Nonlinear Observer for Vehicle Motion Tracking

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Abstract— **This paper focuses on the development and use of a nonlinear observer for tracking of vehicle motion trajectories on highways while using a radar or laser sensor. Previous results on vehicle tracking have typically used an interacting multiple model filter that needs different models for different modes of vehicle motion. This paper uses a single nonlinear vehicle model that can be used for all modes of vehicle motion. A corresponding exponentially stable nonlinear observer is needed. Previous nonlinear observer design results do not work for the nonlinear system under consideration. Hence, a new nonlinear observer that utilizes better bounds on the coupled nonlinear functions in the dynamics is developed. The observer design with the developed technique is implemented in both simulations and experiments. Experimental results show that the observer can simultaneously estimate longitudinal position, lateral position, velocity and orientation variables for the vehicle from radar measurements during highway driving.**

I. INTRODUCTION

Vehicle motion tracking is an important problem that is frequently encountered in autonomous driving, as well as in collision avoidance and adaptive cruise control (ACC) applications [1-6]. Collision avoidance and ACC systems typically use radar or laser sensors for measuring distances and azimuth angles [1-6] to vehicles.

In the case of vehicles in urban traffic, the raw radar-measured variables are inadequate in order to predict the trajectories of vehicles. Both lateral and longitudinal distances and orientation of the vehicles are needed in order to accurately predict vehicle motion and provide appropriate warnings or automated driving actuation. Previous work in vehicle tracking has typically utilized interacting multiple model (IMM) filters for estimation of vehicle trajectories [6-10]. The models used in the IMM filter typically include a "straight line driving" model and a "constant turn rate" model. Each model can be used for its respective driving scenario and is not applicable for the other scenario.

This paper develops a vehicle tracking algorithm that uses a single model to represent all possible vehicle motions involving both longitudinal and lateral maneuvers. This reduces the computational effort in estimating trajectories of multiple vehicles on the road and also makes tracking a wider range of vehicle motions possible in the future.

Since the proposed vehicle model is nonlinear, an effective nonlinear observer design technique is required to ensure an exponentially stable observer. Two nonlinear observer design techniques from literature that are designed for bounded Jacobian systems are first used in an attempt to obtain a stable observer. However, their design procedures fail to yield stable observer gains for this application, due to feasibility of the associated Linear Matrix Inequities (LMIs). A new nonlinear observer design technique suitable for this application is therefore developed. The stability of the observer is proved and an observer gain for the vehicle model application is obtained.

The developed observer is used to track the lateral and longitudinal positions and orientation of a vehicle in both simulations and experiments. A radar sensor that measures polar distance and azimuth angle is used as the measurement unit. Vehicles maneuvers that include straight driving cars, a lane change maneuver and a double lane change maneuver are considered in this conference paper.

The outline of the rest of the paper is as follows: In the next section, the proposed nonlinear observer design technique is presented. In Section III, we propose a single vehicle model that can represent both longitudinal and lateral maneuvers, and discuss vehicle motion tracking using the vehicle model and developed observer. Then in Section IV and V, the proposed vehicle motion tracking algorithm is validated in both simulations and experiments. Conclusions are presented in Section VI.

II. NONLINEAR OBSERVER DESIGN

A. Problem Statement

We consider the class of nonlinear systems described by

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= Cx \end{aligned} \tag{1}$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the input vector and $y \in R^p$ is the output vector. $C \in R^{p \times n}$ is a matrix of appropriate dimensions. $f(x,u)$: $R^n \times R^m \to R^n$ is a vector of differentiable nonlinear function.

The following Luenberger observer will be studied:

$$\dot{\hat{x}} = f(\hat{x},u) + L(y - C\hat{x}) \tag{2}$$

where L is the observer gain matrix to be designed such that exponential convergence of the estimation error $\tilde{x} = x - \hat{x}$ towards zero is obtained.

B. Existing Methods for Bounded Jacobian Systems

Previous researchers have developed nonlinear observer design techniques suitable for bounded Jacobian systems using the mean value theorem [11, 12]. These studies define and use Jacobian bounds based on element-wise minimum and maximum values of the Jacobian. By considering the nonlinear system and observer described in (1) and (2), the following results can be summarized from the existing methods in literature.

This research was supported in part by a research grant from the National Science Foundation (NSF Grant PFI-1631133).

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Lemma 1: Differential Mean Value Theorem using Canonical Basis [11, 12].

Let $f(x): R^n \to R^n$ be a function continuous on $[a,b] \in R^n$ and differentiable on convex hull of the set (a,b) with Lipschitz continuous gradient. For $s_1, s_2 \in [a,b]$, there exists $z \in (a,b)$ such that

$$f(s_2) - f(s_1) = \left(\sum_{i,j=1}^{n,n} e_n(i) e_n^T(j) \frac{\partial f_i}{\partial x_j}(z_i) \right) (s_2 - s_1) \quad (3)$$

where $e_n(i) = (0,\ldots,0,1,0,\ldots 0)^T \in R^n$ is a vector of the canonical basis of R^n with 1 at i_{th} component.

Theorem 1 [11]: If an observer gain matrix L can be chosen such that

$$\begin{aligned} &P(\overline{H}_{ij}^{\max}) + (\overline{H}_{ij}^{\max})^T P - C^T L^T P - PLC < 0 \\ &P(\overline{H}_{ij}^{\min}) + (\overline{H}_{ij}^{\min})^T P - C^T L^T P - PLC < 0 \\ &P > 0 \end{aligned} \quad (4)$$

$\forall i = 1,\ldots,n$, and $\forall j = 1,\ldots,n$, where

1) $h_{ij}^{\max} \geq \max(\partial f_i / \partial x_j)$ and $h_{ij}^{\min} \leq \min(\partial f_i / \partial x_j)$;
2) $H_{ij}^{\max} = e_n(i) e_n^T(j) h_{ij}^{\max}$ and $H_{ij}^{\min} = e_n(i) e_n^T(j) h_{ij}^{\min}$;
3) $Z_H = n \times n$ is the state scaling factor, n being dimension of the state vector;
4) $\overline{H}_{ij}^{\max} = Z_H H_{ij}^{\max}$ and $\overline{H}_{ij}^{\min} = Z_H H_{ij}^{\min}$;

then this choice of L leads to asymptotically stable estimates by the observer for the system.

Theorem 2 [12]: The observer estimation error converges exponentially towards zero if there exist matrices $P = P^T > 0$ and R of appropriate dimensions such that the following LMIs are feasible:

$$A^T(\vartheta)P + PA(\vartheta) - C^T R - R^T C < 0 \qquad \forall \vartheta \in v_{H_{n,n}} \quad (5)$$

where

$$A(\vartheta) = \sum_{i,j=1}^{n,n} e_n(i) e_n^T(j) \frac{\partial f_i}{\partial x_j}(z_i) \quad (6)$$

and the domain $v_{H_{n,n}}$ is defined by

$$v_{H_{n,n}} = \{\vartheta = (\vartheta_{11},\ldots,\vartheta_{1n}\ldots,\vartheta_{nn}) \mid \vartheta_{ij} \in \{\underline{h}_{ij}, \overline{h}_{ij}\}\} \quad (7)$$

$$\underline{h}_{ij} = \min(\partial f_i / \partial x_j) \text{ and } \overline{h}_{ij} = \max(\partial f_i / \partial x_j) \quad (8)$$

When these LMIs are feasible, the observer gain L is given by $L = P^{-1} R^T$.

However, these methods do not work for the application of vehicle motion tracking, which is the subject of this paper. Specifically, the LMI toolbox in MATLAB fails to provide a feasible solution for the multiple LMIs that need to be simultaneously satisfied in both Theorems 1 and 2. It should be noted that an observer gain (and an associated P matrix) that satisfy multiple LMIs need to be obtained in both methods.

Hence, a new nonlinear observer design method suitable for the vehicle tracking application is presented herein.

C. *Nonlinear Observer*

In this section, we present a modified nonlinear observer design method for the nonlinear system described in the previous two sub-sections.

Theorem 3: Consider the nonlinear system (1) and observer form (2). If there exist matrices $P \geq I$ and R of appropriate dimensions such that the following problem is solvable:

$$\min \gamma \quad \text{subject to} \quad (9)$$

$$P \geq I \quad (10)$$

$$\begin{bmatrix} P & R^T \\ R & \gamma I \end{bmatrix} \geq 0 \quad (11)$$

$$\mathrm{A}(\zeta,u)^T P + P\mathrm{A}(\zeta,u) - C^T R - R^T C + 2\alpha P \leq 0, \quad \forall u \in u_{grid},\ \forall \zeta \in \zeta_{grid} \quad (12)$$

where $\mathrm{A}(\zeta,u) = \frac{\partial f}{\partial x}(\zeta,u)$, $\zeta \in R^n$

Then, the observer gain L is given by

$$L = P^{-1} R^T \quad (13)$$

With this value of the observer gain, the estimation error of the observer (2) converges exponentially towards zero.

Proof: To show this, let the Lyapunov function candidate for observer design be defined as $V = \tilde{x}^T P \tilde{x}$ where $P = P^T > 0$ and $P \in R^{n \times n}$. We will require the derivative of V satisfies the following differential inequality:

$$\dot{V} \leq -2\alpha V \quad (14)$$

where α is a positive constant. The inequality (14) implies the exponential stability condition [13]:

$$\|x(t) - \hat{x}(t)\| \leq k \|x(0) - \hat{x}(0)\| e^{-\alpha t} \quad (15)$$

where k is a positive constant. From the Lyapunov function candidate, the differential inequality (14) can be represented as

$$\dot{\tilde{x}}^T P \tilde{x} + \tilde{x}^T P \dot{\tilde{x}} + 2\alpha(\tilde{x}^T P \tilde{x}) \leq 0 \quad (16)$$

Based on (1) and (2), the estimation error dynamics are given by

$$\dot{\tilde{x}} = f(x,u) - f(\hat{x},u) - L C \tilde{x} \quad (17)$$

For $x, \hat{x} \in [a,b]$, the difference of the functions $f(x,u) - f(\hat{x},u)$ can be presented by using the differential mean value theorem for vector functions:

$$f(x,u) - f(\hat{x},u) = \left(\frac{\partial f}{\partial x}(\zeta,u) \right)(x - \hat{x}) \quad (18)$$

where $\zeta \in (a,b)$. Using the notation

$$\mathrm{A}(\zeta,u) = \frac{\partial f}{\partial x}(\zeta,u), \quad (19)$$

the dynamics of the estimation error becomes

$$\dot{\tilde{x}} = \left(\mathrm{A}(\zeta,u) - LC\right)\tilde{x} \quad (20)$$

Since ζ varies with the value of x and $\hat{x}$, A is an unknown and continuously time varying matrix. According to the dynamics of the estimation error, (16) becomes

$$\tilde{x}^T\{(\mathrm{A}(\zeta,u) - LC)^T P + P(\mathrm{A}(\zeta,u) - LC) + 2\alpha P\}\tilde{x} \leq 0 \quad (21)$$

(21) is satisfied when following condition is satisfied:

$$(\mathrm{A}(\zeta,u)-LC)^T P + P(\mathrm{A}(\zeta,u)-LC) + 2\alpha P \le 0 \qquad (22)$$

By introducing a new variable $R = L^T P$, (22) can be expressed as

$$\mathrm{A}(\zeta,u)^T P + P\mathrm{A}(\zeta,u) - C^T R - R^T C + 2\alpha P \le 0 \qquad (23)$$

Since ζ and u vary infinitely in a given set, (23) gives us infinitely many LMIs. This can be reduced to a finite number of LMIs using gridding techniques. We fix a finite subset of the ζ and u within its bounds such that

$$\begin{aligned} u_{grid} &\in \{u_1,\dots,u_N\} \\ \zeta_{grid} &\in \{\zeta_1,\dots,\zeta_N\} \end{aligned} \qquad (24)$$

It is noted that the dimension of the grid is proportional to the number of varying variables in ζ and u. Also, the points of the finite subset need to be chosen sufficiently dense so that solving LMIs for the finite subset is equivalent to satisfying the original stability condition. Therefore, the observer design condition (12) can be obtained.

Unfortunately, the observer gain L can be arbitrarily large, if only the design condition (23) is utilized. Hence, we use the following additional specification [14] for L. If (12) with $P = P^T > 0$ has a solution for P and R, the following condition must be satisfied for γ sufficiently large:

$$\|L\| \le \sqrt{\gamma} \qquad (25)$$

This means that $\sqrt{\gamma}$ is an upper bound on the norm of the gain L. Without loss of generality, we can assume that $P \ge I$. Since $L = P^{-1}R^T$ and γ is sufficiently large, the condition (25) becomes

$$L^T L \le L^T P L = R P^{-1} R^T \le \gamma I \qquad (26)$$

From this, we obtain

$$\gamma I - R P^{-1} R^T \ge 0 \qquad (27)$$

Using the Schur complement, (27) can be represented as (11). QED.

III. Vehicle Motion Tracking Problem

A. Proposed Vehicle Motion Model

Previous models used for tracking of vehicle motion in active safety or autonomous driving applications have primarily involved longitudinal motion variables (and sometimes additional lateral position variables) of the vehicle. For instance, a popular approach for radar based vehicle tracking consists of an interacting multiple model filter with two models – a "constant velocity" model and a "nearly coordinated turn" model [17].

It should be noted that the constant velocity model is only applicable to straight line motion and the coordinated turn model only applies while turning. Neither model can be used for both scenarios and the coordinated turn model, in fact, becomes singular when the rotation rate becomes zero.

Another disadvantage of the above approach is that all the three degrees of freedom have independent unknown inputs – lateral, longitudinal and orientation variables are all driven by unknown terms.

This paper proposes the use of a single nonlinear model that encompasses both straight line and turning motions. Considering only planar motion for the vehicle, the motion of the vehicle can be described by X, Y and ψ as shown in Fig. 1. X and Y are coordinates of longitudinal and lateral locations of the vehicle with respect to the sensor (radar or LIDAR) location, and ψ is orientation angle of the vehicle with respect to the X axis. Assuming that the slip angles at the tires are zero (but the slip angle of the vehicle itself is not zero), the model equations can be described by

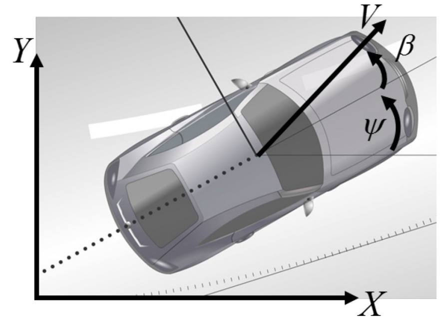

Fig. 1. Vehicle motion model.

$$\dot{X} = V\cos(\psi+\beta) \qquad (28)$$

$$\dot{Y} = V\sin(\psi+\beta) \qquad (29)$$

$$\dot{\psi} = \frac{V\cos(\beta)}{l_f + l_r}\tan(\delta_f) \qquad (30)$$

$$\beta = \tan^{-1}\left(\frac{l_r \tan(\delta_f)}{l_f + l_r}\right) \qquad (31)$$

and

$$\dot{V} = a \qquad (32)$$

where V is the speed of the vehicle, β is the slip angle of the vehicle. l_f and l_r are the distances from the center of gravity of the vehicle to the front and rear wheelbases of the vehicle, δ_f is steering angle of front wheel and a is the acceleration of the vehicle. δ_f and a are unknown inputs. More understanding of the nonlinear vehicle model can be found by reading [15].

The location of the vehicle is assumed to be measured by using a radar or LIDAR sensor. Therefore, the output equations can be written as

$$y = \begin{bmatrix} X \\ Y \end{bmatrix} = Cx \qquad (33)$$

where

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \text{ and } x = \begin{bmatrix} X & Y & V & \psi \end{bmatrix}^T \qquad (34)$$

Then, the form of the system and output equations above is the same as the form of (1) used for the nonlinear observer development.

It should be noted that the "zero slip angle at the tires" assumption could be avoided, and a model that assumes the tire force as a function of the slip angle could be utilized. However, such a model becomes a function of a large number of tire and vehicle parameters. Since the vehicle that is encountered and is being tracked is unknown, the values of these parameters cannot be known. Hence, the above model is more appropriate, in spite of the zero-slip-at-tire assumption.

It should be noted that this model has two unknown inputs (steering angle δ_f and longitudinal acceleration a). Both of these unknown inputs can be assumed to be constants (or slowly changing) and equations with their derivates as zero can be appended to the dynamic model used for tracking.

B. Observer Design for Vehicle Motion Tracking

The nonlinear observer is designed by the proposed method in section II. C with the nonlinear vehicle motion model (28) – (32). First, $A(\zeta,u)$ in (12) can be computed from the Jacobian of the vehicle motion model:

$$A(\zeta,u)=\begin{bmatrix} 0 & 0 & \cos(\psi+\beta) & -V\sin(\psi+\beta) \\ 0 & 0 & \sin(\psi+\beta) & V\cos(\psi+\beta) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{\cos(\beta)}{l_f+l_r}\tan(\delta_f) & 0 \end{bmatrix} \tag{35}$$

The Jacobian has 3 varying variables: V, δ_f and ψ which gridding is required. We aim to track the vehicle motion under the following conditions: 1) $5\,m/s \le V \le 15\,m/s$, 2) $\pi/9 \le \delta_f \le \pi/9$ and 3) $\pi/3 \le \psi \le \pi/3$. Based on these conditions, a finite subset (3-dimensional grid) is found. Then, we solve (9) – (13) for the observer gain using the LMI toolbox in MATLAB. The example of an observer gain without the exponential stability condition (i.e., $\alpha=0$) is

$$L=\begin{bmatrix} 34.2214 & -1.5711\text{e-}14 \\ -5.1932\text{e-}14 & 30.5531 \\ 2.3666\text{e-}05 & -4.9153\text{e-}19 \\ 2.1132\text{e-}14 & 14.7771 \end{bmatrix} \tag{36}$$

and with the exponential stability condition using $\alpha=0.5$ is

$$L=\begin{bmatrix} 317.7892 & 2.2971\text{e-}10 \\ -1.2979\text{e-}10 & 561.7626 \\ 2685.1585 & 2.2204\text{e-}09 \\ -1.023\text{e-}10 & 518.4968 \end{bmatrix} \tag{37}$$

Despite the fact that the observer gain is obtained by using the vehicle motion model, the observer (2) cannot be directly used with the vehicle motion model since the inputs are unknown and not measurable. In order to handle this, we simply assume that the inputs in the vehicle motion model are zero and utilize the vehicle motion model with zero inputs to the observer. In other words, the actual vehicle has unknown non-zero inputs, but the observer is assumed to have zero inputs.

IV. SIMULATION RESULTS

The nonlinear observer for vehicle motion tracking described in the previous section has first been evaluated using simulations. The simulation environment is built using MATLAB.

The three scenarios as shown in Fig. 2 are simulated. The trajectories of the vehicle are generated by using the nonlinear vehicle motion model. Each trajectory and the inputs for the trajectory are shown in Fig. 3, 4 and 5. The initial conditions of the observer are set as zero except the values of the outputs $\hat{X}_0$ and $\hat{Y}_0$.

The estimation results using the nonlinear observer are also shown in Fig. 3, 4 and 5. Only small estimation errors are present during transient periods. Overall, the proposed nonlinear observer provides good performance for vehicle motion tracking. The estimates of the vehicle motion converge to the true vehicle motion, even with unknown steering and acceleration inputs.

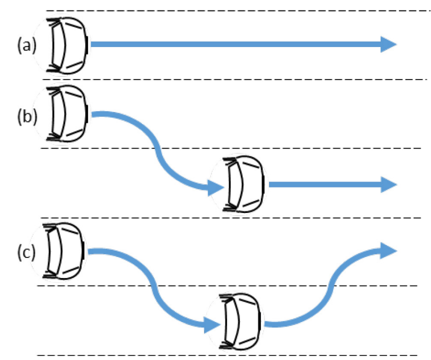

Fig. 2. Three types of vehicle maneuver scenarios. (a) Straight driving, (b) Lane change, (c) Double lane change maneuvers.

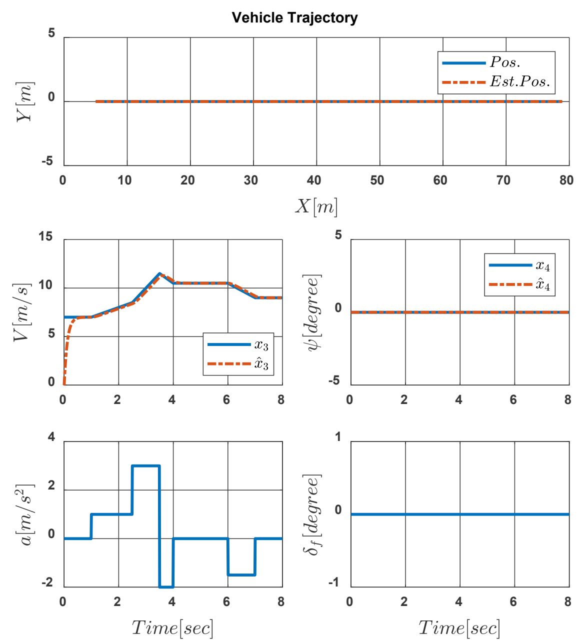

Fig. 3. Simulation results of straight driving maneuver.

V. EXPERIMENTAL RESULTS

Experiments are conducted to validate the proposed nonlinear observer design in situations corresponding to all the three scenarios of Fig. 2, of i) Straight Driving, ii) Lane Change and iii) Double Lane Change maneuvers. The Delphi Electronically Scanning Radar (ESR) shown in Fig. 6 is used for the experimental evaluation. The radar provides vehicle radial position information within 174 meters of maximum range and ± 45 degrees of maximum field of view [16]. Also, the velocity of the target can be obtained from the radar. However, we only use the velocity information as a reference to compare with the velocity estimate, for the validation of the proposed nonlinear observer. A low pass filter is implemented to smoothen the vehicle position data obtained from the radar, before supplying it to the observer. The radar is installed on a

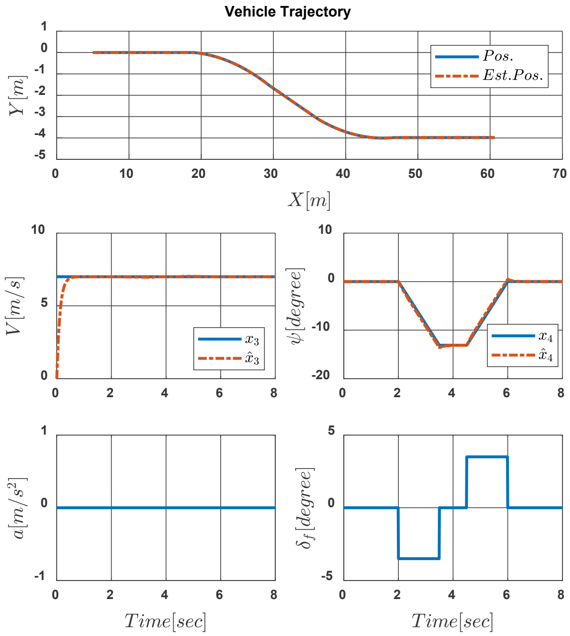

Fig. 4. Simulation results of lane change maneuver.

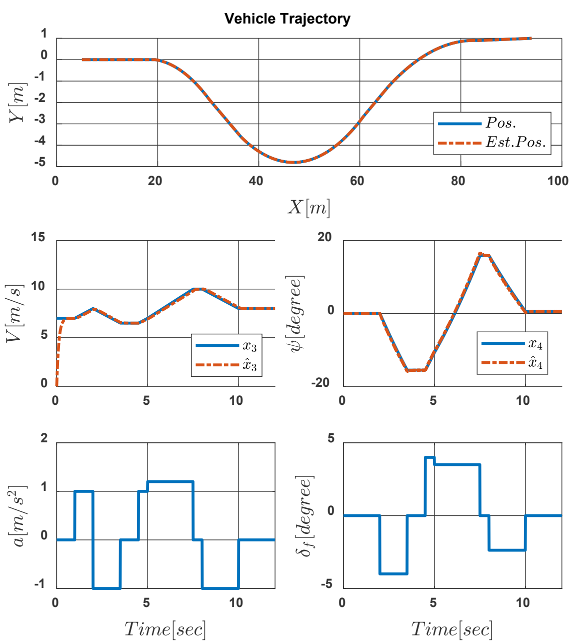

Fig. 5. Simulation results of double lane change maneuver.

tripod and initially located behind the stopped vehicle. Then, the vehicle starts moving to execute the scenarios as shown in Fig. 2.

Fig. 7, 8 and 9 show the experimental results for the three scenarios. It is shown that the proposed nonlinear observer can estimate the vehicle motion based on radar measurements, without knowing the steering and acceleration of the vehicle. The velocity estimates have good match with reference values from the radar system. Also, the estimates from the experimental data provides very reasonable evolutions of the orientation of the vehicle which are quite similar as the orientation results from the simulations.

Fig. 6. Delphi ESR [16].

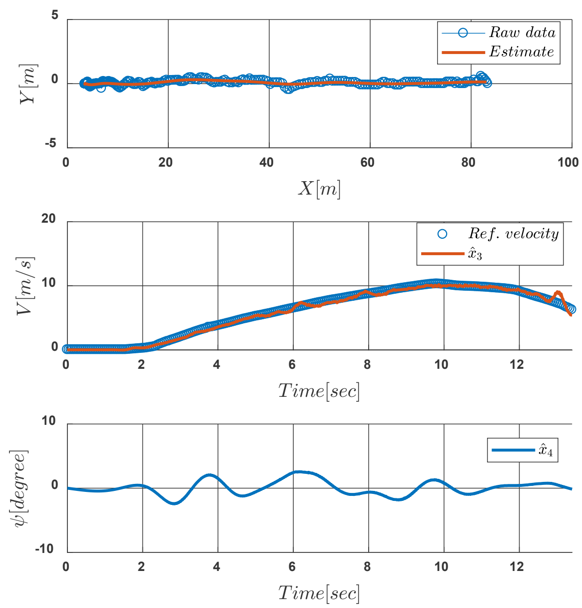

Fig. 7. Experimental results of straight driving maneuver.

VI. Conclusion

A vehicle tracking algorithm that uses a single model to represent all possible vehicle motions is presented in this paper. By using a single vehicle model, nonlinear observer design techniques can be utilized. The developed nonlinear observer guarantees exponential stability of the estimates and requires less computational effort in estimating trajectories of multiple vehicles on the road.

The developed observer is used to track vehicle motion including the lateral and longitudinal positions, velocity and orientation of a vehicle. Simulation results were presented to show the performance of the developed observer in the application of vehicle motion tracking. Experimental results were also presented using a radar sensor that measures polar distance and azimuth angle as the measurement unit. Both simulations and experiments show excellent results in vehicle motion tracking on vehicle maneuvers including straight driving cars, a lane change maneuver and a double lane change maneuver.

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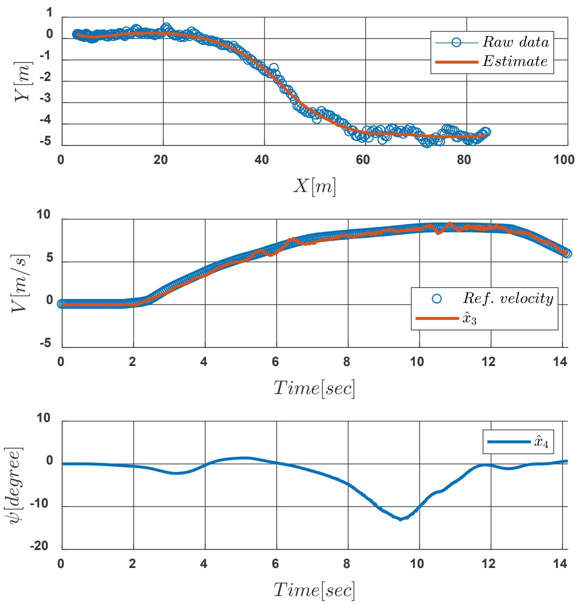

Fig. 8. Experimental results of lane change maneuver.

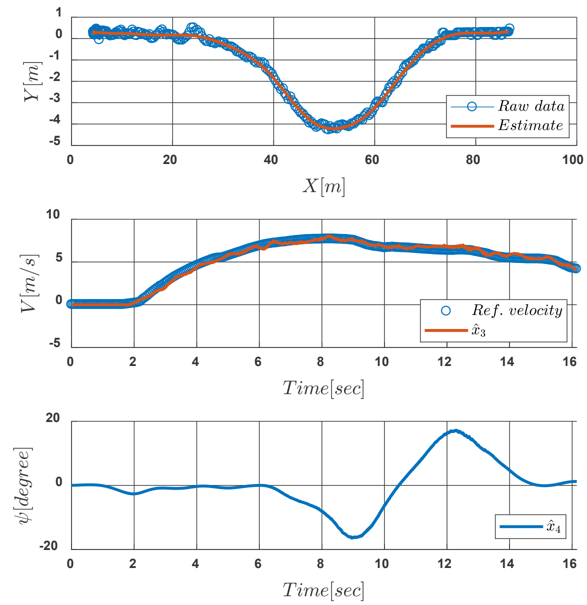

Fig. 9. Experimental results of double lane change maneuver.

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