

Vehicle-to-Vehicle Communication for Autonomous Vehicles: Safety and Maneuver Planning

Anum Ali*, Libin Jiang[†], Shailesh Patil[‡], Junyi Li[†] and Robert W. Heath Jr.*

*The University of Texas at Austin, Austin, Texas.

[†]Qualcomm R&D, Bridgewater, New Jersey.

[‡]Qualcomm R&D, San Diego, California.

{anumali,rheath}@utexas.edu, {libinj,patil,junyi}@qti.qualcomm.com

Abstract—Autonomous vehicles (AVs) have the potential to transform road transportation by improving safety and increasing traffic/fuel efficiency. Currently, AVs rely primarily on line-of-sight sensing technologies to gather information about their surroundings. Some of the surrounding objects (e.g., vehicles, infrastructure, and pedestrians), however, can potentially communicate with the AV to help it create a better digital map of the world around it. In this work, we quantify the gains vehicle-to-vehicle (V2V) communication can provide for AVs. Specifically, we focus on the safety and maneuver planning of the AVs. Our preliminary findings indicate that V2V can reduce a significant fraction of AV collisions. Further, V2V can considerably reduce the maneuver completion time.

I. INTRODUCTION

The information about the surroundings of an autonomous vehicle (AV) comes from several sensors e.g., ultrasonic sensors, radars, lidars and cameras [1], [2]. All sensors are limited by line-of-sight (LOS) and range. Further, a significant part of planning for AVs is future risk assessment that is done by predicting the trajectory of moving obstacles. Sensor based perception is not adequate for accurate long-term trajectory prediction.

The vehicle-to-vehicle (V2V) communication can supplement the sensing ability of an AV. In Fig. 1(a), the V2V enabled ego AV is aware of the vehicles A and B on the main road that are non-line-of-sight (NLOS) and cannot be detected with LOS sensing. Future generations of V2V communication may also support information relaying. As an example, in Fig. 1(b), the V2V enabled ego AV receives information about the oncoming vehicle B through vehicle A. The vehicle B is outside the sensing range of the ego vehicle, but the vehicle A acts as an intermediate node, and effectively extends the sensing horizon of the ego AV. The scenarios in Fig. 1(a) and 1(b), also apply to a non-vehicular object (in place of vehicle B), given that the object is in the sensing range of vehicle A. Vehicles can share their own position/velocity/acceleration information with other vehicles using V2V (as in Fig. 1(c)). This eliminates the error in estimating these quantities through sensor based perception, and in turn results in accurate localization. Finally, vehicles can share their intentions (i.e., future trajectory) with other vehicles, eliminating trajectory prediction, as shown in Fig. 1(d). The intent information helps in risk assessment and can result in better planning for AVs [3].

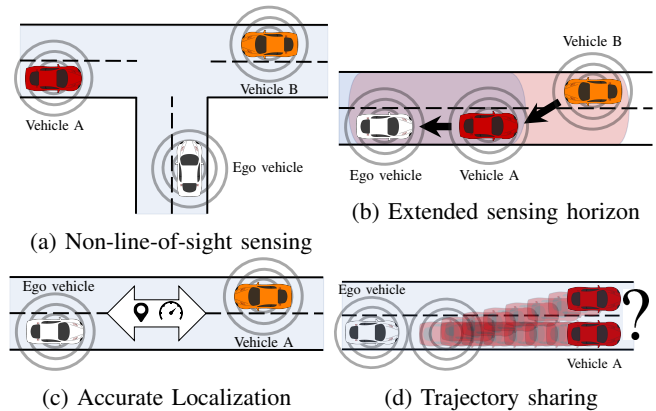


Fig. 1: Improved perception with vehicle-to-vehicle communication.

The high data-rate required by intention sharing, however, is not supported by existing V2V technologies [4].

Despite the numerous envisioned advantages of V2V for AVs, there is little work to quantify the gains that V2V can provide. In a recent article [5], the benefit of V2V to reduce inter-vehicle distance in a platoon is quantified. In comparison, we quantify the impact of V2V communication on two aspects of autonomous driving (i) safety and (ii) maneuver planning. For safety, we study the accidents involving AVs and reason that V2V communication can help in eliminating a significant fraction of these accidents. For maneuver planning, we compare the maneuver completion time of V2V enabled AVs with AVs equipped only with sensors. Specifically, we study (i) the time to reach the left-most lane from the right-most lane while driving on a multi-lane road and (ii) the time to make right-turn or left-turn on a T-intersection. Our findings show that V2V communication can cut the maneuver completion time significantly.

The remainder of this paper is structured as follows: In Section II we discuss the potential impact of V2V on the safety of the AVs in urban conditions. In Section III, we outline the setup used to assess the impact of V2V in maneuver planning and provide numerical results. Finally, Section IV concludes the paper.

II. SAFETY

In the year 2015, conventional vehicles in U.S. drove a total of 3,095,373 million-miles and were involved in 6,296,000

collisions [6], i.e., 203 accidents per 100-million-miles-driven. These collisions caused 2.4 million injuries and 35,092 fatalities. Of the total fatalities, 29% were caused by alcohol-impaired drivers, 27% by over-speeding drivers, and 10% by distracted drivers. AVs have the potential to dramatically reduce vehicle collisions caused by irresponsible/negligent human behavior.

In California, U.S., the manufacturers authorized to test AVs on public roads are required to provide the Department of Motor Vehicles (DMV) with a report of traffic accidents involving an AV [7], [8], and an annual report summarizing the disengagements of the technology during testing [7], [9]. The DMV defined disengagements as deactivations of the autonomous mode in two situations: (i) “when a failure of the autonomous technology is detected”, or (ii) “when the safe operation of the vehicle requires that the AV test driver disengages the autonomous mode and take immediate manual control of the vehicle”. According to the 2016 disengagement reports [9], Waymo (i.e., formerly the Google self-driving car project) AV fleet drove 65x more miles and 8x more miles-per-disengagement compared to the closest competitor. As such, the Waymo technology can be considered state-of-the-art. Specifically, Waymo AV fleet drove a total of 635,868 miles in 2016 [10] and was involved in 7 accidents [8], implying 1101 accidents per 100-million-miles-driven. Thus, despite the potential of the AVs to dramatically reduce vehicle collisions, present-day AVs have a more than 5x higher accident rate compared to the conventional vehicles.

Rear-end collisions: In 2016/17, 25 accidents involving AVs were reported to DMV, 15 of which were rear-end collisions [8]. Rear-end collisions typically happen because AVs’ behavior and human drivers’ expectations do not match. Consider for example the accident on 09/07/2016 [8], in which a Waymo AV came to a stop at an intersection, then, in preparation for making a right-turn, began to gradually advance forward in order to gain a better view of cross-traffic. A 12-passenger shuttle van waiting immediately behind the Waymo AV then advanced forward at approximately 11 km/h and collided with the rear bumper of the Waymo AV. The Waymo AV was traveling at approximately 5 km/h at the time of the collision. In this case, the driver of the passenger shuttle van arguably expected the AV to move at a higher speed than it actually did, i.e., erroneous intention prediction.

Sensor based technologies, e.g., forward collision warning (FCW) systems, may not eradicate rear-end collisions completely. The reason is that rear-end collisions happen at low-speeds and the detection systems are typically deactivated at low-speeds to reduce false alarms. For example, the minimum speed for Chrysler 300 FCW system activation is 16 km/h [11], and for some manufacturers can be as high as 32 km/h [12]. Further, the blind spot indicator of 2016 Honda pilot is turned off below 32 km/h to prevent false alarms [13]. Rear-end collisions, however, can be avoided by intention sharing via V2V.

Red-light running: A leading cause of urban crashes in the U.S., red-light running was responsible for 771 deaths and

137,000 injuries in 2015 [14]. A Waymo AV was involved in a red-light running accident on 09/23/2017 [8]. While the Waymo AV was traveling northbound, a vehicle heading westbound ran a red-light and collided with the right-side of the AV. At the time of the accident the AV was traveling at 35 km/h and the conventional vehicle was traveling at 48 km/h.

At 35 km/h, the stopping distance of the AV (after deciding to stop) is about 14.3 m [15, pp. 112]. The AV will travel at its initial speed for some time after coming in the LOS of the other vehicle and before deciding to stop i.e., data-processing time. Let us assume a 0.4 s processing time (i.e., a typical time for algorithms with acceptable performance on GPUs [16]). Note that, the 0.4 s processing time for an AV is much less than the human reaction time of 2.5 s [15, pp. 112]. With this, the total distance traveled by the AV after coming in the LOS of the other vehicle and before coming to a complete stop could be as high as 18.3 m.

The guidelines for “signal controlled intersection design” from American Association of State Highway and Transportation Officials (AASHTO) require only that the first vehicle stopped on each approach should be able to see the first vehicle on all other approaches [15, pp. 675]. Further, consider a 3.6 m lane-width [15, pp. 675] and the observation that vehicles stop 3 m behind the edge of main road [15, pp. 662-663]. Thus, on a road with one lane each for traffic approaching from right and left, the vehicle approaching from right will become visible to the AV just 6.6 m behind the intersection. This distance is much smaller than the vehicle stopping distance at 35 km/h, i.e., 18.3 m. The V2V NLOS communication, however, can have a 107 m range [17]. Simple calculations show that with 107 m sensing range, an AV can make a safe stop before entering the intersection for speeds up-to 90 km/h.

Safety using V2V: Four accidents (out of the 25 accidents reported to the DMV) occurred when the AV was operating in the conventional mode (i.e., the test driver had assumed control). In one accident, a conventional vehicle had hit another conventional vehicle, which in turn hit the AV. Extending the reasoning from the rear-end collisions and red-light running accidents, it can be argued that V2V could have helped the AVs in the remaining 20 accidents i.e., 80% of the total accidents.

III. MANEUVER PLANNING

In this work, we study three maneuvers. The first maneuver is reaching the left-most lane while starting from the right-most lane, as shown in Fig. 2(a). Successful completion of this maneuver typically ensures that a vehicle can subsequently travel at the maximum permitted speed of the road. The other two maneuvers are right-turn and left-turn on a T-intersection (see Fig. 2(b)). The turn maneuvers are fairly common in urban driving and time-saved in completing these maneuvers can shorten the total time-to-destination significantly.

A. Vehicle model

We use the simple point-mass-model for the dynamics of the vehicle [18]. The point-mass-model abstracts the vehicle

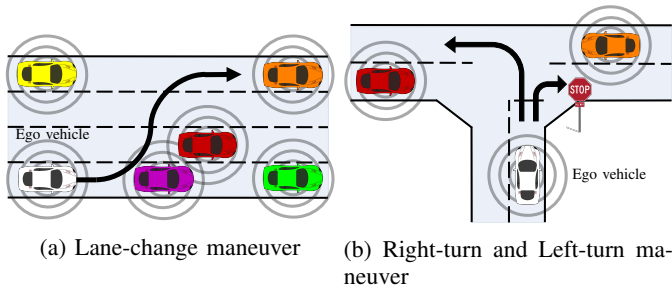


Fig. 2: The lane-change, right-turn, and left-turn maneuvers.

as a point-mass that can be accelerated within bounds. We denote the position in x -direction (i.e., longitude) as s_x , the position in y -direction (i.e., latitude) as s_y , the acceleration in x -direction as a_x , the acceleration in y -direction as a_y , and the acceleration bound as a_{\max} . We use a single dot (and double dot) on top of a variable to denote its derivative (and double-derivative). With this notation, the vehicle dynamics are

$$\ddot{s}_x = a_x, \quad \ddot{s}_y = a_y, \quad \sqrt{a_x^2 + a_y^2} \leq a_{\max}. \quad (1)$$

The ego AV and all other vehicles are described uniquely by their global center position (s_x, s_y) . Further, we assume that a vehicle occupies a rectangular region (called the vehicle occupancy region) of a certain length and width around its center position. Considering typical consumer vehicles, we use 4.8 m length and 1.8 m width. For humans, 3.4 m/s^2 is a comfortable deceleration rate [15, pp. 112], thus allowing slightly higher acceleration/deceleration, we use $a_{\max} = 4 \text{ m/s}^2$.

B. Perception and trajectory prediction

LOS sensing: The ego AV and any other vehicle (henceforth called the subject vehicle) are LOS when the global center position of the ego AV and the subject vehicle is unobstructed. As the distance between the vehicles increases, the accuracy of LOS sensing decreases. To capture this effect, we assume that vehicles can be detected using LOS sensing for only up-to a certain range (we use 100 m sensing range). We implemented the LOS sensing using Bresenham's algorithm [19].

Once the current position/velocity has been obtained, there is uncertainty in the intention of the subject vehicle. To plan the maneuvers safely, this uncertainty in the future needs to be taken into consideration. The uncertainty implies that the ego AV needs to assume worst case and plan as if the occupancy of the subject vehicle expands with time. The uncertainty in the behavior of other vehicles is specified by maximum possible acceleration/deceleration i.e., $a_{\max} = 4 \text{ m/s}^2$. Finally, while accounting for the uncertainty in the behavior of other vehicles, it is fair to assume that the vehicles will behave reasonably (i.e., they will not cause accidents). Thus, for LOS sensing, we assume that the intention of the vehicles in the same lane as the AV can be predicted fairly well.

V2V: The V2V enabled transmitting vehicle broadcasts its information and hence any V2V enabled receiving vehicle within the communication range is aware of the transmitting

vehicle. The broadcasted basic safety message (BSM) may contain information about vehicle's heading, location, and velocity. For V2V, we assume that the ego AV is aware of all the vehicles within the communication range.

The intention of the subject vehicle for up-to a few seconds can potentially be included in the BSM. Thus, we assume that the future trajectory of the V2V enabled transmitting vehicles is known to a reasonable accuracy. This implies that for V2V, the occupancy of the subject vehicle does not grow with time.

Information update interval: We assume that the ego AV feeds the new information about the environment to the maneuver planning algorithm every 100 ms. This is reasonable for V2V as typically the BSM is broadcasted every 100 ms [20]. For consistency, we also use the same value for LOS sensing, though sensors typically work at a higher-rate.

C. Reaching the left-most lane

Road: For the lane-change maneuver, we consider a straight 4-lane road with all vehicles traveling in the same direction as in Fig. 2(a). The speed of the lanes from the right-most to left-most is 40, 50, 60, and 70 km/h. All subject vehicles in a lane operate at the speed of the lane. Note that the point mass vehicle model allows for acceleration/deceleration but fixed speed in any lane simplifies the simulation scenario. The ego AV is initially in the right-most lane and intends to reach the left-most lane as quickly as possible. As the ego AV starts in the right-most lane, it has a speed of 40 km/h, which is also the minimum possible speed for the ego AV. To overtake the vehicles operating in the left-most lane, we choose the maximum possible speed of the ego AV to be 80 km/h. This is reasonable as temporarily exceeding the speed limit is typically allowed for overtaking. We assume a 3.6 m lane-width [15, pp. 675]. For simulations, we place a number of vehicles in a 1 km road segment, ensuring at-least 1 s separation between successive vehicles in the same lane.

Lane-change maneuver: A typical lane-change takes approximately 5 s [21]. During a single lane-change, the new information about the environment is fed to the planning algorithm several times. Thus, it is fair to assume that a lane-change maneuver is a sequence of several actions (or sub-maneuvers). Specifically, we assume the following: while in the center of the lane, the AV chooses one of three possible next actions, (i) stay in lane, (ii) initiate lane-change to the right, and (iii) initiate lane-change to the left. Depending on the position of the vehicle, some of these actions may not be feasible e.g., changing lane to the right from the right-most lane.

Once out of the center of a lane, the AV subsequently chooses from one of the following two actions till the lane-change is complete (i) continue the previously initiated lane-change, or (ii) revert its decision and initiate going back to the center of the lane that it came from. It is necessary to provide an option to revert because as the vehicle starts a lane-change, its lateral position changes and it can discover an object that may render previously planned maneuver infeasible (this is particularly true for LOS sensing). Further, once out of the

center of the lane, there is no option to maintain the same lateral position. This ensures that the AV always strives to keep center of a lane and does not travel consistently on other lateral positions.

All actions can be performed at the current speed of the vehicle, or with acceleration/deceleration (i.e., three choices of speed). Thus effectively, from center of a lane, the AV can choose from up-to 9 actions and from other lateral positions 6 actions.

Graph-based maneuver planning: To complete a lane-change maneuver, the AV needs to decide “a sequence of actions” that is optimal in the sense that it minimizes the time to the left-most lane. To find the optimal sequence, we formulate the maneuver planning problem as a graph-search problem. Taking the current state (i.e., position and velocity) as the starting node, we consider all feasible states after 100 ms as the neighboring nodes. We check which actions are feasible (i.e., do not result in a crash with some other vehicle) using forward simulation (based on the vehicle dynamic model). For each feasible action, there can be several subsequent feasible actions, resulting in a graph.

For graph-search, we use the A* algorithm [22] (i.e., an extension of the Dijkstra’s algorithm [23]). The A* algorithm uses a heuristic to guide its search. If the heuristic function does not overestimate the cost-to-destination (i.e., the heuristic function is admissible), A* has been shown to be optimally efficient and is guaranteed to return an optimal solution [24]. We choose the heuristic cost (of any node) to be the minimum time required to reach the left-most lane from the current node (or lateral position).

Results: The time-to-completion (TTC) results for this experiment are shown in Fig. 3. The curve “LOS” shows the TTC for an ego AV equipped only with LOS sensing, and the curve “V2V” shows the TTC for a V2V enabled ego AV. The double arrows between data-points represent the percent reduction from the top data-point to the bottom data-point. As the number of vehicles increase, the TTC increases for both “V2V” and “LOS”. The increase in “LOS”, however, is more rapid and for high vehicle density the time saved by “V2V” can be as high as 42%. To isolate the gains provided by intention sharing and NLOS sensing, we include a third curve “LOS+Traj.”. This is a genie aided LOS sensing case, where the intent of other vehicles is assumed known. From the results (zoomed in on the top-left corner) we see that most of the gains of “V2V” come from intention sharing.

‘Path prediction’ has already been defined in BSMs, but the prediction is based on vehicle’s past positions/trajectory and may not be accurate. ‘Intention sharing’ by an AV is expected to provide accurate and detailed information about the AV’s future trajectory, as computed and planned by the AV itself. ‘Intention sharing’, however, is expected to increase the amount of data to transmit and motivates future generations of V2V technologies based, for example, on 5G NR.

For the “LOS” in Fig. 3, we assumed perfect knowledge of the position/velocity of the other vehicles. In practice, however, sensors provide noisy measurements and the esti-

mation procedure is subject to errors. Thus, we perform a numerical sensitivity analyses of “LOS” in Fig. ?? against the error in position, and position plus velocity. This experiment is performed with 120 vehicles, using “V2V” case as a “Baseline”. In Fig. 4, the TTC for LOS increases from 17.9s to 25.2s with 3m position error. For position plus velocity errors, the x-axis represent both errors combined i.e., the point 3 on x-axis means a position error of 3 m and a velocity error of 3 m/s. With position plus velocity errors, the TTC goes up-to 38s.

D. Right-turn and left-turn

Road: For the turn maneuvers, we consider a stop sign controlled T-intersection as in Fig. 2(b). We consider a two-lane main road with traffic in opposite direction in each lane. The vehicles on the main road have a speed of 50 km/h and do not have a stop sign. The AV approaches the intersection from bottom, is the first vehicle at the intersection, and waits at the stop sign for its turn to make right-turn or left-turn. Thus, the starting speed of the AV is 0 km/h. We make a fair assumption that at the end of the maneuver, the speed of the AV matches the speed of the traffic in the main lane.

Turn maneuvers: To make the right-turn or left-turn on a T-intersection, the AV will enter the path of the traffic on the main road. Thus, the maneuver has to be initiated only when the AV can safely complete the maneuver. As such, it is reasonable to assume that the right-turn or left-turn is a single action, in which a preset trajectory is followed. The AV will initiate the turn maneuver if the occupancy region of the AV (based on the planned trajectory) does not intersect with the occupancy region of any other vehicle. Typically, a right-turn takes 6.5s [15, pp. 668] and a left-turn takes 7.5s [15, pp. 664].

Clear sight triangle: The LOS sensing and V2V communication for turn maneuvers is the same as explained in Section III-B. For the turn maneuvers, however, the view on each side can be obstructed by buildings/trees etc. To ensure that humans can see far enough to make safe turns, the intersection are designed to provide clear vision for a certain distance through “clear sight triangle” [15, pp. 655] (see Fig. 6). The length of the sight triangle sides along the main road (i.e., b_{left} and b_{right}) is a function of the permitted speed on the main road. For 50 km/h, $b_{\text{right}} = 90$ m and $b_{\text{left}} = 105$ m. Only the vehicles in the “clear sight triangle” can be detected by LOS sensing. Further, for the left-turn, the vehicles approaching the intersection from right can be partially or completely occluded by the vehicles approaching the intersection from left. As such, it is expected the AV will sense for some time (and keep track of sensed vehicles) before initiating the left-turn maneuver. The right-turn maneuver, however, can be completed safely based on instantaneous information. To model the uncertainty in the intention of other vehicles, the ego AV assumes that the other vehicles may speed up-to 70 km/h and down-to 30 km/h.

Results: We assume that the vehicles arrive at the intersection with an exponential rate and in Fig. 5, we plot the TTC

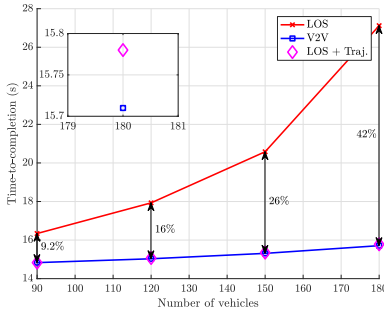


Fig. 3: Time-to-completion of the lane-change maneuver without error.

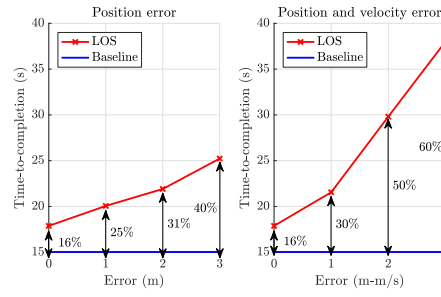


Fig. 4: Time-to-completion of the lane-change maneuver with error.

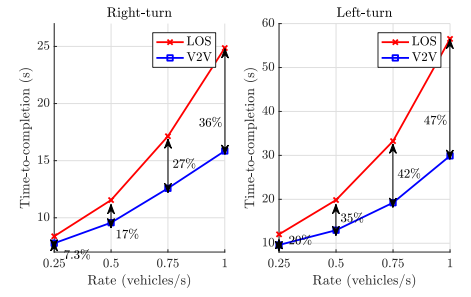


Fig. 5: Time-to-completion of right-turn and left-turn maneuvers.

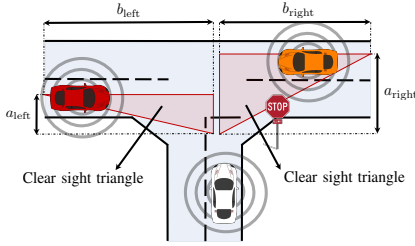


Fig. 6: Clear sight triangles on a T-intersection

against the mean vehicle arrival rate. The TTC for both the left-turn maneuver and the right-turn maneuver increases with the rate, but the increase for LOS sensing is higher, and the percent gap between the “LOS” and “V2V” can be as high 36% for the right-turn and 47% for left-turn.

IV. CONCLUSION

We quantified the benefit of V2V communication for the safety and maneuver planning of AVs. Based on the collisions of present-day AVs, it was argued that the V2V communication can help in reducing the accidents. Further, through numerical results for maneuver planning, it was shown that the V2V communication can help reduce the time-to completion of the lane-change maneuver by up-to 42%, right-turn by 36%, and left-turn by 47%. Most of the time-savings came from intention sharing, which requires high-rate data transmission and motivates advanced vehicular communications, e.g., 5G NR, in the future.

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REFERENCES

- [1] U. Ozguner, C. Stiller, and K. Redmill, “Systems for safety and autonomous behavior in cars: The DARPA Grand Challenge experience,” *Proc. IEEE*, vol. 95, no. 2, pp. 397–412, 2007.
- [2] T. Luettel, M. Himmelsbach, and H.-J. Wuensche, “Autonomous ground vehicles - Concepts and a path to the future,” *Proc. IEEE*, vol. 100, no. Special Centennial Issue, pp. 1831–1839, 2012.
- [3] S. Lefevre, “Risk estimation at road intersections for connected vehicle safety applications,” Ph.D. dissertation, Université de Grenoble, 2012.
- [4] J. Choi *et al.*, “Millimeter wave vehicular communication to support massive automotive sensing,” *IEEE Commun. Mag.*, vol. 54, no. 12, pp. 160–167, Dec. 2016.
- [5] S. Darbha, S. Konduri, and P. R. Pagilla, “Benefits of V2V communication for autonomous and connected vehicles,” *arXiv preprint arXiv:1803.02900*, 2018.
- [6] “2015 motor vehicle crashes: Overview,” U.S. Department of Transportation, National Highway Traffic Safety Administration, Aug. 2016, Report No. DOT HS 812 318.
- [7] Article 3.7 (Autonomous Vehicles) of Title 13, Division 1, Chapter 1, California Code of Regulations. California Department of Motor Vehicles.
- [8] Report of Traffic Accident Involving an Autonomous Vehicle (OL 316). [Online]. Available: https://www.dmv.ca.gov/portal/dmv/detail/vr/autonomous/autonomousveh_ol316+
- [9] Autonomous Vehicle Disengagement Reports 2016. [Online]. Available: https://www.dmv.ca.gov/portal/dmv/detail/vr/autonomous/disengagement_report_2016
- [10] Waymo, “Report on autonomous mode disengagements for Waymo self-driving vehicles in California,” Dec. 2016.
- [11] *Chrysler 300, owner’s manual*, 2013. [Online]. Available: <http://www.chrysler.com/eg/docs/300-om/files/assets/basic-html/page242.html>
- [12] J. B. Cicchino, “Effectiveness of forward collision warning systems with and without autonomous emergency braking in reducing police-reported crash rates,” Jan. 2016, insurance Institute for Highway Safety.
- [13] 2016 Honda Pilot - Safety and Driver Assistance. [Online]. Available: <http://news.honda.com/newsandviews/article.aspx?id=8493-en>
- [14] Red light running. Insurance Institute for Highway Safety. [Online]. Available: <http://www.iihs.org/iihs/topics/t/red-light-running/qanda>
- [15] *A Policy on geometric design of highways and streets*. American Association of State Highway and Transportation Officials (AASHTO), Washington, DC, 2001.
- [16] J. Janai *et al.*, “Computer Vision for Autonomous Vehicles: Problems, Datasets and State-of-the-Art,” *arXiv preprint arXiv:1704.05519*, 2017.
- [17] Accelerating C-V2X commercialization. Qualcomm. [Online]. Available: <https://www.qualcomm.com/media/documents/files/the-path-to-5g-cellular-vehicle-to-everything-c-v2x.pdf>
- [18] M. Althoff, M. Koschi, and S. Manzingler, “CommonRoad: Composable benchmarks for motion planning on roads,” in *Proc. IEEE Intell. Veh. Symp. (IV)*, Jun. 2017, pp. 719–726.
- [19] J. E. Bresenham, “Algorithm for computer control of a digital plotter,” *IBM Syst. J.*, vol. 4, no. 1, pp. 25–30, 1965.
- [20] *J2735 Dedicated Short Range Communications (DSRC) Message Set Dictionary*, Society of Automotive Engineers (SAE) Std., 2009.
- [21] T. Toledo and D. Zohar, “Modeling duration of lane changes,” *Trans. Res. Rec.*, no. 1999, pp. 71–78, 2007.
- [22] P. E. Hart, N. J. Nilsson, and B. Raphael, “A formal basis for the heuristic determination of minimum cost paths,” *IEEE Trans. Syst. Sci. Cybern.*, vol. 4, no. 2, pp. 100–107, 1968.
- [23] E. W. Dijkstra, “A note on two problems in connexion with graphs,” *Numer. Math.*, vol. 1, no. 1, pp. 269–271, 1959.
- [24] R. Dechter and J. Pearl, “Generalized best-first search strategies and the optimality of A*,” *J. ACM*, vol. 32, no. 3, pp. 505–536, 1985.