Sum-Rate Maximization of Uplink Rate Splitting Multiple Access (RSMA) Communication

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***Abstract*—In this paper, the problem of maximizing sum-rate for uplink rate splitting multiple access (RSMA) communications is studied. In the considered model, each user transmits two messages to the base station (BS) with separate transmit power and the BS will use a successive decoding technique to decode the received messages. To maximize each user's transmission rate, the users must adjust their transmit power and the BS must determine the decoding order of the messages transmitted from the users to the BS. This problem is formulated as a sum-rate maximization problem with proportional rate constraints by adjusting the users' transmit power and the BS's decoding order. However, since the decoding order variable in the optimization problem is discrete, the original minimization problem with transmit power and decoding order variables can be transformed into a problem with only the rate splitting variable. Then, the optimal rate splitting of each user is determined. Given the optimal rate splitting of each user and a decoding order, the optimal transmit power of each user is determined. Next, the optimal decoding order is determined by an exhaustive search method. To further reduce the complexity of the optimization algorithm used for sum-rate maximization in RSMA, a user pairing based algorithm is introduced, which enables two users to use RSMA in each pair and also enables the users in different pairs to be allocated with orthogonal frequency. Simulation results show that RSMA can achieve up to 10.0%, 22.2%, and 83.7% gains in terms of rate compared to non-orthogonal multiple access (NOMA), frequency division multiple access (FDMA), and time division multiple access (TDMA).**

***Index Terms*—Rate splitting multiple access (RSMA), decoding order, power management.**

I. Introduction

Driven by the rapid development of advanced multimedia applications, next-generation wireless networks must support high spectral efficiency and massive connectivity [1]–[3]. In consequence, rate splitting multiple access (RSMA) has been recently proposed as an effective approach to provide more general and robust transmission framework compared to non-orthogonal multiple access (NOMA) [4] and space-division multiple access (SDMA). However, implementing RSMA in wireless networks also faces several challenges [5] such as decoding order design and resource management for message transmission.

Recently, a number of existing works such as in [5]–[11] have studied a number of problems related to the implementation of RSMA in wireless networks. In [5], the authors outlined the opportunities and challenges of using RSMA for multiple input multiple output (MIMO) based wireless networks. The authors in [6] developed an algorithm to optimize the users' sum-rate in downlink RSMA under imperfect channel state information (CSIT). The work in [7] evaluated that RSMA can achieve better performance than NOMA and SDMA. In [8], the application of linearly-precoded rate splitting is studied for multiple input single output (MISO) simultaneous wireless information and power transfer (SWIPT) broadcast channel systems. The authors in [9] investigated the rate splitting-based robust transceiver design problem in a multi-antenna interference channel with SWIPT under the norm-bounded errors of CSIT. The work in [10] developed a transmission scheme that combines rate splitting, common message decoding, clustering and coordinated beamforming so as to maximize the weighted sum-rate of users. Our previous work in [11] investigated the power management and rate splitting scheme to maximize the users sum data rates. However, most of the existing works such as in [5]–[11] studied the use of RSMA over downlink transmission rather than in the uplink. In fact, using RSMA for uplink data transmission can significantly improve the transmission rate. Moreover, none of the existing works in [5]–[11] jointly considered the optimization of power management and message decoding order for uplink RSMA. In essence, message decoding order will affect the transmission rate of the uplink users.

The main contribution of this paper is a novel framework for optimizing power control and message decoding for the users that use RSMA over uplink. Our key contributions include:

- We consider the uplink of a wireless network that uses RSMA, in which each user transmits two messages with different power levels and the base station (BS) uses a successive interference cancellation (SIC) technique to decode the received messages. The power control and decoding order problem is formulated as an optimization problem whose goal is to maximize the sum-rate of all

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users under proportional rate constraints.

- The non-convex sum-rate maximization problem with discrete decoding variable and transmit power variable is first transformed into an equivalent problem with only the rate splitting variable. Then, the optimal solution of the rate splitting is obtained in closed form. Based on the optimal rate splitting of each user, the optimal transmit power can be derived under a given decoding order. Finally, the optimal decoding order is determined by exhaustive search. To reduce the computation complexity, a low-complexity RSMA scheme based on user pairing is proposed.
- Simulation results show that RSMA achieves better performance than NOMA, frequency division multiple access (FDMA), and time division multiple access (TDMA) in terms of sum-rate.

The rest of this paper is organized as follows. The system model and problem formulation are described in Section II. The optimal solution is presented in Section III. Section IV presents a low-complexity sum-rate maximization scheme. Simulation results are analyzed in Section V. Conclusions are drawn in Section VI.

II. System Model and Problem Formulation

Consider a single cell uplink network with one BS serving a set $\mathcal{M}$ of K users using RSMA. In uplink RSMA, each user first transmits a message which is split into two sub-messages in order to dynamically manage interference using rate-splitting. Then, the BS uses a SIC technique to decode the messages of all users [12].

The transmitted message s_k of user $k \in \mathcal{M}$ is given by:

$$s_k = \sum_{j=1}^{2} \sqrt{p_{kj}} s_{kj}, \quad \forall k \in \mathcal{M}, \tag{1}$$

where p_{kj} is the transmit power of message s_{kj} from user k.

The total received message s_0 at the BS can be given by:

$$s_0 = \sum_{k=1}^{K} \sqrt{h_k} s_k + n = \sum_{k=1}^{K} \sum_{j=1}^{2} \sqrt{h_k p_{kj}} s_{kj} + n, \tag{2}$$

where h_k is the channel gain between user k and the BS and n is the additive white Gaussian noise. Each user k has a maximum transmission power limit P_k, i.e., $\sum_{j=1}^{2} p_{kj} \le P_k$.

To decode all messages s_{kj} from the received message s_0, the BS will use SIC. The decoding order at the BS is denoted by a permutation $\boldsymbol{\pi}$. The permutation $\boldsymbol{\pi}$ belongs to Π, which is the set of all possible decoding orders of all $2K$ messages from K users. The decoding order of message s_{kj} from user k is π_{kj}. The achievable rate of decoding message s_{kj} is:

$$r_{kj} = B \log_2 \left(1 + \frac{h_k p_{kj}}{\sum_{\{(l \in \mathcal{K}, m \in \mathcal{J}) | \pi_{lm} > \pi_{kj}\}} h_l p_{lm} + \sigma^2 B} \right), \tag{3}$$

where B is the bandwidth of the BS, σ^2 is the power spectral density of the Gaussian noise. The set $\{(l \in \mathcal{M}, m \in \mathcal{K}) | \pi_{lm} > \pi_{kj}\}$ in (3) represents the messages s_{lm} that are decoded after message s_{kj}.

Since the transmitted message of user k includes messages s_{k1} and s_{k2}, the achievable rate of user k is given by:

$$r_k = \sum_{j=1}^{2} r_{kj}. \tag{4}$$

Our objective is to maximize the sum-rate of all users with proportional rate constraints. Mathematically, the sum-rate maximization problem can be formulated as:

$$\max_{\boldsymbol{\pi}, \boldsymbol{p}} \quad \sum_{k=1}^{K} r_k, \tag{5}$$

$$\text{s.t.} \quad r_1 : r_2 : \cdots : r_K = D_1 : D_2 : \cdots D_K, \tag{5a}$$

$$\sum_{j=1}^{2} p_{kj} \le P_k, \quad \forall k \in \mathcal{M}, \tag{5b}$$

$$\boldsymbol{\pi} \in \Pi, p_{kj} \ge 0, \quad \forall k \in \mathcal{M}, j \in \mathcal{K}, \tag{5c}$$

where $\boldsymbol{p} = [p_{11}, p_{12}, \cdots, p_{K1}, p_{K2}]^T$, r_k is defined in (4), and $\mathcal{K} = \{1, 2\}$. $D_1, \cdots, D_K$ is a set of predetermined nonnegative values that are used to ensure proportional fairness among users. With proper unitization, we set

$$\sum_{k=1}^{K} D_k = 1. \tag{6}$$

The fairness index is defined as

$$\frac{\left(\sum_{k=1}^{K} D_k\right)^2}{K \sum_{k=1}^{K} D_k^2} \tag{7}$$

with the maximum value of 1 to be the greatest fairness case in which all users would achieve the same data rate [13].

Although it was stated in [12] that RSMA can reach the optimal rate region, no practical algorithm was proposed to compute the decoding order and power allocation.

Due to non-linear equality constraint (5a) and discrete variable $\boldsymbol{\pi}$, problem (5) is a non-convex mixed integer problem. Hence, it is generally hard to solve problem (5). Despite the non-convexity and discrete variable, we provide an algorithm to obtain the globally optimal solution to problem (5) in the following section.

III. Optimal Power Control and Decoding Order

In this section, an effective algorithm is proposed to obtain the optimal power control and decoding order of sum-rate maximization problem (5).

A. Optimal Sum-Rate Maximization

Denote τ as the sum-rate of all K users. Introducing a new variable τ, problem (5) can be rewritten as:

$$\max_{\tau, \boldsymbol{\pi}, \boldsymbol{p}} \quad \tau, \tag{8}$$

$$\text{s.t.} \quad r_k = D_k \tau, \quad \forall k \in \mathcal{M}, \tag{8a}$$

$$\sum_{j=1}^{2} p_{kj} \le P_k, \quad \forall k \in \mathcal{M}, \tag{8b}$$

$$\boldsymbol{\pi} \in \Pi, p_{kj} \ge 0, \quad \forall k \in \mathcal{M}, j \in \mathcal{K}, \tag{8c}$$

where τ is the sum-rate of all users since $\tau = \sum_{k=1}^{K} D_k \tau = \sum_{k=1}^{K} r_k$ according to (6) and(8a).

Problem (8) is challenging to solve due to the decoding order variable $\boldsymbol{\pi}$ with discrete value space. To handle this difficulty, we provide the following lemma, which can be used for transforming problem (8) into an equivalent problem without decoding order variable $\boldsymbol{\pi}$.

Lemma 1: In RSMA, under proper decoding power order $\boldsymbol{\pi}$ and splitting power allocation $\boldsymbol{p}$, the optimal rate region can be fully achieved, i.e.,

$$\sum_{k\in\mathcal{K}'} r_k \geq B\log_2\left(1+\frac{\sum_{k\in\mathcal{K}'} h_k P_k}{\sigma^2 B}\right), \forall \mathcal{K}' \subseteq \mathcal{K}\setminus\emptyset, \quad (9)$$

where $\emptyset$ is an empty set and $\mathcal{K}' \subseteq \mathcal{K}\setminus\emptyset$ means that $\mathcal{K}'$ is a non-empty subset of $\mathcal{K}$.

Lemma 1 follows directly from [12, Theorem 1]. Based on Lemma 1, we can use the rate variable to replace the power and decoding variables. In consequence, problem (8) can be equivalently transformed to

$$\max_{\tau,\boldsymbol{r}} \quad \tau, \quad (10)$$

$$\text{s.t.} \quad r_k = D_k\tau, \quad \forall k \in \mathcal{K}, \quad (10a)$$

$$\sum_{k\in\mathcal{K}'} r_k \geq B\log_2\left(1+\frac{\sum_{k\in\mathcal{K}'} h_k P_k}{\sigma^2 B}\right), \forall \mathcal{K}' \subseteq \mathcal{K}\setminus\emptyset, \quad (10b)$$

where $\boldsymbol{r}=[r_1, r_2, \cdots, r_K]^T$. In problem (10), the dimension of the variable is smaller than that in problem (8). Moreover, the discrete decoding order variable is replaced by rate variable in problem (10). Regarding the optimal solution of problem (10), we have the following lemma.

Lemma 2: For the optimal solution $(\tau^*, \boldsymbol{r}^*)$ of problem (10), there exists at least one $\mathcal{K}' \subseteq \mathcal{K}\setminus\emptyset$ such that $\sum_{k\in\mathcal{K}'} r_k^* = B\log_2\left(1+\frac{\sum_{k\in\mathcal{K}'} h_k P_k}{\sigma^2 B}\right)$.

Proof: See Appendix A. □

Theorem 1: The optimal solution of problem (10) is

$$\tau^* = \min_{\mathcal{K}'\subseteq\mathcal{K}\setminus\emptyset} \frac{B\log_2\left(1+\frac{\sum_{k\in\mathcal{K}'} h_k P_k}{\sigma^2 B}\right)}{\sum_{k\in\mathcal{K}'} D_k}, r_k^* = D_k\tau^*, \ \forall k \in \mathcal{K} \quad (11)$$

Proof: See Appendix B. ■

From (11), one can directly obtain the optimal sum-rate of problem (10) in closed form, which can be helpful in characterizing the rate performance of RSMA.

Having obtained the optimal solution $(\tau^*, \boldsymbol{r}^*)$ of problem (10), we still need to calculate the optimal $(\boldsymbol{\pi}^*, \boldsymbol{p}^*)$ of the original problem (8). Next, we introduce a new algorithm to obtain the optimal $(\boldsymbol{\pi}^*, \boldsymbol{p}^*)$ of problem (8).

Substituting the optimal solution $(\tau^*, \boldsymbol{r}^*)$ of problem (10) into problem (8), we can obtain the following feasibility problem:

$$\text{find } \boldsymbol{\pi}, \boldsymbol{p} \quad (12)$$

$$\text{s.t.} \sum_{j=1}^{2} B\log_2\left(1+\frac{h_k p_{kj}}{\sum_{\{(l\in\mathcal{K},m\in\mathcal{J})|\pi_{lm}>\pi_{kj}\}} h_l p_{lm}+\sigma^2 B}\right) = r_k^*, \forall k \in \mathcal{K}, \quad (12a)$$

$$\sum_{j=1}^{2} p_{kj} \geq P_k, \quad \forall k \in \mathcal{K}, \quad (12b)$$

$$\boldsymbol{\pi} \in \Pi, p_{kj} \geq 0, \quad \forall k \in \mathcal{K}, j \in \mathcal{J}. \quad (12c)$$

Due to the decoding order constraint (12c), it is challenging to find the optimal solution of problem (12). To solve this problem, we first fix the decoding order $\boldsymbol{\pi}$ to obtain the power allocation and then exhaustively search $\boldsymbol{\pi}$. Given decoding order $\boldsymbol{\pi}$, problem (12) can be simplified as:

$$\text{find } \boldsymbol{p} \quad (13)$$

$$\text{s.t.} \sum_{j=1}^{2} B\log_2\left(1+\frac{h_k p_{kj}}{\sum_{\{(l\in\mathcal{K},m\in\mathcal{J})|\pi_{lm}>\pi_{kj}\}} h_l p_{lm}+\sigma^2 B}\right) \geq r_k^*, \forall k \in \mathcal{K}, \quad (13a)$$

$$\sum_{j=1}^{2} p_{kj} \geq P_k, \quad \forall k \in \mathcal{K}, \quad (13b)$$

$$p_{kj} \geq 0, \quad \forall k \in \mathcal{K}, j \in \mathcal{J}. \quad (13c)$$

Note that the equality in (12a) is replaced by the inequality in (13a). The reason is that any feasible solution to problem (12) is also feasible to problem (13). Meanwhile, for a feasible solution to problem (13), we can always construct a feasible solution to problem (12).

To verify the feasibility of problem (13), we can construct the following problem by introducing a new variable α:

$$\max_{\alpha,\boldsymbol{p}} \quad \alpha \quad (14)$$

$$\text{s.t.} \sum_{j=1}^{2} B\log_2\left(1+\frac{h_k p_{kj}}{\sum_{\{(l\in\mathcal{K},m\in\mathcal{J})|\pi_{lm}>\pi_{kj}\}} h_l p_{lm}+\sigma^2 B}\right) \geq \alpha r_k^*, \forall k \in \mathcal{K}, \quad (14a)$$

$$\sum_{j=1}^{2} p_{kj} \geq P_k, \quad \forall k \in \mathcal{K}, \quad (14b)$$

$$p_{kj}, \alpha \geq 0, \quad \forall k \in \mathcal{K}, j \in \mathcal{J}. \quad (14c)$$

To show the equivalence of problems (13) and (14), we provide the following lemma.

Proposition 1: Problem (13) is feasible if and only if the optimal objective value α^* of problem (14) is equal or larger than 1.

Proof: See Appendix C. □

Problem (14) is non-convex due to constraints (14a). To handle this non-convexity of (14), we adopt the difference of two convex function (DC) method, using which a non-convex problem can be solved suboptimally by converting a non-convex problem into convex subproblems. In order to obtain a near globally optimal solution of problem (14), we can try multiple initial points $(\alpha, \boldsymbol{p})$, which can lead to multiple locally optimal solutions. Thus, a near globally optimal solution can be obtained by choosing the locally optimal solution with the highest objective value among all locally optimal solutions. To construct an initial feasible point, we first arbitrarily generate $\boldsymbol{p}$ that satisfies (14b) and (14c), and then we set:

$$\alpha = \max_{k\in\mathcal{K}} \frac{\sum_{j=1}^{2} B\log_2\left(1+\frac{h_k p_{kj}}{\sum_{\{(l\in\mathcal{K},m\in\mathcal{J})|\pi_{lm}>\pi_{kj}\}} h_l p_{lm}+\sigma^2 B}\right)}{r_k^*}. \quad (15)$$

By using the DC method, the left hand side of (14a) satisfies:

Algorithm 1 Near Optimal Sum-Rate Maximization

1: Obtain the optimal solution $(\tau^*, \boldsymbol{r}^*)$ of problem (10) according to Theorem 1.
2: **for** $\boldsymbol{\pi} \in \Pi$ **do**
3:   **for** $1:1:N$ **do**
4:     Arbitrarily generate a feasible solution $(\alpha^{(0)}, \boldsymbol{p}^{(0)})$ of problem (14), and set $n=0$.
5:     **repeat**
6:       Obtain the optimal solution $(\alpha^{(n+1)}, \boldsymbol{p}^{(n+1)})$ of convex problem (14) by replacing the left term of constraints (14a) with $r_{k,\text{lb}}(\boldsymbol{p}, \boldsymbol{p}^{(n)})$.
7:       Set $n = n+1$.
8:     **until** the objective function (14a) converges
9:   **end for**
10:   Obtain the optimal solution $(\alpha^*, \boldsymbol{p}^*)$ of problem (14) with the highest objective value.
11:   If $\alpha^* \geq 1$, break and jump to step 13.
12: **end for**
13: Obtain the optimal decoding order $\boldsymbol{\pi}^* = \boldsymbol{\pi}$ and power allocation $\boldsymbol{p}^*$ of problem (12).

$$\sum_{j=1}^{2} B\log_2\left(1+\frac{h_k p_{kj}}{\sum_{\{(l\in\mathcal{K},m\in\mathcal{J})|\pi_{lm}>\pi_{kj}\}} h_l p_{lm}+\sigma^2 B}\right)$$

$$\geq \sum_{j=1}^{2} B\log_2\left(\sum_{\{(l\in\mathcal{K},m\in\mathcal{J})|\pi_{lm}\geq\pi_{kj}\}} h_l p_{lm}+\sigma^2 B\right) - \sum_{j=1}^{2} B\log_2\left(\sum_{\{(l\in\mathcal{K},m\in\mathcal{J})|\pi_{lm}>\pi_{kj}\}} h_l p_{lm}^{(n)}+\sigma^2 B\right) - \sum_{j=1}^{2} B\frac{\sum_{\{(l\in\mathcal{K},m\in\mathcal{J})|\pi_{lm}>\pi_{kj}\}} h_l (p_{lm}-p_{lm}^{(n)})}{(\ln 2)\sum_{\{(l\in\mathcal{K},m\in\mathcal{J})|\pi_{lm}>\pi_{kj}\}} h_l p_{lm}^{(n)}+\sigma^2 B}$$

$$\triangleq r_{k,\text{lb}}(\boldsymbol{p}, \boldsymbol{p}^{(n)}),$$

where $p_{lm}^{(n)}$ stands for the value of p_{lm} in iteration n, and the inequality follows from the fact that $\log_2(x)$ is a concave function and a concave function is always no greater than its first-order approximation. By substituting the left term of constraints (14a) with the concave function $r_{k,\text{lb}}(\boldsymbol{p}, \boldsymbol{p}^{(n)})$, problem (14) becomes convex, and can be effectively solved by the interior point method [14].

The optimal sum-rate maximization algorithm for RSMA is provided in Algorithm 1, where N is the number of initial points to obtain a nearly global optimal solution of non-convex problem (14).

B. Complexity Analysis

In Algorithm 1, the major complexity lies in solving problem (10) and problem (12). To solve (10), from Theorem 1, the complexity is $\mathcal{O}(2^K-1)$ since the set $\mathcal{M}$ has 2^K-1 non-empty subsets. According to steps 2-12, a near globally optimal solution of problem (12) is obtained via solving a series of convex problems with different initial points and decoding order strategies. Considering that the dimension of variables in problem (14) is $1+2K$, the complexity of solving convex problem in step 6 by using the standard interior point method is $\mathcal{O}(K^3)$ [14, Pages 487, 569]. Since the network consists of K users and each user is split into two virtual users (there are $2K$ virtual users in total), the decoding order set Π consists of $(2K)!/2^K$ elements. Given N initial points, the total complexity of solving problem (12) is $\mathcal{O}(NK^3(2K)!/2^K)$. As a result, the total complexity of Algorithm 1 is $\mathcal{O}(2^K + NK^3(2K)!/2^K)$.

IV. Low-Complexity Sum-Rate Maximization

According to Section III-A, the computation complexity of sum-rate maximization for RSMA is extremely high. In this section, we propose a low-complexity scheme for RSMA, where users are classified into different pairs[1] and each pair consists of two users. RSMA is used in each pair and different pairs are allocated with different frequency band. Assume that K users are classified into M pairs, i.e., $K=2M$. The set of all pairs is denoted by $\mathcal{O}$. The users in pair m are denoted by u_{m1} and u_{m2}.

For pair m, the allocated fraction of bandwidth is denoted by f_m. Let c_{m1} and c_{m2} respectively denote the data rate of users 1 and 2 in pair m. According to Lemma 1, we have:

$$c_{mj} \leq Bf_m \log_2\left(1+\frac{h_{mj}P_{mj}}{\sigma^2 Bf_m}\right), \quad \forall m \in \mathcal{O}, \forall j \in \mathcal{K}, \tag{16}$$

$$c_{m1}+c_{m2} \leq Bf_m \log_2\left(1+\frac{h_{m1}P_{m1}+h_{m2}P_{m2}}{\sigma^2 Bf_m}\right), \quad \forall m \in \mathcal{O}, \tag{17}$$

where h_{mj} denotes the channel gain between user j in pair m and the BS, and P_{mj} is the maximal transmission power of user j in pair m.

Similar to (8), the sum-rate maximization problem for RSMA with user pairing can be formulated as:

$$\max_{\tau,\boldsymbol{f},\boldsymbol{r}} \quad \sum_{m=1}^{M}\sum_{j=1}^{2} c_{mj}, \tag{18}$$

$$\text{s.t.} \quad c_{11}:c_{12}:\cdots:c_{M2} = D_{11}:D_{12}:\cdots:D_{M2} \tag{18a}$$

$$\sum_{m=1}^{M} f_m = 1, \tag{18b}$$

$$(16), (17), f_m, r_{m1}, r_{m2} \geq 0, \quad \forall m \in \mathcal{O}, \tag{18c}$$

where $\boldsymbol{f} = [f_1, f_2, \cdots, f_M]^T$, $\boldsymbol{c} = [c_{11}, c_{12}, \cdots, c_{M1}, c_{M2}]^T$, and $D_{11}, D_{12}, \cdots, D_{M1}, D_{M2}$ is a set of predetermined non-negative values that are used to ensure proportional fairness among users with $\sum_{m=1}^{M}\sum_{j=1}^{2} D_{mj} = 1$.

Similar to (8), introducing a new variable τ, problem (18) can be rewritten as:

$$\max_{\tau,\boldsymbol{f},\boldsymbol{r}} \quad \tau, \tag{19}$$

$$\text{s.t.} \quad c_{mj} = D_{mj}\tau, \quad \forall m \in \mathcal{O}, j \in \mathcal{K}, \tag{19a}$$

$$\sum_{m=1}^{M} f_m = 1, \tag{19b}$$

$$(16), (17), f_m, r_{m1}, r_{m2} \geq 0, \quad \forall m \in \mathcal{O}. \tag{19c}$$

To solve problem (19) with objective function τ, we utilize the bisection method. For each given τ, we solve a feasibility problem with constraints (19a)-(19c). With given τ, the

[1]In this paper, we assume that the user pairing is given.

Algorithm 2 : Low-Complexity Sum-Rate Maximization

1: Initialize $\tau_{\min}=0$, $\tau_{\max}=\tau^*$, and the tolerance ϵ.
2: Set $\tau=\frac{\tau_{\min}+\tau_{\max}}{2}$, and calculate f_{m1}, f_{m2} and f_{m3} according to (24) and (25), respectively.
3: Check the feasibility condition (3). If problem (20) is feasible, set $\tau_{\min}=\tau$. Otherwise, set $\tau=\tau_{\max}$.
4: If $(\tau_{\max}-\tau_{\min})/\tau_{\max}\leq\epsilon$, terminate. Otherwise, go to step 2.

feasibility problem of (19) becomes

$$\text{find} \quad \boldsymbol{f}, \boldsymbol{r}, \tag{20}$$

$$\text{s.t.} \quad (19a) \quad (19c). \tag{20a}$$

Combing (19a) and (19c), we have:

$$D_{mj}\tau \geq B f_m \log_2\left(1+\frac{h_{mj}P_{mj}}{\sigma^2 B f_m}\right), \quad j \forall \mathcal{K}, \tag{21}$$

$$(D_{m1}+D_{m2})\tau \geq B f_m \log_2\left(1+\frac{h_{m1}P_{m1}+h_{m2}P_{m2}}{\sigma^2 B f_m}\right). \tag{22}$$

It can be proved that $g(x)=x\ln\left(1+\frac{1}{x}\right)$ is a monotonically increasing function. Thus, to satisfy (21) and (22), bandwidth fraction f_m should satisfy:

$$f_m \in \max\{f_{m1}, f_{m2}, f_{m3}\}, \tag{23}$$

where the expressions of f_{m1}, f_{m2}, f_{m3} are given in (24) and (25) at the top of this page.

Based on (23) and (19b), we have:

$$\sum_{m=1}^{M} \max\{f_{m1}, f_{m2}, f_{m3}\} \geq 1. \tag{26}$$

According to (21)-(3), problem (20) is feasible if and only if (3) is satisfied. As a result, the algorithm for obtaining the maximum sum-rate of problem (20) is summarized in Algorithm 2, where τ^* is the optimal sum-rate of (10).

The complexity of the proposed Algorithm 2 in each step lies in checking the feasibility of problem (20), which involves the complexity of $\mathcal{O}(M)$ according to (24)-(25). As a result, the total complexity of the proposed Algorithm 2 is $\mathcal{O}(M\log_2(1/\eta))$, where $\mathcal{O}(\log_2(1/\eta))$ is the complexity of the bisection method with accuracy η.

V. Numerical Results

For our simulations, we deploy K users uniformly in a square area of size 500 m $\times$ 500 m with the BS located at its center. The path loss model is $128.1+37.6\log_{10} d$ (d is in km) and the standard deviation of shadow fading is 8 dB. In addition, and the noise power spectral density is $\sigma^2=-174$ dBm/Hz. We chose an equal maximum transmit power $P_1=\cdots=P_K=1$ dBm, and a bandwidth $B=1$ MHz. All statistical results are averaged over a large number of independent runs.

We compare the sum-rate performance of RSMA with NOMA [15], FDMA [16], and TDMA [17]. An example of the rate region for multiple access schemes is shown in Fig. 1. It should be noted that the channel gain of user 1 is larger than that of user 2 and the BS always first decodes the message of user 1 in uplink NOMA. From Fig 1, it is observed that RSMA has the largest rate region, while TDMA has the smallest rate region.

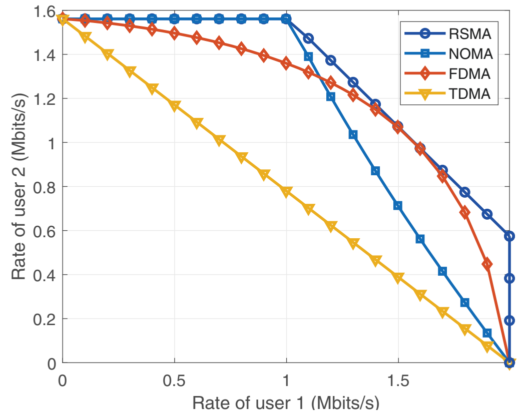

Fig. 1. An example of the rate region for RSMA, NOMA, FDMA, and TDMA.

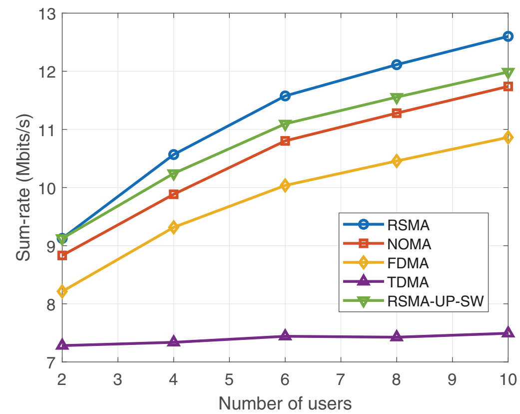

Fig. 2. Sum-rate versus number of users with equal rate allocation parameter $D_1=\cdots=D_K=1/K$.

For the low-complexity RSMA with user pairing, we choose the strong-weak (SW) pair method [15] (labeled as 'RSMA-UP-SW'), where the user with the strongest channel condition is paired with the user with the weakest in one pair, and the user with the second strongest is paired with one with the second weakest in one pair, and so on.

Fig. 2 shows how the sum-rate changes with the number of users. Clearly, the proposed RSMA or RSMA-UP-SW is always better than NOMA, FDMA, and TDMA especially when the number of users is large. In particular, RSMA can increase up to 10.0%, 22.2% and 83.7% sum-rate compared to NOMA, FDMA, and TDMA, respectively, while RSMA-UP-SW can improve up to 4.1%, 11.6% and 66.8% sum-rate compared to NOMA, FDMA, and TDMA, respectively. When the number of users is large, the multiuser gain is more apparent by the proposed RSMA compared to conventional NOMA, FDMA, and TDMA. This is because RSMA can effectively determine the power splitting of each user to achieve the theoretically maximal rate region, while there is no power splitting in NOMA and the allocated bandwidth/time of each user is low for FDMA/TDMA when the number of users is large. It is also found that RSMA-UP-SW achieves better performance in terms of sum-rate than NOMA, FDMA, and TDMA but with low complexity according to Section IV.

VI. Conclusion

In this paper, we have investigated the decoder order and power optimization in an uplink RSMA system. We have for-

$$f_{mk} = \frac{(\ln 2) D_{mk} h_{mk} P_{mk}}{B h_{mk} P_{mk} \tau W \left) \frac{(\ln 2) D_{mk} \sigma^2}{h_{mk} P_{mk} \tau} \mathrm{e}^{-\frac{(\ln 2) D_{mk} \sigma^2}{h_{mk} P_{mk} \tau}} \right(+ (\ln 2) D_{mk} \sigma^2 B}, \quad k = 1, 2, \tag{24}$$

$$f_{m3} = \frac{(\ln 2)(D_{m1} + D_{m2})(h_{m1} P_{m1} + h_{m2} P_{m2})}{B(h_{m1} P_{m1} + h_{m2} P_{m2}) \tau W \left) \frac{(\ln 2)(D_{m1}+D_{m2})\sigma^2}{(h_{m1} P_{m1} + h_{m2} P_{m2})\tau} \mathrm{e}^{-\frac{(\ln 2)(D_{m1}+D_{m2})\sigma^2}{(h_{m1} P_{m1} + h_{m2} P_{m2})\tau}} \right(+ (\ln 2)(D_{m1} + D_{m2}) \sigma^2 B}. \tag{25}$$

mulated the problem as a sum-rate maximization problem. To solve this problem, we have transformed it into an equivalent problem with only rate splitting variables, which has closed-form optimal solution. Given the optimal rate requirement of each user, the optimal transmit power of each user is obtained under given the decoding order and the optimal decoding order is found by an exhaustive search method. To reduce the computation complexity, we have proposed a low-complexity RSMA with user pairing. Simulation results show that RSMA achieves higher sum-rate than NOMA, FDMA, and TDMA.

APPENDIX A
PROOF OF LEMMA 2

Assume that for the optimal solution $(\tau^*, \boldsymbol{r}^*)$ of problem (10), we have $\sum_{k\in\mathcal{K}'} r_k^* < B \log_2 \left) 1 + \frac{\sum_{k\in\mathcal{K}'} h_k P_k}{\sigma^2 B} \right(, \emptyset \mathcal{M} \leq \mathcal{M}\ \mathcal{J}$. In this case, we can construct a new rate solution $\boldsymbol{r}' = [r_1', \times\times\times, r_K']$ with $r_k' = \epsilon r_k^*$ and

$$\epsilon = \min_{\mathcal{K}' \subseteq \mathcal{K} \setminus \emptyset} \frac{B \log_2 \left) 1 + \frac{\sum_{k\in\mathcal{K}'} h_k P_k}{\sigma^2 B} \right(}{\sum_{k\in\mathcal{K}'} r_k^*} > 1. \tag{A.1}$$

According to (A.1), we can show that

$$\sqrt{}_{k\in\mathcal{K}'} r_k' \geq B \log_2 \left) 1 + \frac{\sum_{k\in\mathcal{K}'} h_k P_k}{\sigma^2 B} \right(, \emptyset \mathcal{M} \leq \mathcal{M}\ \mathcal{J}, \tag{A.2}$$

which ensures that $\boldsymbol{r}'$ satisfies constraints (10b).

Based on (10a), we have $\tau^* = \frac{r_k^*}{D_k}, \emptyset k \,\forall\, \mathcal{M}$ We set τ' as

$$\tau' = \frac{r_k'}{D_k} = \frac{\epsilon r_k^*}{D_k} > \tau^*. \tag{A.3}$$

According to (A.2) and (A.3), we can see that new solution $(\tau', \boldsymbol{r}')$ is feasible and the objective value (10) of new solution is better than that of solution $(\tau^*, \boldsymbol{r}^*)$, which contradicts the fact that $(\tau^*, \boldsymbol{r}^*)$ is the optimal solution. Lemma 2 is proved.

APPENDIX B
PROOF OF THEOREM 1

According to Lemma 2, there exists at least one $\mathcal{M} \leq \mathcal{M}\ \mathcal{J}$ such that $\sum_{k\in\mathcal{K}'} r_k^* = B \log_2 \left) 1 + \frac{\sum_{k\in\mathcal{K}'} h_k P_k}{\sigma^2 B} \right($. To ensure the feasibility of (10b), the optimal τ^* is given by (11). Then, according to (10a), the optimal r_k^* is determined as in (11).

APPENDIX C
PROOF OF PROPOSITION 1

(1) On one side, if $\boldsymbol{p}$ is a feasible solution of problem (13), we can show that $(\alpha = 1, \boldsymbol{p})$ is a feasible solution of problem (14), which indicates that the optimal objective value of problem (14) should be larger or equal to than 1. (2) On the other side, if the optimal solution $(\alpha^*, \boldsymbol{p}^*)$ of problem (14) satisfies $\alpha^* \in 1$, we can show that $\boldsymbol{p}^*$ is a feasible solution of problem (13).

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