

Optimizing Pilots and Analog Processing for Channel Estimation in Cell-Free Massive MIMO With One-Bit ADCs

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Abstract—In a cell-free cloud radio access network (C-RAN) architecture, uplink channel estimation is carried out by a centralized baseband processing unit (BBU) connected to distributed remote radio heads (RRHs). When the RRHs have multiple antennas and limited radio front-end resources, the design of uplink channel estimation is faced with the challenges posed by reduced radio frequency (RF) chains and one-bit analog-to-digital converters (ADCs) at the RRHs. This work tackles the problem of jointly optimizing the pilot sequences and the pre-RF chains analog combiners with the goal of minimizing the sum of mean squared errors (MSEs) of the estimated channel vectors at the BBU. The problem formulation models the impact of the ADC operation by leveraging Bussgang's theorem. An efficient solution is developed by means of an iterative alternating optimization algorithm. Numerical results validate the advantages of the proposed joint design compared to baseline schemes that randomly choose either pilots or analog combiners.

Index Terms—Channel estimation, C-RAN, pilot design, analog combining, one-bit ADC, Bussgang's theorem.

I. INTRODUCTION

In a cell-free cloud radio access network (C-RAN) system, a number of remote radio heads (RRHs) are deployed to collectively serve users in the covered area. The RRHs are connected to a baseband processing unit (BBU) that carries out centralized baseband signal processing [1]. In a typical 5G deployment, due to the use of wideband spectrum and massive antenna arrays, it is generally impractical to equip the RRHs with high-precision analog-to-digital converters (ADCs) and with one radio frequency (RF) chain per antenna element due to high cost and power consumption [2]–[5]. Therefore, RRHs typically have a limited number of RF chains with limited resolution ADCs. A well-known solution to the problem of limited RF chains is to deploy a hybrid beamforming archi-

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ture, whereby analog combining is applied prior to ADC operations [2]–[4].

A key task in massive MIMO systems is acquiring channel state information (CSI) at the BBU. This is typically done via uplink training by leveraging channel reciprocity in Time Division Duplex (TDD) systems. With a cell-less architecture and centralized processing, the presence of a large number of users in the covered area implies that the number of resources allocated for training may not be sufficient to allocate orthogonal pilot sequences to all users.

In this work, we study channel estimation for a cell-free C-RAN uplink. Following [4], specifically, we tackle the problem of jointly optimizing the pilot sequences and the distributed analog combiners at the RRHs with the goal of minimizing the sum of mean squared errors (MSEs) of the estimated channel vectors at the BBU. The problem formulation models the impact of the ADC operation by leveraging Bussgang's theorem [6]. We develop an efficient solution by means of an iterative alternating optimization algorithm. Numerical results validate the advantages of the proposed joint design compared to baseline schemes that randomly choose either pilots or analog combiners.

Related works: In [7] [8], the uplink channel estimation problem was studied for a single-cell uplink system with low-resolution ADCs and fully-digital, instead of hybrid, beamforming. The problem of channel estimation for the multi-cell uplink of massive MIMO systems in the presence of pilot contamination was tackled in [4] under the assumptions that the uplink channel is noiseless, the RRHs use high-resolution ADCs, and they do not cooperate with each other. In [9], the design of joint signal and CSI compression for fronthaul transmission was studied for a C-RAN uplink with finite-capacity fronthaul links under the ergodic fading channel model. The work [10] studied the optimization of uplink reception with mixed-ADC front-end under the assumption of perfect CSI.

The rest of the paper is organized as follows. The system model for uplink channel estimation in a cell-free C-RAN system is described in Sec. II. We discuss the problems of jointly optimizing the pilots and analog processing for channel estimation first under the assumption that the RRHs use high-

resolution ADCs in Sec. III and then with one-bit ADCs in Sec. IV. In Sec. V, we provide numerical results that validate the advantages of the proposed joint design, and we conclude the paper in Sec. VI.

Notations: We denote the circularly symmetric complex Gaussian distribution with mean μ and covariance matrix \mathbf{R} as $\mathcal{CN}(\mu, \mathbf{R})$. The set of all $M \times N$ complex matrices is denoted as $\mathbb{C}^{M \times N}$, and $\mathbb{E}(\cdot)$ represents the expectation operator. We denote the transpose, Hermitian transpose and vectorization operations as $(\cdot)^T$, $(\cdot)^H$ and $\text{vec}(\cdot)$, respectively, and $\mathbf{A} \otimes \mathbf{B}$ represents the Kronecker product of matrices \mathbf{A} and \mathbf{B} . We denote by \mathbf{I}_N an N -dimensional identity matrix.

II. SYSTEM MODEL

In this section, we describe the system model under study. We consider the uplink of a cell-free C-RAN system, in which N_U single-antenna user equipments (UEs) communicate with a BBU through N_R RRHs. We assume that every RRH uses M antennas with $L \leq M$ RF chains, each equipped with a one-bit ADC. Each RRH performs analog combining prior to the ADCs. The fronthaul links connecting the RRHs to BBU are assumed to have enough capacity to support the transmission of the ADC outputs. We define the sets $\mathcal{N}_U = \{1, \dots, N_U\}$ and $\mathcal{N}_R = \{1, \dots, N_R\}$ of UEs' and RRHs' indices.

A. Uplink Channel Model for Pilot Transmission

For uplink channel estimation, each UE k sends a pilot sequence $\mathbf{s}_k = [s_{k,1} \dots s_{k,\tau}]^T$ during τ symbols. We impose per-UE transmit power constraints as

$$\frac{1}{\tau} \mathbf{s}_k^H \mathbf{s}_k \leq P_k, \text{ for } k \in \mathcal{N}_U. \quad (1)$$

Assuming a flat-fading channel model, the signal $\mathbf{Y}_t \in \mathbb{C}^{M \times \tau}$ received by RRH i can be modeled as

$$\mathbf{Y}_t = \sum_{k \in \mathcal{N}_U} \mathbf{h}_{i,k} \mathbf{s}_k^T + \mathbf{Z}_t, \quad (2)$$

where $\mathbf{h}_{i,k} \in \mathbb{C}^{M \times 1}$ denotes the channel vector from UE k to RRH i , and \mathbf{Z}_t represents the additive noise matrix distributed as $\mathbf{z}_t = \text{vec}(\mathbf{Z}_t) \sim \mathcal{CN}(\mathbf{0}, \sigma_t^2 \mathbf{I}_{M\tau})$. As in [4], we model each channel vector $\mathbf{h}_{i,k}$ as

$$\mathbf{h}_{i,k} = \sqrt{\rho_{i,k}} \mathbf{Q}_i^{1/2} \mathbf{h}_{i,k}^w, \quad (3)$$

where $\rho_{i,k} = 1/(1 + (D_{i,k}/10)^3)$ denotes the pathloss, with $D_{i,k}$ being the distance between RRH i and UE k , \mathbf{Q}_i represents the receive correlation matrix of RRH i , and $\mathbf{h}_{i,k}^w$ is a spatially white channel vector distributed as $\mathbf{h}_{i,k}^w \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$. We assume that the channel vectors $\mathbf{h}_{i,k}$ are independent across the indices i and k . The discussion can be generalized to the case where the channel vectors from different UEs are correlated [4].

B. Reduced RF Chain and Analog Combining

Since each RRH i uses only L RF chains, analog combining is carried out at RRH i via a matrix $\mathbf{W}_i \in \mathbb{C}^{L \times M}$. Analog

combining maps the M received signals into an L -dimensional vector

$$\tilde{\mathbf{Y}}_t = \mathbf{W}_i \mathbf{Y}_t, \quad (4)$$

with $\tilde{\mathbf{Y}}_t \in \mathbb{C}^{L \times \tau}$. The condition on the analog combining matrix \mathbf{W}_i depends on the specific architecture of the analog network [3][4]. In this work, we consider fully-connected phase shifters network so that the matrix \mathbf{W}_i is subject to constant modulus constraints stated as [2]

$$|\mathbf{W}_i(a, b)|^2 = 1, \text{ for } a \in \mathcal{L}, b \in \mathcal{M}, \quad (5)$$

where $\mathbf{W}_i(a, b)$ indicates the (a, b) th element of \mathbf{W}_i , and $\mathcal{M} = \{1, \dots, M\}$ and $\mathcal{L} = \{1, \dots, L\}$ denote the sets of antennas' and RF chains' indices, respectively.

For mathematical convenience, we also introduce the vectorized version $\tilde{\mathbf{y}}_t \in \mathbb{C}^{L\tau \times 1}$ of the signal $\tilde{\mathbf{Y}}_t$ as

$$\tilde{\mathbf{y}}_t = \text{vec}(\tilde{\mathbf{Y}}_t) = \sum_{k \in \mathcal{N}_U} \mathbf{B}_{k,t} \mathbf{h}_{i,k} + \tilde{\mathbf{z}}_t, \quad (6)$$

where we defined the notations $\mathbf{B}_{k,t} = \mathbf{s}_k \otimes \mathbf{W}_i$ and $\tilde{\mathbf{z}}_t = \text{vec}(\mathbf{Z}_t) = (\mathbf{I}_\tau \otimes \mathbf{W}_i) \text{vec}(\mathbf{Z}_t)$. Here the effective noise vector $\tilde{\mathbf{z}}_t$ is distributed as $\tilde{\mathbf{z}}_t \sim \mathcal{CN}(\mathbf{0}, \mathbf{C}_{\tilde{\mathbf{z}}_t})$ with $\mathbf{C}_{\tilde{\mathbf{z}}_t} = \sigma_t^2 (\mathbf{I}_\tau \otimes \mathbf{W}_i) (\mathbf{I}_\tau \otimes \mathbf{W}_i)^H$. We note that the covariance $\mathbf{C}_{\tilde{\mathbf{y}}_t} = \mathbb{E}[\tilde{\mathbf{y}}_t \tilde{\mathbf{y}}_t^H]$ of vector $\tilde{\mathbf{y}}_t$ is

$$\mathbf{C}_{\tilde{\mathbf{y}}_t} = \sum_{k \in \mathcal{N}_U} \rho_{i,k} \mathbf{B}_{k,t} \mathbf{Q}_i \mathbf{B}_{k,t}^H + \mathbf{C}_{\tilde{\mathbf{z}}_t}. \quad (7)$$

C. One-Bit ADC

Each RRH i quantizes the in-phase and quadrature (IQ) components of the elements of the vector $\tilde{\mathbf{y}}_t$ using one-bit ADCs. As in [7][8], we model the impact of one-bit ADC using Bussgang's theorem [6]. Accordingly, the ADC output vector, denoted by $\hat{\mathbf{y}}_t$, is statistically equivalent to

$$\hat{\mathbf{y}}_t = \mathbf{A}_i \tilde{\mathbf{y}}_t + \mathbf{q}_t, \quad (8)$$

where the transformation matrix \mathbf{A}_i is equal to

$$\mathbf{A}_i = \sqrt{\frac{1}{2}} \Sigma_{\tilde{\mathbf{y}}_t}^{-1/2}, \quad (9)$$

and vector \mathbf{q}_t represents the quantization noise uncorrelated to the input signal $\tilde{\mathbf{y}}_t$. The matrix $\Sigma_{\tilde{\mathbf{y}}_t}$ denotes a diagonal matrix that contains only the diagonal elements of $\mathbf{C}_{\tilde{\mathbf{y}}_t}$. Furthermore, the covariance matrix $\mathbf{C}_{\mathbf{q}_t} = \mathbb{E}[\mathbf{q}_t \mathbf{q}_t^H]$ of vector \mathbf{q}_t is equal to

$$\mathbf{C}_{\mathbf{q}_t} = \mathbf{C}_{\tilde{\mathbf{y}}_t} - \mathbf{A}_i \mathbf{C}_{\tilde{\mathbf{y}}_t} \mathbf{A}_i^H, \quad (10)$$

with the covariance matrix $\mathbf{C}_{\hat{\mathbf{y}}_t} = \mathbb{E}[\hat{\mathbf{y}}_t \hat{\mathbf{y}}_t^H]$ given by

$$\mathbf{C}_{\hat{\mathbf{y}}_t} = \frac{2}{\pi} \left[\begin{array}{l} \arcsin \left(\Sigma_{\tilde{\mathbf{y}}_t}^{-1/2} \Re \{ \mathbf{C}_{\tilde{\mathbf{y}}_t} \} \Sigma_{\tilde{\mathbf{y}}_t}^{-1/2} \right) + \\ j \arcsin \left(\Sigma_{\tilde{\mathbf{y}}_t}^{-1/2} \Im \{ \mathbf{C}_{\tilde{\mathbf{y}}_t} \} \Sigma_{\tilde{\mathbf{y}}_t}^{-1/2} \right) \end{array} \right]. \quad (11)$$

We note that the matrices \mathbf{A}_i and $\mathbf{C}_{\mathbf{q}_t}$ depend both on the pilots $\mathbf{S} = \{\mathbf{s}_k\}_{k \in \mathcal{N}_U}$ and the analog combining matrix \mathbf{W}_i , since the covariance matrix $\mathbf{C}_{\tilde{\mathbf{y}}_t}$ defined in (7) is a function of $\{\mathbf{B}_{k,t}\}_{k \in \mathcal{N}_U}$ with $\mathbf{B}_{k,t} = \mathbf{s}_k \otimes \mathbf{W}_i$.

D. Channel Estimation

The BBU estimates all the channel vectors $\{\mathbf{h}_{t,k}\}_{t \in \mathcal{N}_R, k \in \mathcal{N}_U}$ based on the quantized signals $\hat{\mathbf{y}} = [\hat{\mathbf{y}}_1^H \cdots \hat{\mathbf{y}}_{N_R}^H]^H$ collected from the RRHs:

$$\hat{\mathbf{y}} = \sum_{k \in \mathcal{N}_U} \mathbf{A} \mathbf{B}_k \mathbf{h}_k + \mathbf{A} \tilde{\mathbf{z}} + \mathbf{q}, \quad (12)$$

where we defined the notations $\mathbf{A} = \text{diag}(\mathbf{A}_1, \dots, \mathbf{A}_{N_R})$, $\mathbf{B}_k = \text{diag}(\mathbf{B}_{k,1}, \dots, \mathbf{B}_{k,N_R})$, $\mathbf{h}_k = [\mathbf{h}_{1,k}^H \cdots \mathbf{h}_{N_R,k}^H]^H$, $\tilde{\mathbf{z}} = [\tilde{\mathbf{z}}_1^H \cdots \tilde{\mathbf{z}}_{N_R}^H]^H \sim \mathcal{CN}(\mathbf{0}, \mathbf{C}_{\tilde{\mathbf{z}}})$ and $\mathbf{q} = [\mathbf{q}_1^H \cdots \mathbf{q}_{N_R}^H]^H \sim \mathcal{CN}(\mathbf{0}, \mathbf{C}_q)$ with $\mathbf{C}_{\tilde{\mathbf{z}}} = \text{diag}(\mathbf{C}_{\tilde{\mathbf{z}},1}, \dots, \mathbf{C}_{\tilde{\mathbf{z}},N_R})$ and $\mathbf{C}_q = \text{diag}(\mathbf{C}_{q,1}, \dots, \mathbf{C}_{q,N_R})$.

As in [4], we assume that the BBU applies a linear channel estimator to the signal $\hat{\mathbf{y}}$ so that the estimate $\hat{\mathbf{h}}_k$ of \mathbf{h}_k is given as

$$\hat{\mathbf{h}}_k = \mathbf{F}_k \hat{\mathbf{y}}, \quad (13)$$

with a linear filter matrix $\mathbf{F}_k \in \mathbb{C}^{MN_R \times L\tau N_R}$. For given \mathbf{S} , $\mathbf{W} = \{\mathbf{W}_t\}_{t \in \mathcal{N}_R}$ and \mathbf{F}_k , the MSE $\varepsilon_k = \mathbb{E}[||\hat{\mathbf{h}}_k - \mathbf{h}_k||^2]$ is equal to

$$\begin{aligned} \varepsilon_k &= e_k(\mathbf{S}, \mathbf{W}, \mathbf{F}_k) \\ &= \text{tr} \left((\mathbf{F}_k \mathbf{A} \mathbf{B}_k - \mathbf{I}_{MN_R}) \Theta_k (\mathbf{F}_k \mathbf{A} \mathbf{B}_k - \mathbf{I}_{MN_R})^H \right) \\ &\quad + \sum_{l \in \mathcal{N}_U \setminus \{k\}} \text{tr} \left(\mathbf{F}_k \mathbf{A} \mathbf{B}_l \Theta_l \mathbf{B}_l^H \mathbf{A}^H \mathbf{F}_k^H \right) \\ &\quad + \text{tr} \left(\mathbf{F}_k \mathbf{A} \mathbf{C}_{\tilde{\mathbf{z}}} \mathbf{A}^H \mathbf{F}_k^H \right) + \text{tr} \left(\mathbf{F}_k \mathbf{C}_q \mathbf{F}_k^H \right), \end{aligned} \quad (14)$$

with the definition $\Theta_k = \text{diag}(\{\rho_{t,k} \mathbf{Q}_t\}_{t \in \mathcal{N}_R})$. We aim at minimizing the sum-MSE $\varepsilon_{\text{sum}} = \sum_{k \in \mathcal{N}_U} \varepsilon_k$ over the pilots \mathbf{S} , the analog combiners \mathbf{W} and the digital filter matrices $\mathbf{F} = \{\mathbf{F}_k\}_{k \in \mathcal{N}_U}$.

III. OPTIMIZATION WITH HIGH-RESOLUTION ADCS

In this section, we discuss the joint optimization of the pilots \mathbf{S} and analog processing \mathbf{W} under the assumption that the RRHs use high-resolution ADCs (i.e., $\mathbf{A}_t = \mathbf{I}_{L\tau}$, $\mathbf{C}_{q,t} = \mathbf{0}$ and $\hat{\mathbf{y}}_t = \tilde{\mathbf{y}}_t$ for $t \in \mathcal{N}_R$). Furthermore, as in [4], we assume that the uplink channel is noise-free, i.e., $\sigma_t^2 = 0$, $t \in \mathcal{N}_R$.

Define the channel vector $\mathbf{h}_{R,t} = [\mathbf{h}_{t,1}^H \cdots \mathbf{h}_{t,N_U}^H]^H$ for RRH t and whole channel vector $\mathbf{h}_R = [\mathbf{h}_{R,1}^H \cdots \mathbf{h}_{R,N_R}^H]^H$. Following the same steps as in [4, Sec. III], we can write the MMSE estimate of the whole channel vector \mathbf{h}_R as

$$\hat{\mathbf{h}}_{R,\text{MMSE}} = \mathbf{R} \mathbf{B}_R^H (\mathbf{B}_R \mathbf{R} \mathbf{B}_R^H)^{-1} \hat{\mathbf{y}}, \quad (15)$$

where we have defined the notations $\mathbf{R} = \text{diag}(\{\mathbf{R}_t\}_{t \in \mathcal{N}_R})$ and $\bar{\mathbf{B}}_R = \text{diag}(\{\mathbf{B}_{R,t}\}_{t \in \mathcal{N}_R})$ with $\mathbf{R}_t = \mathbf{P}_t \otimes \mathbf{Q}_t$, $\mathbf{P}_t = \text{diag}(\{\rho_{t,k}\}_{k \in \mathcal{N}_U})$, $\mathbf{B}_{R,t} = \mathbf{S} \otimes \mathbf{W}_t$ and $\mathbf{S} = [\mathbf{s}_1 \cdots \mathbf{s}_{N_U}]$. The estimate in (15) can be decoupled across RRHs, i.e.,

$$\hat{\mathbf{h}}_{R,t,\text{MMSE}} = \mathbf{R}_t \mathbf{B}_{R,t}^H (\mathbf{B}_{R,t} \mathbf{R}_t \mathbf{B}_{R,t}^H)^{-1} \hat{\mathbf{y}}_t, \quad (16)$$

for $t \in \mathcal{N}_R$, due to the independence of the channel vectors $\mathbf{h}_{R,1}, \dots, \mathbf{h}_{R,N_R}$ and distributed analog processing at RRHs.

The sum-MSE $\varepsilon_{\text{sum}} = \sum_{t \in \mathcal{N}_R} \mathbb{E}[||\mathbf{h}_{R,t} - \hat{\mathbf{h}}_{R,t,\text{MMSE}}||^2]$ can hence be decomposed as

$$\varepsilon_{\text{sum}} = \sum_{t \in \mathcal{N}_R} [\text{tr}(\mathbf{R}_t) - \text{tr}(\mathbf{J}_t) \cdot \text{tr}(\mathbf{K}_t)], \quad (17)$$

with matrices $\mathbf{J}_t = \mathbf{W}_t \mathbf{Q}_t^2 \mathbf{W}_t^H (\mathbf{W}_t \mathbf{Q}_t \mathbf{W}_t)^{-1}$ and $\mathbf{K}_t = \mathbf{S} \mathbf{P}_t^2 \mathbf{S}^H (\mathbf{S} \mathbf{P}_t \mathbf{S}^H)^{-1}$. Since the covariance matrices \mathbf{R}_t are fixed, the problem of minimizing the sum-MSE in (17) is equivalent to that of maximizing $\sum_{t \in \mathcal{N}_R} \text{tr}(\mathbf{J}_t) \cdot \text{tr}(\mathbf{K}_t)$.

In order to minimize the sum-MSE ε_{sum} , the analog combiner \mathbf{W}_t of each RRH t can be separately optimized according to the problem:

$$\underset{\mathbf{W}_t}{\text{maximize}} \text{tr}(\mathbf{J}_t) \text{ s.t. (5).} \quad (18)$$

The problem (18) is the same as that in [4, Eq. (16)] and hence can be tackled by using the approach proposed in [4, Sec. IV].

Given the optimal analog combiners, the optimization over the pilots \mathbf{S} amounts to the maximization of $\sum_{t \in \mathcal{N}_R} w_t \cdot \text{tr}(\mathbf{K}_t)$, where $w_t = \text{tr}(\mathbf{J}_t)$ is now a fixed constant. To the best of our knowledge, as was also reported in [4], there is no known solution to this problem except for special cases with $N_R = 1$ or $\tau = 1$ or $\mathbf{P}_t = \mathbf{P}$ for all $t \in \mathcal{N}_R$. Instead, we propose to adopt the greedy sum of ratio traces maximization (GSRTM) algorithm [4, Sec. V-C] to find an efficient solution of the problem.

IV. OPTIMIZATION WITH ONE-BIT ADCS

In this section, we tackle the problem of jointly optimizing the pilots \mathbf{S} , the analog combiners \mathbf{W} and the digital filter matrices \mathbf{F} under the more challenging scenario with one-bit, instead of high-resolution, ADCs. Also, unlike Sec. III, we assume that the uplink channel is noisy, i.e., $\sigma_t^2 > 0$, $t \in \mathcal{N}_R$. The problem at hand can be stated as

$$\underset{\mathbf{S}, \mathbf{W}, \mathbf{F}}{\text{minimize}} \sum_{k \in \mathcal{N}_U} e_k(\mathbf{S}, \mathbf{W}, \mathbf{F}_k) \quad (19a)$$

$$\text{s.t. } \frac{1}{\tau} \mathbf{s}_k^\dagger \mathbf{s}_k \leq P_k, \text{ for } k \in \mathcal{N}_U, \quad (19b)$$

$$|\mathbf{W}_t(a, b)|^2 = 1, \text{ for } a \in \mathcal{L}, b \in \mathcal{M}, t \in \mathcal{N}_R. \quad (19c)$$

We note that, with the channel noise and quantization distortion, the sum-MSE in (19a) does not decouple as in (17) even if we plug the optimal (MMSE) filter \mathbf{F} into (19a). Therefore, we propose here to solve the problem alternately over the variables \mathbf{S} , \mathbf{W} and \mathbf{F} .

A. Proposed Optimization

To start, we observe that, if we fix in (19) the transformation matrices \mathbf{A} and the covariance matrices \mathbf{C}_q and relax the constraint (19c) as $|\mathbf{W}_t(a, b)|^2 \leq 1$ for $a \in \mathcal{L}$, $b \in \mathcal{M}$ and $t \in \mathcal{N}_R$, the problem becomes separately convex with respect to the variables \mathbf{S} , \mathbf{W} and \mathbf{F} [11]. This observation motivates us to derive an alternating optimization algorithm. Note that fixing matrices \mathbf{A} and \mathbf{C}_q ignores their dependence on variables \mathbf{S} and \mathbf{W} as in (9) and (10).

The algorithm, which is described in Algorithm 1, solves sequentially the convex problems obtained from (19) by restricting the optimization variables only to \mathbf{W} , \mathbf{S} and \mathbf{F} . When solving the convex problems with respect to \mathbf{W} , constraint (19c) is relaxed as $|\mathbf{W}_t(a, b)|^2 \leq 1$ for $a \in \mathcal{L}$, $b \in \mathcal{M}$ and $t \in \mathcal{N}_R$, and the resulting problem can be solved separately for every RRH t without loss of optimality. A feasible solution

Algorithm 1 Iterative optimization algorithm for problem (19)**Initialization:**

1. Initialize the pilot sequence $\mathbf{S}^{(1)}$ and analog combining variables $\mathbf{W}^{(1)}$ such that the conditions (19b) and (19c) are satisfied.
2. Initialize the matrices $\mathbf{A}_i^{(1)}$ and $\mathbf{C}_{\mathbf{q}_i}^{(1)}$, $i \in \mathcal{N}_R$, according to (9) and (10), respectively, for fixed $\mathbf{S}^{(1)}$ and $\mathbf{W}^{(1)}$, and set $t \leftarrow 1$.
3. Initialize the filter matrices $\mathbf{F}_k^{(1)}$, $k \in \mathcal{N}_U$, according to (20) for fixed $\mathbf{S}^{(1)}$, $\mathbf{W}^{(1)}$, $\mathbf{A}^{(1)}$ and $\mathbf{C}_{\mathbf{q}}^{(1)}$.

Iteration:

4. Update the pilot sequences $\mathbf{S}^{(t+1)}$ as $\mathbf{S}^{(t+1)} \leftarrow \mathbf{S}^{(t)} + \gamma^t(\mathbf{S}' - \mathbf{S}^{(t)})$, where \mathbf{S}' denotes a solution of the problem (19) for fixed $\mathbf{W}^{(t)}$, $\mathbf{A}^{(t)}$, $\mathbf{C}_{\mathbf{q}}^{(t)}$ and $\mathbf{F}^{(t)}$.
5. Update the analog combiners $\mathbf{W}^{(t+1)}$ as $\mathbf{W}^{(t+1)} \leftarrow \text{proj}(\mathbf{W}^{(t)} + \gamma^t(\mathbf{W}' - \mathbf{W}^{(t)}))$, where \mathbf{W}' denotes a solution of the problem (19) for fixed $\mathbf{S}^{(t+1)}$, $\mathbf{A}^{(t)}$, $\mathbf{C}_{\mathbf{q}}^{(t)}$ and $\mathbf{F}^{(t)}$, and $\text{proj}(\cdot)$ denotes the projection onto the space of matrices that satisfy (19c).
6. Update the matrices $\mathbf{A}_i^{(t+1)}$ and $\mathbf{C}_{\mathbf{q}_i}^{(t+1)}$, $i \in \mathcal{N}_R$, according to (9) and (10), respectively, for fixed $\mathbf{S}^{(t+1)}$ and $\mathbf{W}^{(t+1)}$.
7. Update the filter matrices $\mathbf{F}_k^{(t+1)}$, $k \in \mathcal{N}_U$, according to (20) for fixed $\mathbf{S}^{(t+1)}$, $\mathbf{W}^{(t+1)}$, $\mathbf{A}^{(t+1)}$ and $\mathbf{C}_{\mathbf{q}}^{(t+1)}$.
8. Stop if a convergence criterion is satisfied. Otherwise, set $t \leftarrow t + 1$ and go back to Step 4.

is obtained by using the projection approach in [4, Eq. (18)]. Also, the optimal linear filter \mathbf{F}_k , $k \in \mathcal{N}_U$, for fixed other variables is obtained in closed form as

$$\begin{aligned} \mathbf{F}_{k, \text{MMSE}} &= \mathbb{E}[\mathbf{h}_k \mathbf{y}^H] \mathbb{E}[\mathbf{y} \mathbf{y}^H]^{-1} \\ &= \Theta_k \mathbf{B}_k^H \mathbf{A}^H \left(\sum_{l \in \mathcal{N}_U} \mathbf{A} \mathbf{B}_l \Theta_l \mathbf{B}_l^H \mathbf{A}^H + \mathbf{A} \mathbf{C}_{\bar{\mathbf{z}}} \mathbf{A}^H + \mathbf{C}_{\mathbf{q}} \right). \end{aligned} \quad (20)$$

The step size sequence γ^t is selected to be decreasing with the iteration number t as in [12, Eq. (5)], as a means to improve the empirical convergence properties of the algorithm.

V. NUMERICAL RESULTS

In this section, we provide numerical results that validate the effectiveness of the proposed joint design of the pilot sequences and analog combining matrices for the uplink of cell-free C-RAN with one-bit ADCs. For performance evaluation, we assume that N_U UEs and N_R RRHs are uniformly distributed within a square area of the side length equal to 100 m. As in [13][14], the correlation matrix $\mathbf{R}_{t,k}$ in (3) is given as $\mathbf{R}_{t,k}(a, b) = J_0(2\pi |a - b| \sin(d_t/\lambda_t)/\Delta_t)$, where $J_0(\cdot)$ denotes the zero-th order Bessel function, and we set $d_t/\lambda_t = 0.5$ and $\Delta_t = 25$ [14].

For comparison, we consider the performance of the following reference schemes: *i) Fully random*: Pilot sequences \mathbf{S} and analog combining matrices \mathbf{W} are randomly chosen; *ii) Optimized analog combining with random pilots*: Analog combining matrices \mathbf{W} are optimized for randomly selected pilot sequences \mathbf{S} ; *iii) Optimized pilots with random analog combining*: Pilot signals \mathbf{S} are optimized for randomly selected

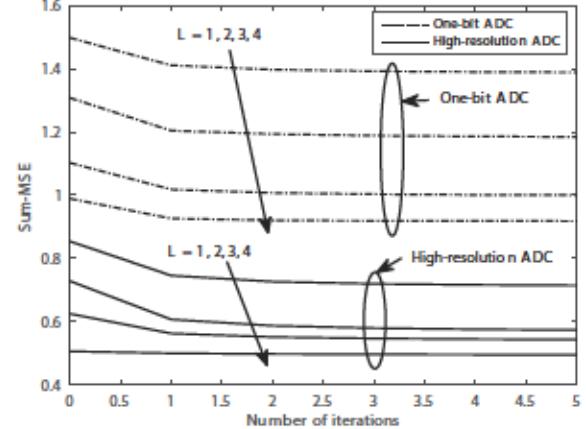


Fig. 1. Average sum-MSE versus the number of iterations ($N_U = 6$, $N_R = 2$, $M = 4$, $\tau = 2$ and 10 dB SNR).

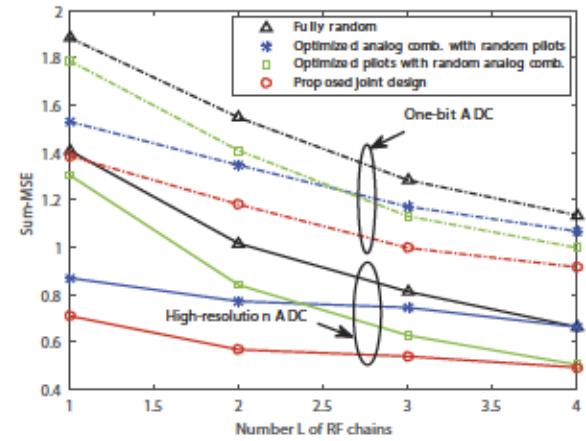


Fig. 2. Average sum-MSE versus the number L of RF chains ($N_U = 6$, $N_R = 2$, $M = 4$, $\tau = 2$ and 10 dB SNR).

analog combining matrices \mathbf{W} ; and *iv) Proposed joint design*: Pilot sequences \mathbf{S} and analog combining matrices \mathbf{W} are jointly optimized.

The algorithm proposed in Sec. IV-A is used for the last scheme, while the other reference approaches are implemented adding the indicated linear constraints to the optimization problem (19).

We first observe in Fig. 1 the convergence behavior of the proposed algorithm by plotting the average sum-MSE versus the number of iterations for $N_U = 6$ UEs, $N_R = 2$ RRHs, $M = 4$ RRH antennas, $\tau = 2$ pilot symbols and 10 dB SNR. From the figure, we observe that the proposed algorithm converges within a few iterations.

In Fig. 2, we investigate the impact of the number L of RF chains for the same configuration as in the previous figure. A first observation is that optimizing analog combiner yields larger performance gain for smaller values of L , where fewer signal dimensions are available for channel estimation at the receiver. In contrast, optimizing the pilots only provides more

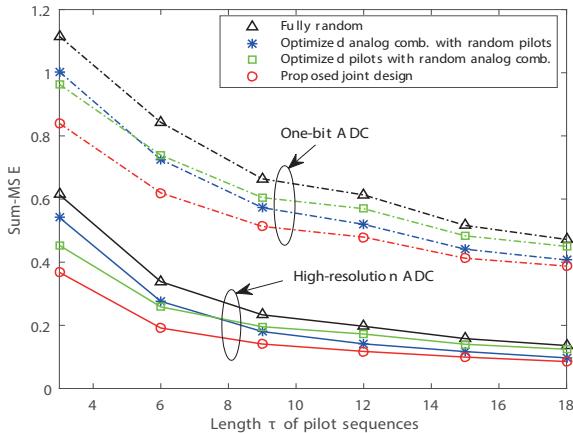


Fig. 3. Average sum-MSE versus the pilot length τ ($N_U = 6$, $N_R = 2$, $M = 4$, $L = 3$ and 10 dB SNR).

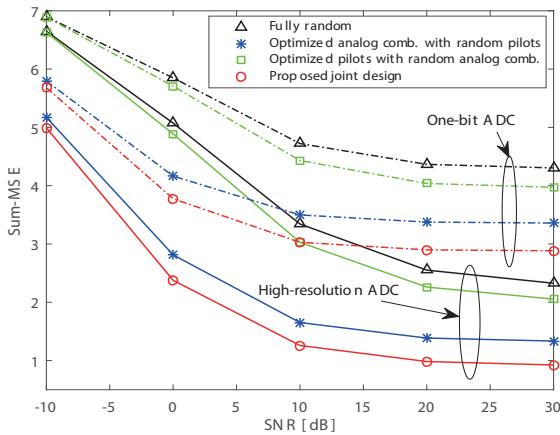


Fig. 4. Average sum-MSE versus the SNR ($N_U = 6$, $N_R = 2$, $M = 10$, $L = 2$ and $\tau = 3$).

significant performance gain for larger values of L . In this regime, the channel estimation performance is dominated by the variance due to channel noise rather than by the bias caused by a small number L of RF chains. Joint optimization allows both gains of optimizing pilots and analog combiners to be harnessed. Finally, we note that, with one-bit ADCs, optimized analog combining design offers performance gains even when $L = M$. This is because the analog combiners can pre-process the received signal in order to enable the one-bit ADCs to extract the most useful information for channel estimation.

In Fig. 3, we plot the average sum-MSE versus the pilot length τ for $N_U = 6$ UEs, $N_R = 2$ RRHs, $M = 4$ RRH antennas, $L = 3$ RF chains and 10 dB SNR. It is observed that the impact of pilot optimization is more significant in the regime where τ is smaller, which calls for the use of well-designed pilot signals.

Lastly, Fig. 4 plots the average sum-MSE versus the SNR for $N_U = 6$ UEs, $N_R = 2$ RRHs, $M = 10$ RRH antennas, $L = 2$ RF chain and $\tau = 3$ pilot symbols. We note that the

pilot optimization has a negligible impact on the performance in the low SNR regime, where the performance is limited by additive noise. In contrast, the design of analog combiners provides relevant performance gains even for low SNRs, since it can provide array beamforming gains to increase the effective SNR at the combiners' output signals.

VI. CONCLUSIONS

The joint design of the pilot signals and analog combining matrices was addressed for a cell-free C-RAN system with the goal of minimizing the sum-MSE metric of all the channel vectors in the presence of high-resolution or one-bit ADCs. We observed that, with high-resolution ADCs and noiseless uplink channel, the analog combining matrix of each RRH can be separately optimized. For the optimization with one-bit ADCs, we modeled the impact of ADC by leveraging Bussgang's theorem, and proposed an iterative algorithm that alternately optimizes the pilots, analog combiners and digital filter matrices. As a future work, we mention the analysis of the impact of fronthaul compression techniques for cell-free massive MIMO systems with finite-capacity fronthaul links.

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