DISTRIBUTED MULTIPLE GAUSSIAN FILTERING FOR MULTIPLE TARGET LOCALIZATION IN WIRELESS SENSOR NETWORKS

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ABSTRACT

Indoor target tracking appears in several engineering problems and is a key enabler to a myriad of new applications. Localization in such global navigation satellite system (GNSS)-denied environments typically relies on the use of existing infrastructures and already deployed technologies. In this paper, we are interested in received signal strength (RSS)-based multiple target tracking (MTT) in wireless sensor networks (WSN). From an estimation standpoint, two problems arise: i) standard Bayesian filtering techniques are not able to cope with high-dimensional systems, and ii) WSN are typically built with resource-constrained low-cost sensors, which implies the need for distributed algorithms. A possible solution is to use a multiple Bayesian filtering approach, where the state-space is partitioned in several lower dimensional subspaces, and then a set of parallel filters are used to characterize the marginal subspace posteriors. In this work, we propose a new distributed multiple Gaussian filtering (MGF) formulation, to solve both the curse-of-dimensionality in high-dimensional systems and the need of distributed algorithms in network localization applications.

***Index Terms*—** Network localization, distributed Gaussian filtering, multiple target tracking, state partitioning.

1. INTRODUCTION

It is foreseen that a large number of sensors will be available in the context of the Internet-of-Things (IoT). The pervasiveness of such technology will enable more accurate and widespread localization applications in a variety of disciplines such as smart cities, smart grids, and intelligent transportation systems [1]. Particularly, this relates to topics in indoor tracking, multiple target tracking (MTT), and network localization [2]. In this paper, we focus on decentralized approaches to those challenges. Given that the number of sensing devices is expected to be large in the IoT context, there is a clear need to develop new methodology that is scalable and does not collapse due to the increase in dimensionality of the network and system. Additionally, decentralized networks of sensors have the feature of being inherently robust to node failures, due to the redundancy in a dense network. Finally, decentralized approaches require cost-efficient algorithms whose implementation does not involve high computational complexity.

In the context of Bayesian filtering, a well-known problem is the curse-of-dimensionality, that is, the computational complexity increase and associated performance degradation in high-dimensional systems [3,4]. Among the possible solutions, a promising approach is the multiple state-partitioning framework [5], where the state-space is partitioned in several lower dimensional subspaces, and then a set of parallel filters are used to characterize the marginal subspace posteriors. This has been applied to both particle filters [5,6] and sigma-point Gaussian filters [7–9], the latter named multiple Gaussian filters (MGFs). But the formal derivation of the distributed multiple Bayesian filtering is still an open problem.

In this article we propose a new distributed MGF formulation, to solve both the curse-of-dimensionality in high-dimensional systems, and the need of distributed algorithms in network localization applications.

2. STATE-SPACE FORMULATION

The general nonlinear Gaussian state-space model (SSM) of interest can be written as

$$\mathbf{x}_t = \mathbf{f}_{t-1}(\mathbf{x}_{t-1}) + \boldsymbol{\nu}_{k-1}\,, \qquad \boldsymbol{\nu}_{t-1} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_{t-1}), \qquad (1)$$

$$\mathbf{y}_t = \mathbf{h}_t(\mathbf{x}_t) + \mathbf{n}_t\,, \qquad \mathbf{n}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t), \qquad (2)$$

where $\mathbf{x}_t \in \mathbb{R}^{n_x}$ and $\mathbf{y}_t \in \mathbb{R}^{n_y}$ are the hidden state of the system and the measurements at time t, respectively; $\mathbf{f}_{t-1}(\cdot)$ and

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$\mathbf{h}_t(\cdot)$ are known (possibly nonlinear) functions of the state; and the white Gaussian sequences $\boldsymbol{\nu}_{t-1}$ and $\mathbf{n}_t$ are assumed to be independent. The system in (1) is assumed to be separable into S non-overlapping subspaces as $\mathbf{x}_k = [\mathbf{x}_k^{(1)}, \ldots, \mathbf{x}_k^{(S)}]$ such that $\mathbf{Q}_{k-1} = \text{diag}(\mathbf{Q}_{k-1}^{(1)}, \ldots, \mathbf{Q}_{k-1}^{(S)})$,[1]

$$\begin{pmatrix} \mathbf{x}_t^{(1)} \\ \mathbf{x}_t^{(2)} \\ \vdots \\ \mathbf{x}_t^{(S)} \end{pmatrix} = \begin{pmatrix} \mathbf{f}_{t-1}^{(1)}(\mathbf{x}_{t-1}^{(1)}, \mathbf{x}_{t-1}^{(-1)}) \\ \mathbf{f}_{t-1}^{(2)}(\mathbf{x}_{t-1}^{(2)}, \mathbf{x}_{t-1}^{(-2)}) \\ \vdots \\ \mathbf{f}_{t-1}^{(S)}(\mathbf{x}_{t-1}^{(S)}, \mathbf{x}_{t-1}^{(-S)}) \end{pmatrix} + \begin{pmatrix} \boldsymbol{\nu}_{t-1}^{(1)} \\ \boldsymbol{\nu}_{t-1}^{(2)} \\ \vdots \\ \boldsymbol{\nu}_{t-1}^{(S)} \end{pmatrix}, \quad (3)$$

where each function $\mathbf{f}_{t-1}^{(s)}(\cdot)$ can be different from one another, and the s-th noise process is distributed as $\boldsymbol{\nu}_{t-1}^{(s)} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_{t-1}^{(s)})$. Considering a set of observations taken at N different clusters of sensors given by

$$\mathbf{y}_{j,t} = \mathbf{h}_{j,t}(\mathbf{x}_t) + \mathbf{n}_{j,t}, \text{ for } j = 1, \ldots, N. \quad (4)$$

with $\mathbf{n}_{j,t} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_{j,t})$, the aim is to use a set of individual filters, each one handling a subspace $\mathbf{x}_t^{(s)}$, in order to characterize at each cluster of sensors j the subspace marginal distributions, $p(\mathbf{x}_t^{(s)}|\mathbf{y}_{j,1:t})$. We can consider the centralized filtering problem (i.e., a single filter with access to the full set of observations collected by all the sensors), which may be implemented using deterministic sigma-point Gaussian filters (SPGF) [10, 11]. The extension of these SPGFs within the multiple state-partitioning framework has been proposed in [7, 8]. In this case, the exchange of information among filters and thus the proper marginalization of subspaces can be tackled via a nested sigma-point approximation [8]. The main assumption is that the joint distributions may be written as

$$p(\mathbf{x}_t|\mathbf{y}_{1:t-1}) = p(\mathbf{x}_t^{(s)}|\mathbf{y}_{1:t-1})p(\mathbf{x}_t^{(-s)}|\mathbf{y}_{1:t-1}), \quad (5)$$
$$p(\mathbf{x}_t|\mathbf{y}_{1:t}) = p(\mathbf{x}_t^{(s)}|\mathbf{y}_{1:t})p(\mathbf{x}_t^{(-s)}|\mathbf{y}_{1:t}). \quad (6)$$

In the following we consider the standard distributed SPGF formulation, where each cluster of nodes provides an estimate of the complete state of the system $\mathbf{x}_t$.

3. DISTRIBUTED SIGMA-POINT GAUSSIAN FILTERING BACKGROUND

The information filter (IF) is an algebraically equivalent form of the Kalman filter (KF), where instead of propagating the state vector and its associated estimation error covariance, the so-called information vector and information matrix (i.e., the inverse of the covariance) is propagated. The main advantage is in terms of information fusion, because the aggregation of information provided by different clusters of sensors is just a sum of individual information vectors [12, 13].

[1] $\mathbf{x}^{(s)}$ denotes the s-th element (possibly a vector) in a vector $\mathbf{x}$ and $\mathbf{x}^{(-s)}$ is the vector of all elements in $\mathbf{x}$ except for $\mathbf{x}^{(s)}$. The dimension of each subspace $n_x^{(s)} = \dim\{\mathbf{x}_t^{(s)}\}$ is defined such that $\sum_{s=1}^{S} n_x^{(s)} = n_x$, $s \in \mathcal{S} = \{1, \ldots, S\}$, and $n_x^{(-s)} = \dim\{\mathbf{x}_t^{(-s)}\}$.

3.1. Standard Information Filtering

Considering linear/Gaussian systems, i.e., $\mathbf{f}_{t-1}(\mathbf{x}_{t-1}) = \mathbf{F}_{t-1}\mathbf{x}_{t-1}$ and $\mathbf{h}_t(\mathbf{x}_t) = \mathbf{H}_t\mathbf{x}_t$, to reformulate the KF as an IF, we define the information vector and matrix as,

$$\hat{\mathbf{z}}_{t|t} = \boldsymbol{\Sigma}_{x,t|t}^{-1}\hat{\mathbf{x}}_{t|t} = \mathbf{Z}_{t|t}\hat{\mathbf{x}}_{t|t} \; ; \; \mathbf{Z}_{t|t} = \boldsymbol{\Sigma}_{x,t|t}^{-1}, \quad (7)$$

and then the standard KF recursions are rewritten as

$$\hat{\mathbf{z}}_{t|t-1} = \mathbf{L}_t\hat{\mathbf{z}}_{t-1|t-1}, \quad (8)$$
$$\mathbf{Z}_{t|t-1} = \left(\mathbf{F}_{t-1}\mathbf{Z}_{t-1|t-1}^{-1}\mathbf{F}_{t-1}^{\top} + \mathbf{Q}_{t-1}\right)^{-1}, \quad (9)$$
$$\hat{\mathbf{z}}_{t|t} = \hat{\mathbf{z}}_{t|t-1} + \mathbf{i}_t, \quad (10)$$
$$\mathbf{Z}_{t|t} = \mathbf{Z}_{t|t-1} + \mathcal{I}_t. \quad (11)$$

with $\mathbf{L}_t = \mathbf{Z}_{t|t-1}\mathbf{F}_{t-1}\mathbf{Z}_{t|t-1}^{-1}$, and the information contributions to the updates $\mathbf{i}_t = \mathbf{H}_t^{\top}\mathbf{R}_t^{-1}\mathbf{y}_t$ and $\mathcal{I}_t = \mathbf{H}_t^{\top}\mathbf{R}_t^{-1}\mathbf{H}_t$. Considering a set of observations taken at N different clusters of sensors as in (4), each cluster computes its own estimate and then the global estimate can be updated simply as

$$\hat{\mathbf{z}}_{t|t} = \hat{\mathbf{z}}_{t|t-1} + \sum_{j=1}^{N}\mathbf{i}_{j,t} \; ; \; \mathbf{Z}_{t|t} = \mathbf{Z}_{t|t-1} + \sum_{j=1}^{N}\mathcal{I}_{j,t},$$

with $\mathbf{i}_{j,t} = \mathbf{H}_{j,t}^{\top}\mathbf{R}_{j,t}^{-1}\mathbf{y}_{j,t}$ and $\mathcal{I}_{j,t} = \mathbf{H}_{j,t}^{\top}\mathbf{R}_{j,t}^{-1}\mathbf{H}_{j,t}$. Notice that the filter complexity (e.g., inversion of matrices) is translated from the measurement update to the state prediction, which is substantially lower dimensional in WSN.

3.2. Sigma-point Information Filtering

For nonlinear/Gaussian systems as the ones of interest in this work, deterministic sampling sigma-point-based information filters (SPIFs) have been proposed [12, 13]. In this case, the prediction step can be implemented as in the standard SPGF, and the information vector and matrix contributions to construct the measurement update at cluster j are given by

$$\mathbf{i}_{j,t} = \mathbf{Z}_{j,t|t-1}\boldsymbol{\Sigma}_{j,xy,t|t-1}\mathbf{R}_{j,t}^{-1}\left(\mathbf{y}_{j,t} - \hat{\mathbf{y}}_{j,t|t-1} + \boldsymbol{\Sigma}_{j,xy,t|t-1}^{\top}\hat{\mathbf{z}}_{j,t|t-1}\right) \quad (12)$$
$$\mathcal{I}_{j,t} = \mathbf{Z}_{j,t|t-1}\boldsymbol{\Sigma}_{j,xy,t|t-1}\mathbf{R}_{j,t}^{-1}\left(\mathbf{Z}_{j,t|t-1}\boldsymbol{\Sigma}_{j,xy,t|t-1}\right)^{\top},$$

where from the SPGF formulation we have that [11]

$$\hat{\mathbf{y}}_{j,t|t-1} = \int \mathbf{h}_{j,t}(\mathbf{x}_t)p(\mathbf{x}_t|\mathbf{y}_{j,1:t-1})d\mathbf{x}_t \approx \sum_{i=1}^{L}\omega_i\mathbf{h}_{j,t}(\mathbf{x}_{i,t|t-1}),$$
$$\boldsymbol{\Sigma}_{j,xy,t|t-1} = \int \mathbf{x}_t\mathbf{h}_{j,t}^{\top}(\mathbf{x}_t)p(\mathbf{x}_t|\mathbf{y}_{1:t-1})d\mathbf{x}_t - \hat{\mathbf{x}}_{j,t|t-1}\left(\hat{\mathbf{y}}_{j,t|t-1}\right)^{\top}$$
$$\approx \sum_{i=1}^{L}\omega_i\mathbf{x}_{i,t|t-1}\mathbf{h}_{j,t}(\mathbf{x}_{i,t|t-1})^{\top} - \hat{\mathbf{x}}_{j,t|t-1}\left(\hat{\mathbf{y}}_{j,t|t-1}\right)^{\top}, \quad (13)$$

with $\{\boldsymbol{\xi}_i, \omega_i\}_{i=1,\ldots,L}$ a set of sigma-points and weights, $\mathbf{x}_{i,t|t-1} = \mathbf{S}_{j,x,t|t-1}\boldsymbol{\xi}_i + \hat{\mathbf{x}}_{j,t|t-1}$, and $\mathbf{S}_{j,x,t|t-1}$ the square-root Cholesky factorization of $\boldsymbol{\Sigma}_{j,x,t|t-1}$. Notice that after computing the global estimate at the fusion center, the information vector and matrix are retransmitted to each cluster.

4. DISTRIBUTED MULTIPLE GAUSSIAN INFORMATION FILTERING FORMULATION

In this paper we leverage on the previously introduced SPIF results to extend the information filtering theory to the distributed subspace posterior characterization of interest. The partitioned state and j-th cluster measurement equations are given in (3) and (4). Considering the filter in charge of the s-th subspace at cluster j, the Bayesian solution is given by the s-th subspace marginal predictive and posterior distributions,

$$p(\mathbf{x}_t^{(s)}|\mathbf{y}_{j,1:t-1}) = \mathcal{N}\left(\mathbf{x}_t^{(s)}; \hat{\mathbf{x}}_{j,t|t-1}^{(s)}, \boldsymbol{\Sigma}_{j,x,t|t-1}^{(s)}\right) \quad (14)$$

$$p(\mathbf{x}_t^{(s)}|\mathbf{y}_{j,1:t}) = \mathcal{N}\left(\mathbf{x}_t^{(s)}; \hat{\mathbf{x}}_{j,t|t}^{(s)}, \boldsymbol{\Sigma}_{j,x,t|t}^{(s)}\right). \quad (15)$$

We want to formulate an IF-type approximation of these Gaussian distributions. In the sequel we detail the new distributed multiple Gaussian information filter (DMGIF).

Consider that at time t, each filter at each cluster knows the global (complete) information estimates, $\hat{\mathbf{z}}_{t-1|t-1}$ and $\mathbf{Z}_{t-1|t-1}$, provided by the fusion center. In general, we have that the full state estimate is the concatenation of individual subspace estimates, $\hat{\mathbf{x}}_{t|t} = [\hat{\mathbf{x}}_{t|t}^{(1)}, \ldots, \hat{\mathbf{x}}_{t|t}^{(S)}]^\top$, and the block diagonal covariance is $\boldsymbol{\Sigma}_{x,t|t} = \text{blkdiag}(\boldsymbol{\Sigma}_{x,t|t}^{(1)}, \ldots, \boldsymbol{\Sigma}_{x,t|t}^{(S)})$, then the information matrix turns to be also block diagonal, $\mathbf{Z}_{t|t} = \text{blkdiag}(\mathbf{Z}_{t|t}^{(1)}, \ldots, \mathbf{Z}_{t|t}^{(S)})$, with $\mathbf{Z}_{t|t}^{(i)} = (\boldsymbol{\Sigma}_{t|t}^{(i)})^{-1}$, and the information vector is again the concatenation of individual subspace information vectors, $\hat{\mathbf{z}}_{t|t} = [\hat{\mathbf{z}}_{t|t}^{(1)}, \ldots, \hat{\mathbf{z}}_{t|t}^{(S)}]^\top$. Notice that each filter within the DMGIF shares with the other filters the individual subspace predicted information estimates before the measurement update, $\hat{\mathbf{z}}_{j,t|t-1}^{(s)}$ and $\mathbf{Z}_{j,t|t-1}^{(s)}$.

4.1. Subspace State Prediction

The subspace marginal predictive distribution of interest at the filter in charge of the s-th subspace and cluster j is

$$p(\mathbf{x}_t^{(s)}|\, \mathbf{y}_{j,1:t-1}) = \int\int p(\mathbf{x}_t^{(s)}|\mathbf{x}_{t-1}^{(s)}, \mathbf{x}_{t-1}^{(-s)}) \times p(\mathbf{x}_{t-1}^{(s)}|\mathbf{y}_{j,1:t-1}) p(\mathbf{x}_{t-1}^{(-s)}|\mathbf{y}_{j,1:t-1}) d\mathbf{x}_{t-1}^{(s)} d\mathbf{x}_{t-1}^{(-s)}.$$

Taking into account the approximation in (5) and (6), the mean and corresponding prediction error covariance are[2],

$$\hat{\mathbf{x}}_{j,t|t-1}^{(s)} = \int\int \mathbf{f}(\mathbf{x}_{t-1}^{(s)}, \mathbf{x}_{t-1}^{(-s)}) \times p(\mathbf{x}_{t-1}^{(s)}|\mathbf{y}_{j,1:t-1}) p(\mathbf{x}_{t-1}^{(-s)}|\mathbf{y}_{j,1:t-1}) d\mathbf{x}_{t-1}^{(s)} d\mathbf{x}_{t-1}^{(-s)}, \quad (16)$$

$$\boldsymbol{\Sigma}_{j,x,t|t-1}^{(s)} = \int\int \mathbf{f}^2(\mathbf{x}_{t-1}^{(s)}, \mathbf{x}_{t-1}^{(-s)}) p(\mathbf{x}_{t-1}^{(s)}|\mathbf{y}_{j,1:t-1}) \quad (17)$$
$$\times p(\mathbf{x}_{t-1}^{(-s)}|\mathbf{y}_{j,1:t-1}) d\mathbf{x}_{t-1}^{(s)} d\mathbf{x}_{t-1}^{(-s)} - \left(\hat{\mathbf{x}}_{j,t|t-1}^{(s)}\right)^2 + \mathbf{Q}_{t-1}^{(s)}.$$

[2]We write $(\mathbf{x})^2$, $(\mathbf{y})^2$, $\mathbf{f}^2(\cdot)$ and $\mathbf{h}^2(\cdot)$ as the shorthand for $\mathbf{x}\mathbf{x}^T$, $\mathbf{y}\mathbf{y}^T$, $\mathbf{f}(\cdot)\mathbf{f}^T(\cdot)$ and $\mathbf{h}(\cdot)\mathbf{h}^T(\cdot)$, respectively. We omitted the dependence with time and the superscript $^{(s)}$ of $\mathbf{f}_{t-1}^{(s)}(\cdot)$ and $\mathbf{h}_t(\cdot)$, for the sake of clarity.

These integrals can be approximated using two sets of sigma-points [8], $\{\boldsymbol{\xi}_i^{(s)}, \omega_i^{(s)}\}_{i=1,\ldots,L_s}$ and $\{\boldsymbol{\xi}_j^{(-s)}, \omega_j^{(-s)}\}_{j=1,\ldots,L_{-s}}$, where L_s and L_{-s} depend on the sigma-point rule and the dimensions $n_x^{(s)}$ and $n_x^{(-s)}$, respectively. The corresponding transformed sets which capture the mean and covariance of $p(\mathbf{x}_{t-1}^{(s)}|\mathbf{y}_{j,1:t-1})$ and $p(\mathbf{x}_{t-1}^{(-s)}|\mathbf{y}_{j,1:t-1})$ are

$$\mathbf{x}_{i,t-1|t-1}^{(s)} = \mathbf{S}_{x,t-1|t-1}^{(s)} \boldsymbol{\xi}_i^{(s)} + \hat{\mathbf{x}}_{t-1|t-1}^{(s)}, \; i = 1, \ldots, L_s,$$
$$\mathbf{x}_{l,t-1|t-1}^{(-s)} = \mathbf{S}_{x,t-1|t-1}^{(-s)} \boldsymbol{\xi}_l^{(-s)} + \hat{\mathbf{x}}_{t-1|t-1}^{(-s)}, \; l = 1, \ldots, L_{-s},$$

with $\mathbf{Z}_{t-1|t-1}^{(s)} = \mathbf{S}_{z,t-1|t-1}^{(s)} \left(\mathbf{S}_{z,t-1|t-1}^{(s)}\right)^\top$, $\mathbf{S}_{x,t-1|t-1}^{(s)} = \left(\mathbf{S}_{z,t-1|t-1}^{(s)}\right)^{-\top}$, and the subspace estimates are obtained as $\hat{\mathbf{x}}_{t-1|t-1}^{(s)} = \left(\mathbf{Z}_{t-1|t-1}^{(s)}\right)^{-1} \hat{\mathbf{z}}_{t-1|t-1}^{(s)}$. Then (16) and (17) are approximated (at cluster of nodes j) by

$$\hat{\mathbf{x}}_{j,t|t-1}^{(s)} = \sum_{l=1}^{L_{-s}} \omega_l^{(-s)} \sum_{i=1}^{L_s} \omega_i^{(s)} \mathbf{f}(\mathbf{x}_{i,t-1|t-1}^{(s)}, \mathbf{x}_{l,t-1|t-1}^{(-s)}),$$

$$\boldsymbol{\Sigma}_{j,x,t|t-1}^{(s)} = \sum_{l=1}^{L_{-s}} \omega_l^{(-s)} \sum_{i=1}^{L_s} \omega_i^{(s)} \mathbf{f}^2(\mathbf{x}_{i,t-1|t-1}^{(s)}, \mathbf{x}_{l,t-1|t-1}^{(-s)}) - \left(\hat{\mathbf{x}}_{j,t|t-1}^{(s)}\right)^2 + \mathbf{Q}_{t-1}^{(s)}.$$

At the end of the subspace prediction step, this mean and covariance are shared with the other filters. We can go back to the information space as $\hat{\mathbf{z}}_{j,t|t-1}^{(s)} = (\boldsymbol{\Sigma}_{j,x,t|t-1}^{(s)})^{-1} \hat{\mathbf{x}}_{j,t|t-1}^{(s)}$.

4.2. Subspace Information Contribution

The s-th filter information vector and matrix contributions to construct the measurement update at cluster j are [13]

$$\mathbf{i}_{j,t}^{(s)} = \mathbf{Z}_{j,t|t-1}^{(s)} \boldsymbol{\Sigma}_{j,xy,t|t-1}^{(s)} \mathbf{R}_{j,t}^{-1} \left(\mathbf{y}_{j,t} - \hat{\mathbf{y}}_{j,t|t-1}^{(s)} + \left(\boldsymbol{\Sigma}_{j,xy,t|t-1}^{(s)}\right)^\top \hat{\mathbf{z}}_{j,t|t-1}^{(s)}\right) \quad (18)$$

$$\mathcal{I}_{j,t}^{(s)} = \mathbf{Z}_{j,t|t-1}^{(s)} \boldsymbol{\Sigma}_{j,xy,t|t-1}^{(s)} \mathbf{R}_{j,t}^{-1} \left(\mathbf{Z}_{j,t|t-1}^{(s)} \boldsymbol{\Sigma}_{j,xy,t|t-1}^{(s)}\right)^\top \quad (19)$$

where $\mathbf{Z}_{j,t|t-1}^{(s)} = (\boldsymbol{\Sigma}_{j,x,t|t-1}^{(s)})^{-1}$, and we need to obtain the predicted measurement and cross-covariance matrix, which are defined as

$$\hat{\mathbf{y}}_{j,t|t-1}^{(s)} = \int\int \mathbf{h}_{j,t}(\mathbf{x}_t^{(s)}, \mathbf{x}_t^{(-s)}) p(\mathbf{x}_t^{(s)}|\mathbf{y}_{j,1:t-1}) \times p(\mathbf{x}_t^{(-s)}|\mathbf{y}_{j,1:t-1}) d\mathbf{x}_t^{(s)} d\mathbf{x}_t^{(-s)}, \quad (20)$$

$$\boldsymbol{\Sigma}_{j,xy,t|t-1}^{(s)} = \int\int \mathbf{x}_t^{(s)} \mathbf{h}_{j,t}^T(\mathbf{x}_t^{(s)}, \mathbf{x}_t^{(-s)}) p(\mathbf{x}_t^{(s)}|\mathbf{y}_{j,1:t-1}) \times p(\mathbf{x}_t^{(-s)}|\mathbf{y}_{j,1:t-1}) d\mathbf{x}_t^{(s)} d\mathbf{x}_t^{(-s)} - \hat{\mathbf{x}}_{t|t-1}^{(s)} \left(\hat{\mathbf{y}}_{j,t|t-1}^{(s)}\right)^T.$$

These integrals can be again approximated using nested sigma-point rules [8]. The transformed sets are now

$$\mathbf{x}_{i,t|t-1}^{(s)} = \mathbf{S}_{j,x,t|t-1}^{(s)} \boldsymbol{\xi}_i^{(s)} + \hat{\mathbf{x}}_{j,t|t-1}^{(s)},\ i = 1, \ldots, L_s,$$
$$\mathbf{x}_{l,t|t-1}^{(-s)} = \mathbf{S}_{j,x,t|t-1}^{(-s)} \boldsymbol{\xi}_l^{(-s)} + \hat{\mathbf{x}}_{j,t|t-1}^{(-s)},\ l = 1, \ldots, L_{-s},$$

with $\mathbf{S}_{j,x,t|t-1}^{(s)}$ the square-root Cholesky factorization of $\boldsymbol{\Sigma}_{j,x,t|t-1}^{(s)}$, and both $\hat{\mathbf{x}}_{j,t|t-1}^{(-s)}$ and $\mathbf{S}_{j,x,t|t-1}^{(-s)}$ constructed from the predictions of the other filters running in parallel. The integrals of interest are then approximated by

$$\hat{\mathbf{y}}_{j,t|t-1}^{(s)} = \sum_{l=1}^{L_{-s}} \omega_l^{(-s)} \sum_{i=1}^{L_s} \omega_i^{(s)} \mathbf{h}(\mathbf{x}_{i,t|t-1}^{(s)}, \mathbf{x}_{l,t|t-1}^{(-s)}), \quad (21)$$

$$\boldsymbol{\Sigma}_{j,xy,t|t-1}^{(s)} = \sum_{l=1}^{L_{-s}} \omega_l^{(-s)} \sum_{i=1}^{L_s} \omega_i^{(s)} \mathbf{x}_{i,k|k-1}^{(s)}$$
$$\times\, \mathbf{h}(\mathbf{x}_{i,t|t-1}^{(s)}, \mathbf{x}_{j,t|t-1}^{(-s)})^\top - \hat{\mathbf{x}}_{j,t|t-1}^{(s)} \left(\hat{\mathbf{y}}_{j,t|t-1}^{(s)}\right)^\top. \quad (22)$$

4.3. Global Information Update

The different filters within the cluster share the different information contributions to build the complete $\mathbf{i}_{j,t}$ and $\mathcal{I}_{j,t}$, constructed as $\mathbf{i}_{j,t} = [\mathbf{i}_{j,t}^{(1)}, \ldots, \mathbf{i}_{j,t}^{(S)}]^\top$ and $\mathcal{I}_{j,t} = \text{blkdiag}(\mathcal{I}_{j,t}^{(1)}, \ldots, \mathcal{I}_{j,t}^{(S)})$, which are transmitted to the fusion center to compute the global estimates as

$$\hat{\mathbf{z}}_{t|t} = \hat{\mathbf{z}}_{t|t-1} + \sum_{j=1}^{N} \mathbf{i}_{j,t}\ ;\ \mathbf{Z}_{t|t} = \mathbf{Z}_{t|t-1} + \sum_{j=1}^{N} \mathcal{I}_{j,t}, \quad (23)$$

which are transmitted back to each node to perform the following prediction step.

5. RESULTS: MULTIPLE TARGET LOCALIZATION

We consider a RSS 2D multiple target localization case study, where K targets are localized using a set of N clusters of sensors. For each target i, we estimate its 2D position and velocity, $\mathbf{x}_t^{(i)} = [p_{x,t}^{(i)}, p_{y,t}^{(i)}, v_{x,t}^{(i)}, v_{y,t}^{(i)}]^\top$. At time t, the m-th sensor RSS is

$$y_{m,t} = \sum_{i=1}^{K} 10 \log_{10} \left(\frac{1}{|\mathbf{r}_m - \mathbf{l}_{i,t}|^2} \right) + n_{m,t},$$

with $n_{m,t} \sim \mathcal{N}(0, \sigma_m^2)$, $\mathbf{l}_{i,t} = [p_{x,t}^{(i)}, p_{y,t}^{(i)}]^\top$, known grid sensor position $\mathbf{r}_m$, and

$$\mathbf{x}_t^{(i)} = \begin{pmatrix} \mathbf{I}_2 & T_s \cdot \mathbf{I}_2 \\ \mathbf{0} & \mathbf{I}_2 \end{pmatrix} \mathbf{x}_{t-1}^{(i)} + \boldsymbol{\nu}_{t-1}^{(i)},\ \boldsymbol{\nu}_{t-1}^{(i)} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}),$$

with $\mathbf{Q} = \text{diag}(\sigma_{p_x}^2, \sigma_{p_y}^2, \sigma_{v_x}^2, \sigma_{v_y}^2)$, T_s the sampling period and $\mathbf{I}_2$ the 2×2 identity matrix. We consider the following setup: $T_s = 1$s, $\sigma_m^2 = 10^{-3}$ for all sensors, and $\sigma_{p_x}^2 = \sigma_{p_y}^2 = 0.0025$, $\sigma_{v_x}^2 = \sigma_{v_y}^2 = 0.01$, for all targets. The new distributed method, named DMGIF, is compared to the centralized MGF having access to the complete set of observations, $\mathbf{y}_t$. We consider one subspace per target, then the MGF runs K filters at the central fusion center, and the DMGIF runs K filters at each cluster of nodes j. To assess the new method's performance the following filters are tested:

- MGF using 100 sensors in a 900×900 m^2 grid.
- DMGIF-1 using $N = 4$ clusters of 25 sensors.
- DMGIF-2 using $N = 9$ clusters of 9 sensors.

Figure 1 plots the results obtained for a $K = 3$ target tracking example using the new DMGIF, where we show how the sensor space is partitioned into N clusters of neighbouring sensors. These clusters first process the information independently, and then a fusion center estimates the global estimate shown in the figure. Figure 2 provides the root mean square error (RMSE) of the position obtained with the three methods for $K = 2$ and $K = 3$ targets and 100 Monte Carlo runs. We can see that the performance of the decentralized filter is almost equivalent to its centralized counterpart, confirming the good behavior of the new methodology.

6. CONCLUSIONS

This paper presented a new distributed multiple sigma-point filter formulation to reduce the curse-of-dimensionality appearing both in the state and the observation dimension in applications such as multiple target localization using large sensor networks. The proposed solution allows to avoid the standard Bayesian filtering performance loss in high-dimensional state-space models. The original state is partitioned into several low dimensional subspaces, and a set of individual sigma-point information filters running in parallel cope with the subspace estimation using only a subset of observations. Each cluster of nodes in the system computes an individual information contribution from the different subspace state estimations, which are transmitted to a fusion center in charge of the global estimation. Numerical results support the discussion and show the promising capabilities of such distributed multiple filtering approach.

7. REFERENCES

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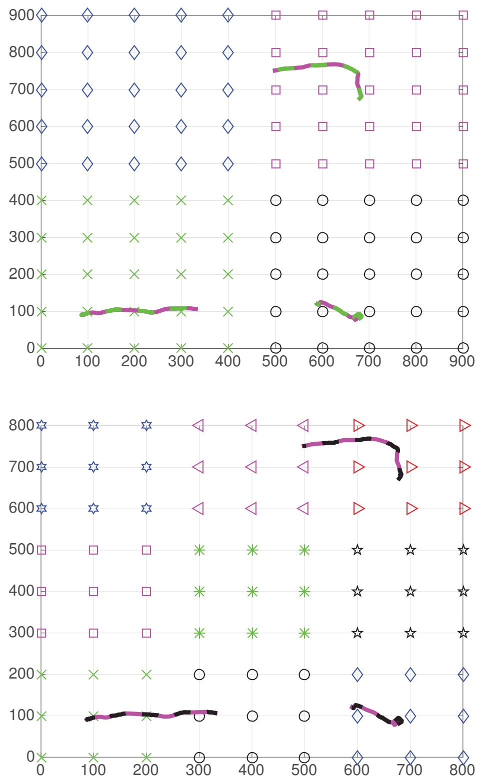

Fig. 1. DMGIF estimation example for $K = 3$ targets using $N = 4$ clusters of 25 sensors (top), and $N = 9$ clusters of 9 sensors (bottom). True trajectory in magenta, and estimated trajectories in green (DMGIF-1) and black (DMGIF-2).

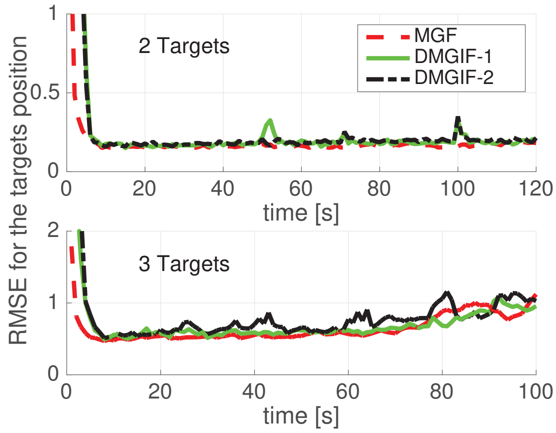

Fig. 2. RMSE for 2 targets (top) and 3 targets (bottom), considering the centralized MGF, and both decentralized DMGIF-1 and DMGIF-2.

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