Demonstration of four-party 32-dimensional Greenberger-Horne-Zeilinger entangled state

Poolad Imany,[1,2,†,*] Mohammed S. Alshaykh,[1,2,†] Joseph M. Lukens,[3]
Alexandria J. Moore, [1,2] Daniel E. Leaird,[1,2] and Andrew M. Weiner[1,2]
[1] *School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN 47907, USA*
[2] *Purdue Quantum Science and Engineering Institute, Purdue University, West Lafayette, IN 47907, USA*
[3] *Quantum Information Science Group, Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA*
[†] These authors contributed equally to this work
**pimany@purdue.edu*

Abstract: By utilizing two-qudit SUM gates between time and frequency degrees of freedom in photons, we demonstrate a high-dimensional non-separable state between two degrees of freedom of photonic states, and realize a four-party 32-dimensional GHZ state.

OCIS codes: (270.0270) Quantum optics; (270.5585) Quantum information and processing; (190.4410) Nonlinear optics, parametric processes.

Generating non-separable states between multiple degrees of freedom (DoFs) of photonic states gives us the ability to encode multiple qubits in a photon and manipulate them deterministically. This also increases the quantum information capacity of photons which is favorable for quantum information processing purposes. Entanglement between binary DoFs of photons and deterministic two-qubit gates where qubits are encoded into different DoFs have been demonstrated [1]. In this abstract, we take advantage of high-dimensional DoFs in photons—namely time and frequency—to encode two qudits in each photon. Using a previously demonstrated two-qudit modulo SUM gate [2], we correlate these two qudits and measure the amount of non-separability between them via entanglement of formation. In the second demonstration, we start with two 32-dimensional frequency-bin entangled photons and perform the SUM gates on both twin photons to entangle their time and frequency DoFs, resulting in a four-party 32-dimensional Greenberger-Horne-Zeilinger (GHZ) entangled state. The demonstrated GHZ state occupies a Hilbert space with $32^4 = 1{,}048{,}576$ dimensions, equivalent to that of 20 qubits. It has been only recently that a three-party three-dimensional GHZ state was experimentally demonstrated, the first demonstration of a high-dimensional GHZ state [3]. The realization of such large GHZ states [4] indicates the potential of our time-frequency platform for near-term quantum technologies such as cluster-state quantum computation [5,6].

Previously, we have encoded high-dimensional quantum information in time bins and frequency bins of single photons [2]. If the spacing between the time bins (δt) and frequency bins (δf) exceed the Fourier transform limit ($\delta t \delta f \gg 1$), the information in these two DoFs can be manipulated independently, allowing us to encode two qudits in a single photon. We have demonstrated a d-dimensional modulo SUM gate between these two DoFs which adds the frequency qudit to the time qudit modulo d, $(|m\rangle_f |n\rangle_t \rightarrow |m\rangle_f |m \oplus n\rangle_t$. To construct the SUM gate, we have used a dispersion module (frequency-dependent delay) followed by an imbalanced interferometer which consists of fast switches instead of beam splitters at its input and output [2]. Here, we take advantage of a three-dimensional SUM gate to generate a high-dimensional state that is non-separable between the time and frequency DoFs of a single

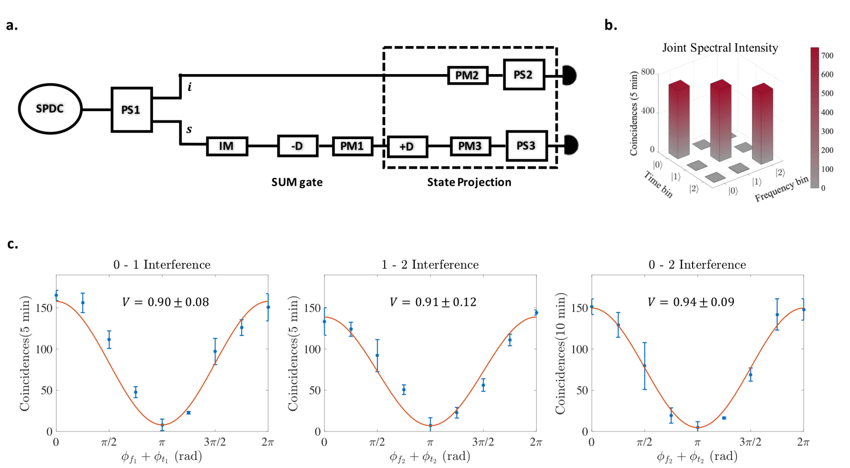

Fig. 1. Measurement of a 3-dimensional maximally non-separable state in time and frequency. **a**, The experimental setup. SPDC: spontaneous parametric down conversion, PS: pulse shaper, IM: intensity modulator, D and -D: dispersion modules with +2 ns/nm and -2 ns/nm, respectively, PM: phase modulator. **b**, Joint spectral intensity of the time-frequency non-separable state. **c**, two-dimensional interference patterns showing the coherence between all three time-frequency modes of the non-separable state. The frequency-bin and time-bin phases are varied using PS1 and PM1, respectively. Both phases are swept together from 0 to π, for a total phase sweep from 0 to 2π. The data shown in **b** and **c** are with accidentals subtracted and coincidence to accidentals ratio of about 30 and 1, respectively.

photon. As our source, we use a periodically-poled lithium niobate (PPLN) crystal which generates broadband (~5 THz) time-frequency entangled photons via a spontaneously parametric down conversion (SPDC) process (Fig. 1a). The entangled photons are then filtered into six energy-matched frequency bins spaced by 75 GHz with a pulse shaper which also spatially separates the idler photons from signal photons to be used as heralding single photons. The signal photons are then carved into one time bin with an intensity modulator, resulting in a single photon state $|\psi\rangle_{in} = 1/\sqrt{3}\left(|0\rangle_f + |1\rangle_f + |2\rangle_f\right)|0\rangle_t$. We then operate the SUM gate on the signal photon using a dispersion module, which results in a three-dimensional non-separable state $|\psi\rangle_{out} = 1/\sqrt{3}\left(|0,0\rangle_{ft} + |1,1\rangle_{ft} + |2,2\rangle_{ft}\right)$. To quantify the dimensionality of this state, we use an entanglement certification measure called *entanglement of formation* (E_{of}) [7]. To obtain E_{of}, we first perform a joint spectral intensity (JSI) measurement (Fig. 1b) followed by two-dimensional projection of both signal and idler photons into indistinguishable time and frequency states. This projection allows us to measure the interference between time and frequency of the signal photons (Fig. 1c). From the visibility of these interference patterns and the JSI values, we obtain $E_{of} \geq 1.19 \pm 0.12$ ebits, where 1 ebit corresponds to a maximally non-separable pair of qubits, our state thus possesses true high-dimensional non-separability.

To generate the GHZ state, we apply the SUM gate on both signal and idler photons to correlate their time bins and frequency bins, while starting from a frequency-bin entangled state between the two photons [8]. In this case, we start with 32 frequency bin pairs for signal and idler photons and only one time bin, $|\psi\rangle_{in} = 1/\sqrt{32}\ |0,0\rangle_{t_s t_i} \sum_{m=0}^{31} |m,m\rangle_{f_s f_i}$. After performing a 32-dimensional SUM gate on both photons, the resulting state is a 4-party 32-dimensional GHZ state in the form of $|\psi\rangle_{out} = 1/\sqrt{32}\ \sum_{m=0}^{31} |m,m,m,m\rangle_{f_s t_s f_i t_i}$, where coincidences only exist where the time-bin and frequency-bin states of both photons are the same. The GHZ state is measured in the computational basis; results are shown in Fig. 2. Remarkably, the demonstrated GHZ state resides in a Hilbert space with $1{,}048{,}576$ (32^4) dimensions. Our measurement comprises coincidence counts recorded at an equal number of distinct signal and idler time-frequency settings.

Encoding high-dimensional quantum information in multiple degrees of freedom of photons can substantially increase their information capacity. Such qudits can be manipulated deterministically via single- and two-qudit gates, in contrast to probabilistic photon-photon gates. Operating these deterministic two-qudit gates on entangled photons can result in generation of large GHZ states, which can be useful for quantum information processing protocols.

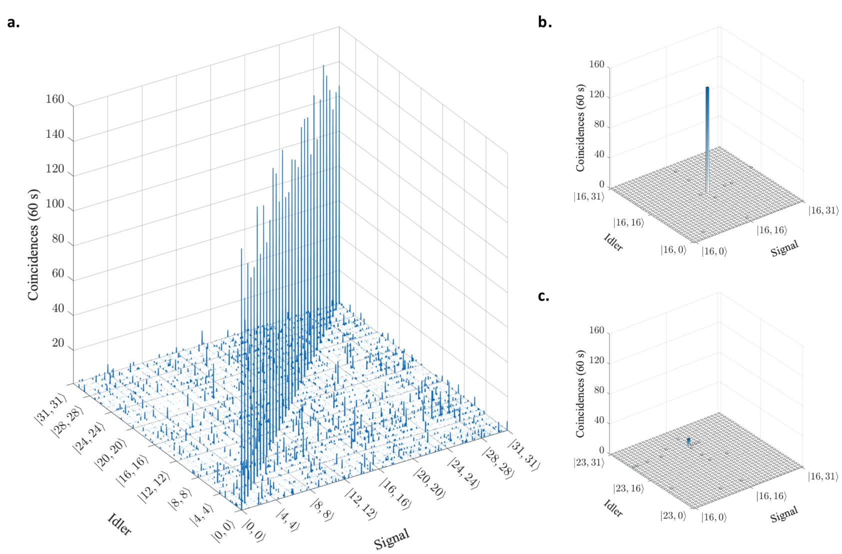

Fig. 2. **a**, Measurement of the four-party 32-dimensional GHZ state in the computational basis. The states $|m,n\rangle$ shown on the signal and idler axes correspond to frequency-bin m and time bin n. The large coincidence peaks exist only for states with the same time-bin and frequency-bin indices for both signal and idler (32 peaks). **b**-**c**, Zoomed-in 32×32 submatrices of the matrix shown in **a**. **b**, matched signal and idler frequency bins: large peak is observed for $|16,16,16,16\rangle_{f_s t_s f_i t_i}$. **c**, Unmatched signal and idler frequency bins. The data are shown with accidentals subtracted (coincidence to accidentals ratio of ~ 4).

References

[1] M. Fiorentino, and F. N. C. Wong "Deterministic controlled-NOT gate for single-photon two-qubit quantum logic." *Phys. Rev. Lett.* **93**, 070502 (2004).
[2] P. Imany *et al.* "Deterministic optical quantum logic with multiple high-dimensional degrees of freedom in a single photon." arXiv:1805.04410 (2018).
[3] M. Erhard *et al.* "Experimental Greenberger-Horne-Zeilinger entanglement beyond qubits." *Nat. Photonics* **12**, 759 (2018).
[4] X. L. Wang *et al.* "18-Qubit Entanglement with Six Photons' Three Degrees of Freedom." *Phys. Rev. Lett.* **120,** 260502 (2018).
[5] M. Pant *et al.* "Percolation thresholds for photonic quantum computing." arXiv:1701.03775 (2017).
[6] C. Reimer *et al.* "High-dimensional one-way quantum processing implemented on d-level cluster states." *Nat. Phys.* **15**, 148 (2019).
[7] A. Martin *et al.* "Quantifying Photonic High-Dimensional Entanglement." *Phys. Rev. Lett.* **118**, 110501 (2017).
[8] P. Imany *et al.* "Characterization of coherent quantum frequency combs using electro-optic phase modulation." *Physical Review A* **97**, 013813 (2018).