Dirac-Maxwell correspondence: Spin-1 bosonic topological insulator

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Abstract: We introduce photonic Dirac monopoles and strings, proving the existence of a spin-1 bosonic topological phase for light. Fundamentally different from pseudo-spin-1/2 based photonic crystals, we discover quantized spin in symmetry protected helical edge states of continuous media.
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Topological phases of electronic materials exhibit a host of fascinating phenomena such as protected edge states, spin-momentum locking, quantized magneto-electric effect, Weyl points and Fermi arcs[1]. The phenomena in non-interacting electronic systems can be traced back to the time-reversal and parity symmetry properties of the band-structure and underlying Hamiltonian. A fundamental ingredient is the spin-1/2 of the electron which admits the definition of topological invariants such as the spin Chern number and $\mathbb{Z}_2$ invariant in the quantum spin Hall phase, which can be related to experimentally observed electronic transport properties (eg: Hall conductivity[1]).

Recent interest in photonics has focused on mimicking topological phenomena using photonic crystals that exploit the correspondence between Schrodinger's equation and Maxwell's equations[2]. This requires a pseudo-spin-1/2 electromagnetic field for systems with time-reversal (T) symmetry and synthetic gauge fields (artificial vector potential) for those without T symmetry[2]. However, these systems do not take into account the fundamental spin-1 nature of the photon, nor the central differences in time-reversal between bosons and fermions. Furthermore, the topological invariants cannot be defined for continuous natural media or metamaterials but necessarily rely on band-structure similar to electronic crystals.

Our major contribution in this paper is the foundation of **bosonic topological insulators (bTI)** for light. We provide the first definition of topological invariants utilizing the spin-1 vector fields of the photon, marking a distinct departure from previous pseudo-spin-1/2 based works. We achieve this by introducing a Dirac-Maxwell correspondence principle for topological photonics, a paradigm shift from existing Schrodinger-Maxwell analogies. This paper also introduces for the first time - **Dirac monopoles, Dirac strings and skyrmions in photonics** as well as bosonic time-reversal and parity symmetry based topological quantum numbers. Furthermore, we show the existence of a practical bosonic topological phase employing degenerate optical chirality which achieves the Quantum spin-1 Hall effect of light (QS[1]HE). Finally, **we discover a quantized photonic spin** in symmetry protected helical edge states which does not occur in any existing photonic crystal or metamaterial designs.

Dirac's equation for a massive electron is an energy eigenvalue problem $E\psi = H_e\psi$, where the Hamiltonian is $H_e = \begin{bmatrix} \mathbf{k}\cdot\boldsymbol{\sigma} & m_e \\ m_e & -\mathbf{k}\cdot\boldsymbol{\sigma} \end{bmatrix} = \sigma_z \otimes (\mathbf{k}\cdot\boldsymbol{\sigma}) + m_e\sigma_x\otimes I_2$. For the electron, $\boldsymbol{\sigma}$ is the set of SU(2) Pauli matrices representing the generators of spin $s = 1/2$; m_e being the electron mass. We utilize the Reimann-Silberstein (R-S) basis to recast Maxwell's equations into a remarkably similar form $\omega\Psi = H_{ph}\Psi$, with the Hamiltonian written as $H_{ph} = \begin{bmatrix} \mathbf{k}\cdot\mathbf{S} & 0 \\ 0 & -\mathbf{k}\cdot\mathbf{S} \end{bmatrix} = \sigma_z\otimes(\mathbf{k}\cdot\mathbf{S})$. Note the fundamental difference for the photon, $\mathbf{S}$ is the set of SO(3) antisymmetric matrices representing the generators of spin $s = 1$. Besides the mass term, the two equations are phenomenologically identical since both $\boldsymbol{\sigma}$ and $\mathbf{S}$ obey the same Lie algebra. Indeed, the massless photon has an equivalent Dirac monopole $\mathbf{F} = \boldsymbol{k}/k^3$ corresponding to a string of singularities in the Berry potential $\mathbf{A} = -\hat{\boldsymbol{\phi}}\cot\theta\,/k$ (Fig. 1(a)). To harness this topological feature, we introduce an effective mass for the photon $m_{ph} = \omega\gamma = m_{\mathrm{Dirac-Maxwell}}$ that behaves analogously to the electron mass (Fig. 1(b)). Here, γ is the degenerate optical chirality that couples the electric and magnetic fields. This enters the constitutive relations as $\begin{bmatrix} \mathbf{D} \\ \mathbf{B} \end{bmatrix} = \begin{bmatrix} \epsilon & \gamma S_z \\ \gamma S_z & \mu \end{bmatrix}\begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix}$, where S_z is the SO(3) generator along $\hat{\mathbf{z}}$ and we may assume ϵ and μ are all-dielectric (positive >1) scalar constants that only effect the apparent speed of light. Importantly, due to the reality condition on the electromagnetic field, $\gamma = -\gamma(-\omega)$ must be odd in ω which ensures the effective mass $\partial_\omega m_{ph} \approx 0$ is approximately constant when sufficiently far off resonance. Tbl. 1 shows the equivalent 2-D system in the R-S basis. We find that this degenerate optical chirality supports counter-propagating photonic QS[1]HE edge states $\Psi_\pm$, which are completely transverse and orthogonal $\Psi_+^\dagger\,\Psi_- = 0$; therefore immune to back-scattering (Fig. 2(b)). Furthermore, we discover that they are linearly dispersing and **helically quantized** along the direction of propagation $\sigma_z \otimes (\hat{\mathbf{k}} \cdot \mathbf{S}) \Psi_\pm = S_\pm \Psi_\pm$, where $S_\pm = \pm 1$ are the spin-1 eigenvalues for the photon. We emphasize that the hallmarks of observable topological phenomena in electronics are quantized spin and transport parameters which has not been achieved in photonics. This quantized effect in the spin-1 bTI lays the foundations of such endeavors for photonics.

Table 1. We introduce the Dirac-Maxwell correspondence for foundations of spin-1 bosonic topological insulators in photonics.

Property	Dirac	Maxwell
2-D Hamiltonian, H	$H_e = \sigma_z \otimes (k_x\sigma_x + k_y\sigma_y) + m_e\sigma_x \otimes I_2$	$H_{ph} = \sigma_z \otimes (k_x S_x + k_y S_y) + m_{ph}\sigma_y \otimes S_z$
Dispersion relation, ω	$E = \sqrt{k^2 + m_e{}^2}$	$\omega = \sqrt{k^2 + m_{ph}{}^2}, \quad m_{ph} = \omega\gamma$
Parity operator, P	$P = \sigma_x \otimes I_2, \quad P^2 = +1$	$P = \sigma_y \otimes I_3, \quad P^2 = +1$
Time-reversal operator, T	$T = I_2 \otimes \sigma_y K, \quad T^2 = -1$	$T = K, \quad T^2 = +1$
Parity-time operator, PT	$[P,T] = 0, \quad (PT)^2 = -1$	$\{P,T\} = 0, \quad (PT)^2 = -1$
Spin, s	$s = 1/2$	$s = 1$
Monopole charge, Q_s	$Q_{1/2} = s = 1/2$	$Q_1 = s = 1$
Time-reversal invariant	$\mathbb{Z}_2$: $\nu = \{0, 1\}$	$2\mathbb{Z}_2$: $\varkappa = \{0, 2\}$

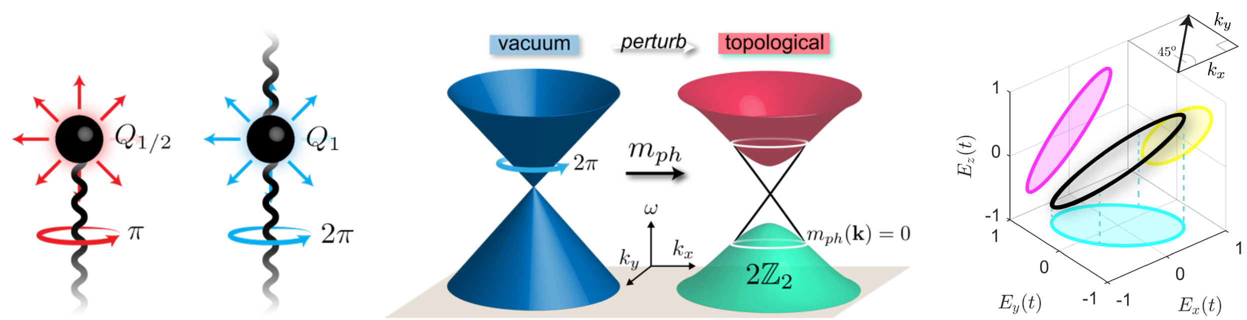

Figure 1. (a) Our theory rigorously puts forth the difference between Dirac monopoles/strings of the massless spin $s = 1/2$ electron and $s = 1$ photon. The electron and photon have monopole charges of $Q_{1/2} = 1/2$ and $Q_1 = 1$ respectively. For massless particles, the energy bands are linearly dispersing around the $\mathbf{k} = 0$ Dirac point (zero energy/frequency). Any closed path around this point produces a quantized Berry phase $\oint \mathbf{A} \cdot d\mathbf{k} = 2\pi Q_s$. Unlike the electron, the photon returns to its initial state after a cyclic evolution around the equator. **(b)** On the left, the Dirac point of the vacuum photon. On the right, we put forth a $2\mathbb{Z}_2$ bosonic topological insulator (bTI) with QS^{1}HE edge states. The spin-1 bTI can be realized by opening the gap with an electronic-like mass term $m_{ph} = m_{\text{Dirac-Maxwell}}$. The edge states (black lines) emerge from any point where $m_{ph}(\mathbf{k}) = 0$ passes through zero (white ring) and these are protected by time-reversal symmetry. **(c)** The massive electromagnetic field is generally desribed by a 3D polarization (black line) as opposed to the 2D polarization of conventional photonic media. The magneta, cyan and yellow lines show the elliptical projections of this 3D polarization in each of the orthogonal planes.

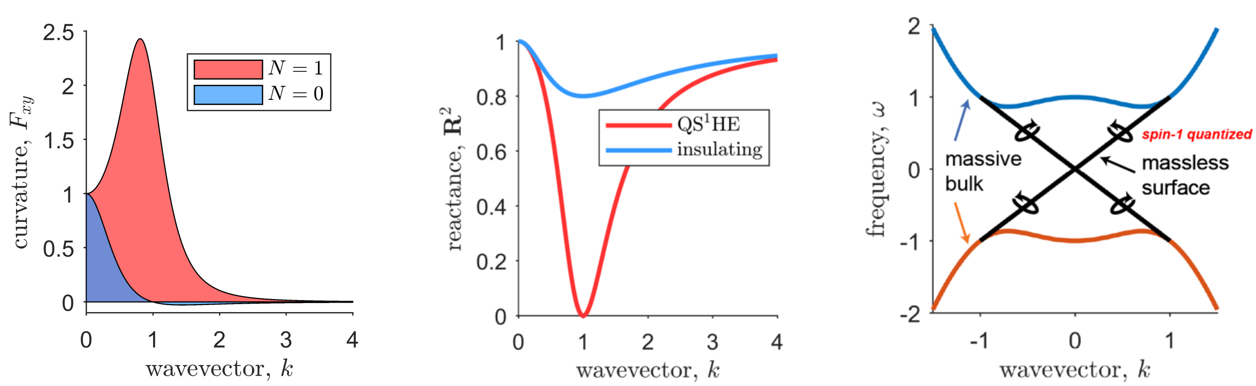

Figure 2. (a) Berry curvature of a trivial $N = 0$ and non-trivial $N = 1$ photonic skyrmion. $C = 2Q_1 N = 2N$ is the corresponding spin-1 Chern number. **(b)** Trivial insulating phase and non-trivial QS^{1}HE phase. The electromagnetic reactance is directly proportional to the effective photonic mass $\mathbf{R}^2 \propto m_{ph}{}^2$. If the reactance/mass passes through zero an odd number of times, the topological phase is non-trivial. **(c)** The linearly dispersing QS^{1}HE edge states $\Psi_\pm$ that emerge from this $m_{ph}(\mathbf{k}) = 0$ point. The states are back-scatter immune, spin-1 quantized $S_\pm = \pm 1$, and can exist at the interface with vacuum. We emphasize that till date, no transport parameters have been quantized in topological photonics, unlike electronics, and our work lays the foundations for this endeavor.

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