Non-Hermitian Nonlinear Optics without Gain and Loss

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Abstract: We propose and demonstrate non-Hermitian but parity-time-symmetric four-wave mixing in cold atoms without linear gain and loss. Besides the occurrence of nontrivial phase transition, efficient nonlinear conversion without phase matching has also been observed.

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1. Introduction

Symmetries, as the fundamental properties of nature, play an essential role in our understanding of the universe. Yet, their studies in the fields of optics have been much limited. Only recently, the notion of parity-time (PT) symmetry [1-3] has started to attract significant attention owing to its potential for novel optical effects that are unattainable with usual Hermitian systems. However, most studies [1-3] insofar have been centered on optical structures with spatially separated gain and loss distributions in order to create PT-symmetric potentials for a propagating classical electromagnetic field. As a result, this confines nearly all the demonstrations to either linear optics or single-mode nonlinear dynamics. Moreover, fabricating such compound photonic structures turns out to be technically challenging in part to physical limitation of gain materials plus the codependency between the real and imaginary parts of refractive-index landscapes ruled by Kramers-Kronig relations. Furthermore, the application of gain and loss makes the existence of any PT-symmetric photonic quantum system questionable. It is thus intriguing to know whether alternative scenarios are available to fulfill PT-symmetric Hamiltonians but without linear optical gain and loss. In this regard, here we would show that nonlinear optics can become a fertile ground for a new generation of PT symmetry for different harmonics of interest by employing the interplay between nonlinearity and non-Hermiticity. Thanks to recent technical advances in 2D optical trapping, here in forward four-wave mixing (FFWM) we have successfully observed PT phase transition as well as remarkable nonlinear conversion between paired Stokes-anti-Stokes modes with a very large phase-mismatch Δk simply through dynamical variation of atomic density (i.e. third-order nonlinearity) [4]. This is in sharp contrast to common wisdom where FWM [5] is very phase-sensitive process in that the efficiency is strongly affected by phase matching conditions.

2. Theory and Experiment

As schematically shown in Fig. 1a, FWM occurs in an atomic ensemble with double-Λ four-level configuration. A strong pump laser (angular frequency ω_p) is blue detuned by Δ_p from the atomic transition $|1\rangle \rightarrow |4\rangle$ and a weak Stokes field (ω_s) follows $|4\rangle \rightarrow |2\rangle$. Another strong coupling laser (ω_c) is on resonance to $|2\rangle \rightarrow |3\rangle$ and renders the atomic medium transparency for the weak anti-Stokes field (ω_{as}), which is on the transition $|3\rangle \rightarrow |1\rangle$. We adopt a large Δ_p so that the atomic population primarily remains on the lower ground state $|1\rangle$. We work under the two-photon resonance condition ω_p-ω_s=ω_{as}-ω_c=ω_{12}, where ω_{12} is the frequency difference between two ground states $|1\rangle$ and $|2\rangle$. We next carefully choose the pump power to make the linear Raman gain on the Stokes field neglectable. With zero dephasing rate (γ_{12}=0) between $|1\rangle$ and $|2\rangle$, perfect electromagnetically induced transparency (EIT) caused by the coupling light makes the anti-Stokes propagation lossless. The optical spatial geometry of the FFWM is illustrated in Fig. 1b, where the anti-Stokes and Stokes modes propagate along the +z axis, with the pump and coupling beams aligned with an angle ±θ=0.4° to the z axis. The evolution of the Stokes and anti-Stokes field amplitudes (E_s and E_{as}), under slowly varying envelope approximations, is described by a Schrödinger-like equation $i\frac{d}{dz}\begin{pmatrix}E_{as}\\E_s\end{pmatrix} = H\begin{pmatrix}E_{as}\\E_s\end{pmatrix}$, with $H = -\frac{\Delta k}{2}\mathcal{I} - \mathcal{PT}\kappa$. Here, $\mathcal{I}$ represents a 2×2 identity operator, $\mathcal{P} = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ is the parity operator, the antilinear time operator $\mathcal{T}$ demands a complex conjugation operation, the real phase mismatch takes the form of $\Delta k = k_{as} + k_s - (k_p + k_c)\cos\theta$ for four involved wavenumbers k_j ($j = s, as, p, c$) in vacuum, and κ stands for the (real) nonlinear coupling coefficient. Interestingly, the Hamiltonian *H* commutes with the joint $\mathcal{PT}$ operator, $[H, \mathcal{PT}] = 0$, the necessary condition of PT symmetry [1]. As a result, one anticipates *H* to permit real eigenvalues. To verify this, we first implement complex conjugate operation and then rewrite the above equation as

$i\frac{d}{dz}\begin{pmatrix}E_{as}\\ E_s^*\end{pmatrix} = \mathcal{H}\begin{pmatrix}E_{as}\\ E_s^*\end{pmatrix}$, where $\mathcal{H} = \begin{pmatrix}-\frac{\Delta k}{2} & -\kappa\\ \kappa^* & \frac{\Delta k}{2}\end{pmatrix}$. Its pair of eigen-propagation constants are $\lambda_{\pm} = \pm\frac{\Delta k}{2}\sqrt{1 - \delta^2}$, where $\delta = \frac{|\kappa|}{\Delta k/2}$ is real. The physical meaning of $\lambda_{\pm}$ is following: the real eigenvalues, resembling the phase constant, define the rate at which the phase changes as the wave propagates; whereas the imaginary ones, referring to as the attenuation (or amplification) constant, characterize the rate at which the fields of the waves are attenuated (or amplified) as they propagate through the medium. The solution can be accordingly expressed as linear superposition of exponential functions of the form $e^{\pm i\lambda z}$. Unlike the perfect phase-matching case ($\Delta k \sim 0$), Stokes and anti-Stokes here experience a striking phase transition at the EP when $\delta=1$ (or $|\kappa|=\Delta k/2$). Detailed calculations show that in the PT-symmetric phase ($\delta<1$) with real eigenvalues, the Stokes and anti-Stokes output powers are bounded and oscillate coherently between two extreme values. When $\delta>1$, however, due to spontaneous PT-symmetry breaking, the two eigenvalues cease to be real and form a complex conjugate pair. Consequently, in this range, one eigenstate undertakes gain while the other vanishes after some propagation distance. Alternatively, in the PT-broken phase, the system instead behaves as optical parametric amplification. Physically, this net gain is a result of coherent power transfer from the input pump and coupling lasers to the Stokes and anti-Stokes modes. Our experiment (Fig. 1c) has confirmed these predictions. For example, Fig. 2 shows the characteristics of real and imaginary parts of eigen-propagation constants of paired Stokes-anti-Stokes supermodes as a function of δ and optical depth (OD) for two different seeding operation. Fig. 3 shows the evolution of PT supermodes in terms of normalized output intensities as the function of δ and optical depth (OD) under different seeding, where the transition from optical parametric oscillation to amplification can be easily identified.

Aside from these compelling properties, of importance, we remark here that distinct from gain-loss PT symmetry, the current scheme is suitable for quantum exploration [6]. Moreover, the demonstrated PT symmetry is the result of simultaneous interaction of two fields where one is produced from vacuum. Given its tight connection with nonlinear optics, quantum optics and laser science, we expect new directions and applications may be opened up towards the region that is fully neglected or unwanted in the past.

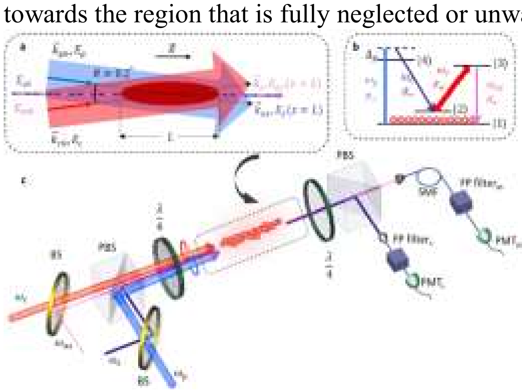

Fig. 1 Experimental setup for PT-symmetric FWM without gain and loss in 2D MOT of ^{85}Rb.

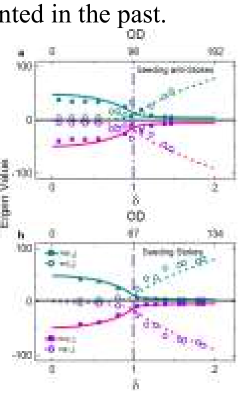

Fig. 2 Characteristics of eigen-propagation constants.

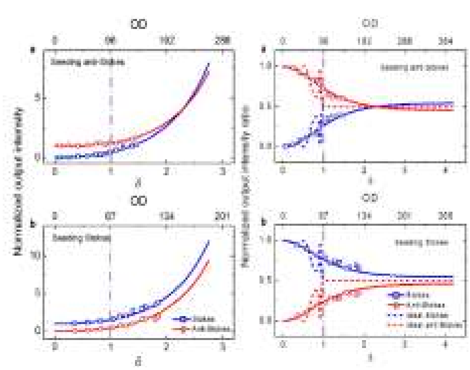

Fig. 3 Evolution of PT supermodes transiting from OPO to OPA.

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