparse network. It means that in a dense network it is much easier to spread the new app among community members quickly than it is in a sparse. Therefore, MLF can also accurately recognize the effect of network structures in the innovation diffusion processes, i.e., a well-organized information sharing platform helps to spread new information to social networks. Understanding the quantitative relationship between network structures and diffusion rates remains as a future research opportunity.

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